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## Real-Time Energy Economy Optimization Based on Nonlinear MPC for Hybrid Electrical Buses

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# Real-Time Energy Economy Optimization Based on Nonlinear MPC for Hybrid Electrical Buses

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**Abstract.** Hybrid electric vehicles (HEVs) which combine at least two energy sources are characterized by high efficiency and seem to strongly support the objective to cut greenhouse gases. In this paper, hybrid electric bus (HEB) with coaxial series-parallel configuration is proposed and very little research on the economy optimization of this powertrain has been done. Considering that predictive control has the mechanism for controlling the nonlinear process while tackling equality or inequality constraints, nonlinear model predictive control (NMPC) is applied to the energy management strategy (EMS) for the HEB. The NMPC-based EMS is formulated and then converted into optimal control problem based on Pontryagin's Minimum Principle (PMP) which can be solved with an iterative method. Finally, the architecture of the EMS problem established and the results of the proposed optimization strategy are illustrated and turn out to be satisfying.

## 1. Introduction

Hybrid electric vehicle (HEV) is currently one of the hot spots of research and development of new energy vehicles due to their high efficiency. The performance of energy management strategy (EMS) directly affect the vehicle's fuel consumption, emissions and dynamic characteristics [1]. The key problem to be solved for the energy management strategy (EMS) of hybrid powertrain system is to distribute the power output between engine and electric machine (EM) and optimize the efficiency of the entire powertrain during the whole driving cycle under the premise of meeting the power demand of the driver. At present the EMS of HEV covers rules-based (RB) EMS, equivalent consumption minimization strategy (ECMS), and global optimization method [2].

Rule-based (RB) strategy defines the operation logic of the powertrain and sets the thresholds of engine torque output, battery state of charge (SOC), speed limit, etc. [3]. However, RB strategy is based primarily on the engineering experience and it cannot achieve the optimal system performance. EMS that based on dynamic programming (DP) could achieve global optimum but cannot be applied in real time since the calculation burden is huge and the whole driving condition cannot be anticipated in advance. ECMS strategy optimizes the vehicle energy flow in real time and minimizes the energy loss of the vehicle in each sampling period [4].

These instantaneous optimization strategy is real-time-oriented with low robustness and have limited optimization effects, yet they can be improved in robustness and optimization performance through predictive control mechanism. Then, model predictive control (MPC) is adopted because this methodology is capable of realizing approximate optimal control within finite time domain. Plus, this



mechanism features the characteristic of processing explicit and active constraints and is suitable for solving EMS problem [5].

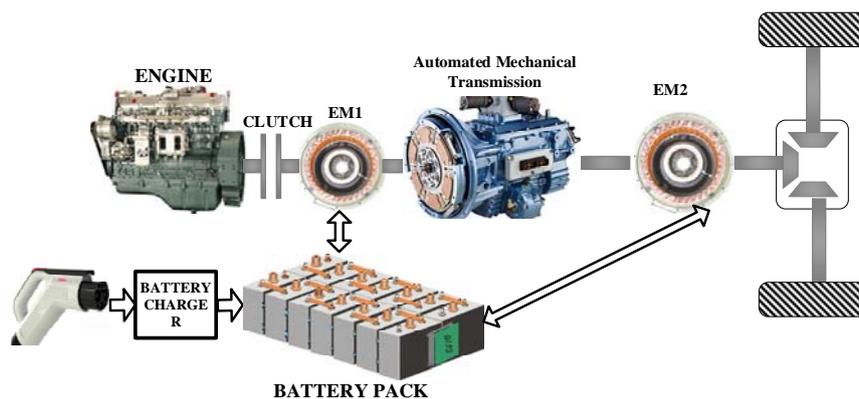
The solution of nonlinear model predictive control (NMPC) has always been difficult since it is difficult to get an explicit expression of the control law and numerical solution method is required. Mathematic methods such as Multiple Shooting (MS) and sequential quadratic programming (SQP) are researched [6]. Some new optimization methods are explored including Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) but the robustness is not guaranteed. In this paper, NMPC problem is converted into optimal control based on PMP and then the control variables are obtained by solving the large scale equations by an iterative method Newton' method. The solution sequence is then obtained and closed-loop control of the whole vehicle powertrain is realized.

The rest of this paper is organized as follows. In Section 2, modeling of the hybrid bus is introduced. And the problem is then described in the second part. In Section 3, the energy management optimization problem is integrated into NMPC framework and the mathematical solution method of NMPC is formulated. In Section4, the results of the proposed control strategy are analyzed and the performance is verified. Finally, conclusions are drawn in Section 5.

## 2. Configuration and modeling

### 2.1. Configuration

The powertrain of the hybrid electric bus (HEB) is shown as follows.



**Figure 1.** The configuration of the HEB

The powertrain comprises a CNG engine, a clutch actuator, two electric machines (EM), an automated mechanical transmission (AMT). This structure can regenerate much energy by coordinating two electric machines and provide more driving power for acceleration.

The main parameters of the HEB are listed in Table 1.

**Table 1.** Main parameters of HEB

Components	Parameters
Engine	YC6G230N,CNG, 6.454L, nominal power:170kW
EM1	Permanent magnet, max torque: 500Nm, nominal power :40kW,peak power:60kW
EM2	Permanent magnet, max torque: 750Nm, nominal power :94kW,peak power:121kW
Battery	Lithium titanate, capacity: 50Ah
Transmission	6-speed AMT, gear ratio: 6.39/3.97/2.4/1.48/1/0.73
Final Drive	Ratio: 5.571

## 2.2. Powertrain modeling

The wheel torque is calculated by equation (1) to obtain the demand torque of the powertrain.

$$T_r = [(T_e + T_{m1}) \cdot \eta_T \cdot i_g + T_{m2}] \cdot i_d + T_b \quad (1)$$

Where  $T_w$ ,  $T_r$ ,  $T_e$ ,  $T_{m1}$ ,  $T_{m2}$ ,  $T_b$  denotes the wheel torque, demand torque of the powertrain, torque of the engine, torque of EM1, torque of EM2 and the braking torque respectively.  $\eta_T$ ,  $i_g$ ,  $i_d$  denotes the transmission efficiency, transmission gear ratio, gear ratio of the differential gear.

The longitudinal dynamics equation of the vehicle is:

$$T_w = [mgf_r \cos \theta + \frac{1}{2} C_D \rho A V_{veh}^2 + mg \sin \theta + \delta m \frac{dV}{dt}] \cdot r \quad (2)$$

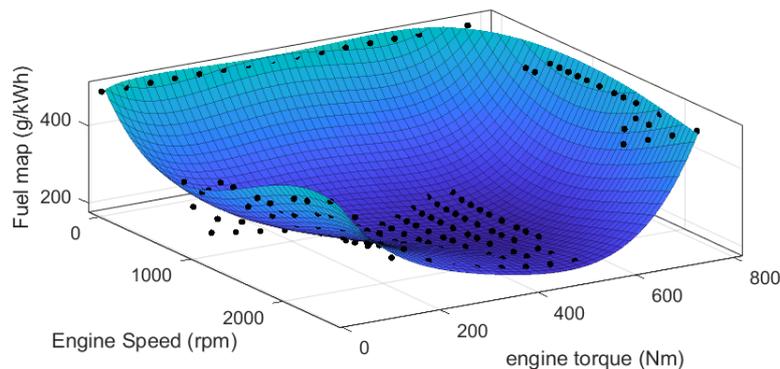
In equation (2),  $m$  denotes the vehicle mass,  $g$  denotes the gravity acceleration,  $f_r$  denotes the rolling resistance of wheel tire,  $\theta$  denotes the angle of the road,  $C_D$  denotes the coefficient of the rolling resistance,  $\rho$  denotes air density,  $\delta$  denotes the correction coefficient of rotating mass,  $A$  denotes the front area of the bus,  $r$  denotes the radius of the wheel. And the rotational speed equation of the powertrain is as follows.

$$\omega_e = \omega_{m1} = i_g \cdot \omega_{m2} = i_d \cdot \omega_w = \frac{V_{veh}}{r} \quad (3)$$

Where  $\omega_e$ ,  $\omega_{m1}$ ,  $\omega_{m2}$  and  $\omega_w$  denote the rotational speed of the engine, EM1, EM2 and the wheel respectively. To measure the fuel consumption of the engine, a steady state model is obtained from the steady state data of CNG engine.

$$\dot{m}_f = \frac{T_e \cdot \omega_e \cdot b_e(T_e, \omega_e)}{367.1 \cdot \rho_g \cdot g} \quad (4)$$

Where  $\dot{m}_f$  denotes the fuel consumption per second,  $T_e$  and  $\omega_e$  denote the torque and rotational speed of the engine respectively.  $b_e$  is the fuel consumption rate of the engine,  $\rho_g$  is the CNG density.



**Figure 2.** Fitting function of the engine fuel rate

The battery model is simplified as an equivalent circuit consists of a voltage source and an internal resistance.

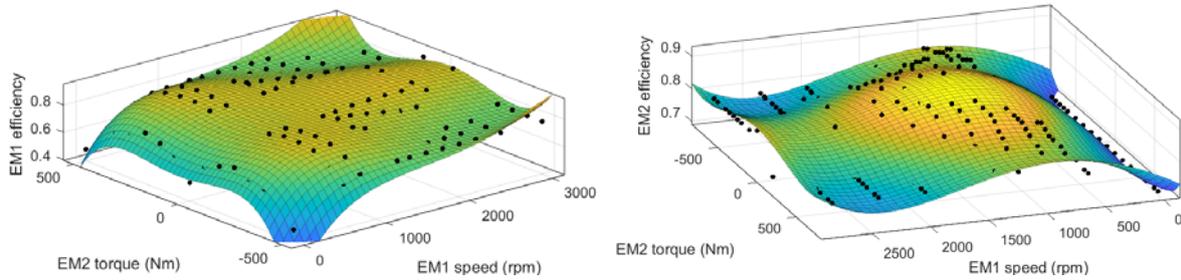
$$\frac{dSOC}{dt} = \frac{U_{bat} - \sqrt{U_{bat}^2 - 4R_{bat}P_{EM}}}{2R_{bat}Q_{bat}} \quad (5)$$

Where  $U_{bat}$  denotes the open circuit voltage of the battery pack.  $R_{bat}$  denotes the internal resistance of the battery pack and  $Q_{bat}$  denotes the capacity of the battery pack.  $R_{bat}$  and  $Q_{bat}$  vary with  $SOC$  and can be calculated by numerical fitting.

In terms of the EMs, they can work as generator and motor so the electric power can be written as

$$P_{EM} = T_{m1}\omega_{m1}\eta_{EM1}^{\text{sgn}(T_{m1})} + T_{m2}\omega_{m2}\eta_{EM2}^{\text{sgn}(T_{m2})} \quad (6)$$

Where  $\eta_{EM}$  denotes the efficiency of the EMs and is obtained by data fitted as follows.



**Figure 3.** Fitting function of the efficiency EM1 and EM2

The electric consumption is calculated by

$$\dot{m}_e = U_{bat}Q \cdot \dot{SOC} \quad (7)$$

Where  $Q$  denotes the battery capacity.

### 3. NMPC Control Scheme

#### 3.1. Control-oriented model

The task of the NMPC energy management strategy is to find an optimal control input to minimize the cost function, that is, to solve the following optimization problem in each sampling period. Nonlinear state-space model is used for NMPC and it can be expressed as

$$\dot{x} = f(x, u) \quad (8)$$

Where the state variable  $x = [SOC \ V]^T$ , the control input  $u = [T_{m1} \ T_{m2}]^T$ . Combining equation (3) and (5) with (1) (2) (4), the nonlinear state space model of this HEB is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{dSOC}{dt} \\ \frac{dV_{veh}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{U_{bat} - \sqrt{U_{bat}^2 - 4R_{bat}(T_{m1}\omega_{m1}/\eta_{m1} + T_{m2}\omega_{m2}/\eta_{m2})}}{2R_{bat}Q_{bat}} \\ \frac{[(Tr - T_{m2}/R) \cdot \eta_T \cdot i_g + T_{m2}] \cdot i_d + T_b}{r\delta m} - \frac{mgf_r \cos\theta + \frac{1}{2}C_D\rho AV_{veh}^2 + mg \sin\theta}{\delta m} \end{bmatrix} \quad (9)$$

The variables in the nonlinear state space model of HEB should be bounded and the corresponding nonlinearity constraints are

$$\begin{cases} T_{e\_min}(\omega_e) \leq T_e \leq T_{e\_max}(\omega_e) \\ T_{m1\_min}(\omega_{m1}) \leq T_{m1} \leq T_{m1\_max}(\omega_{m1}) \\ T_{m2\_min}(\omega_{m2}) \leq T_{m2} \leq T_{m2\_max}(\omega_{m2}) \\ SOC_L \leq SOC \leq SOC_H \end{cases} \quad (10)$$

Where  $SOC_L$  and  $SOC_H$  refer to the limits of battery pack  $SOC$ .

The cost function is set as the equivalent consumption of fuel energy and electricity.

$$\begin{aligned} J &= \int_0^T M(x, u) d\tau + \varphi(x(T)) \\ &= \int_0^T \dot{m}_f(x, u) + \gamma \dot{m}_e(x, u) d\tau + \varphi(x(T)) \end{aligned} \quad (11)$$

Where the terminal constraint  $\varphi(x(T)) = 0$ ,  $\gamma$  is the equivalent conversion factor of the two kinds of energy and can also serve as the adjustable weight coefficient of the cost function.

Then the framework of the NMPC strategy is obtained shown in Figure 4.

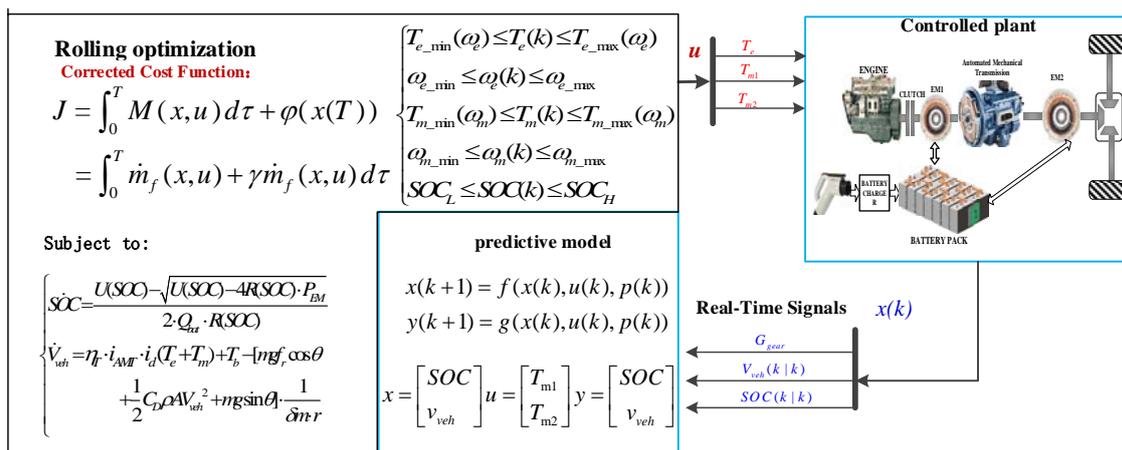


Figure 4. Schematic of NMPC strategy

### 3.2. Numerical solution of NMPC

Constrained optimization problem, especially nonlinear constrained optimization problem is sophisticated to solve in practical scientific research since nonlinear programming problem (NLP) is always difficult to deal with. Considering that it's difficult to obtain an analytical solution of the control law by solving NLP due to the existence of nonlinearity and constraints, numerical method should be utilized at each sampling instant.

In this section, the NLP is converted into Hamilton Jacobi-Bellman (HJB) equation with inequality constraints. According to PMP, inequality constraints are not easy to handle [9]. Therefore, a heuristic method that converts inequality constraints into equality constraints is formulated as

$$\begin{aligned} C_1(x_2, u_1, u_{d1}) &= (u_1 - \frac{u_{1\_min} + u_{1\_max}}{2})^2 + u_{d1}^2 - (\frac{u_{1\_max} - u_{1\_min}}{2})^2 = 0 \\ C_2(x_2, u_2, u_{d2}) &= (u_2 - \frac{u_{2\_min} + u_{2\_max}}{2})^2 + u_{d2}^2 - (\frac{u_{2\_max} - u_{2\_min}}{2})^2 = 0 \\ C_3(x_1, u_{d3}) &= (x_1 - \frac{x_{1\_min} + x_{1\_max}}{2})^2 + u_{d3}^2 - (\frac{x_{1\_max} - x_{1\_min}}{2})^2 = 0 \end{aligned} \quad (12)$$

Where  $x = [SOC \ V]^T$ ,  $u = [T_{m1} \ T_{m2}]^T$ ,  $u_{d1}$ ,  $u_{d2}$  and  $u_{d3}$  are three dummy inputs that guarantee the inequality constraints  $T_{m1\_min}(\omega_{m1}) \leq T_{m1} \leq T_{m1\_max}(\omega_{m1})$ ,  $T_{m2\_min}(\omega_{m2}) \leq T_{m2} \leq T_{m2\_max}(\omega_{m2})$  and  $SOC_L \leq SOC \leq SOC_H$ , respectively.

The modified optimal control problem with the equality constraints and dummy control variables is summarized as follows.

$$\begin{aligned} \min_u J &= \int_0^T M(x, u) d\tau + \varphi(x(T)) \\ \text{with respect to } u &= [u, u_{d1}, u_{d2}]^T \\ \text{subject to } &\begin{cases} \dot{x} = f(x, u) \\ C(x, u) = C(x, u, u_{d1}, u_{d2}) = 0 \end{cases} \end{aligned} \quad (13)$$

Where the fuel flow rate of engine  $\dot{m}_f$  and the electricity consumption  $\dot{m}_e$  are the fitted function of engine map and EM efficiency map.  $\gamma$  denotes the equivalent factor and is set as 0.461.

According to PMP, the corresponding Hamiltonian function is defined as

$$H(x, \lambda, u', \mu) = M(x, u) + \lambda^T f(x, u) + \mu^T C(x, u') \quad (14)$$

Where  $\lambda \in R^n$  denotes co-state,  $\mu \in R^m$  denotes the Lagrange multiplier associated with the equality constraints. The solution of the optimization problem could be found by the solving the equation group below.

$$\begin{cases} \dot{x}(\tau) = f(x(\tau), u(\tau)) \\ C(x(\tau), u'(\tau)) = 0 \\ H_u(x(\tau), \lambda(\tau), u'(\tau), \mu(\tau)) = 0 \\ \dot{\lambda}(\tau) = -H_x(x(\tau), \lambda(\tau), u'(\tau), \mu(\tau)) \end{cases} \quad (15)$$

The first equation and fourth equation of (15) could be discretized as

$$\begin{aligned} x(k+1) &= x(k) + f(x(k), u(k)) \\ \lambda(N) &= \varphi_x^T(x(N)) = 0 \\ \lambda(k) &= \lambda(k+1) + H_x^T(x(k), \lambda(k+1), u'(k), \mu(k)) \Delta\tau \end{aligned} \quad (16)$$

Where  $\Delta\tau = T/N$ ,  $T$  is predictive horizon which is set as 4.8 seconds,  $\Delta\tau$  denotes sampling period which is set as 0.4 seconds.

Then the two sequences  $x(k)$  ( $k = 0 \dots N$ ) and  $\lambda(k)$  ( $k = 0 \dots N$ ) are obtained and is then substituted into formula (15). The following formula is obtained.

$$F(U, x) = \begin{bmatrix} H_u^T(x(0), \lambda(0), u'(0), \mu(0)) \\ C(x(0), u'(0)) \\ \vdots \\ H_u^T(x(N-1), \lambda(N-1), u'(N-1), \mu(0)) \\ C(x(N), u'(N)) \end{bmatrix} = 0 \quad (17)$$

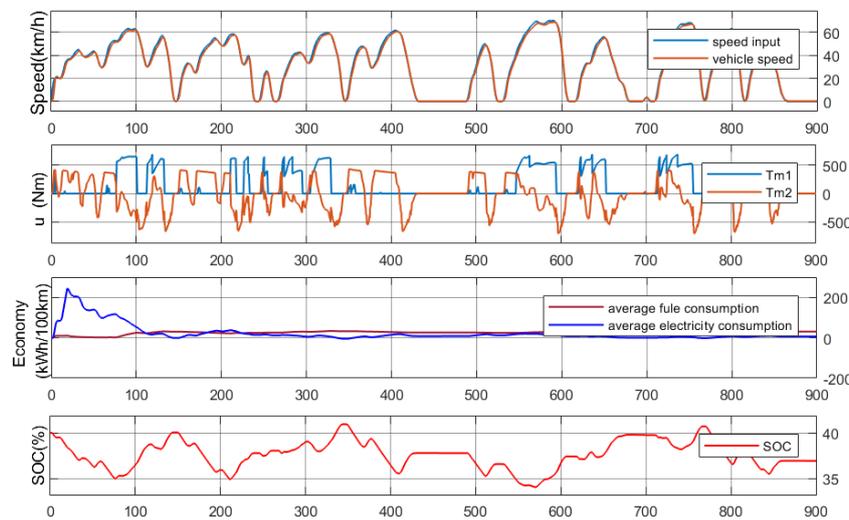
Where the solution  $U = [u'(0), \mu^T(0), \dots, u'(N), \mu^T(N)]^T$ . This equation is solved at each sampling instant by the Newton's method which is one iterative method and then the solution  $U$  which contains the required control variables  $u(k)=[T_{m1} \ T_{m2}]^T$  is obtained.

**4. Validation Results**

One certain energy management strategy must have a balance between optimization performance and the ability of practical application. So this section concentrates mainly on two aspects which are the computational performance of NMPC and the analysis of the comprehensive energy consumption.

The computation test is performed in Simulink on an Intel Core i7 6700HQ CPU with a maximum computation capacity of 3.3 GHz. The length of predictive horizon is 12, the computation time of each iteration is 0.35 second less than the sample time of 0.4 second.

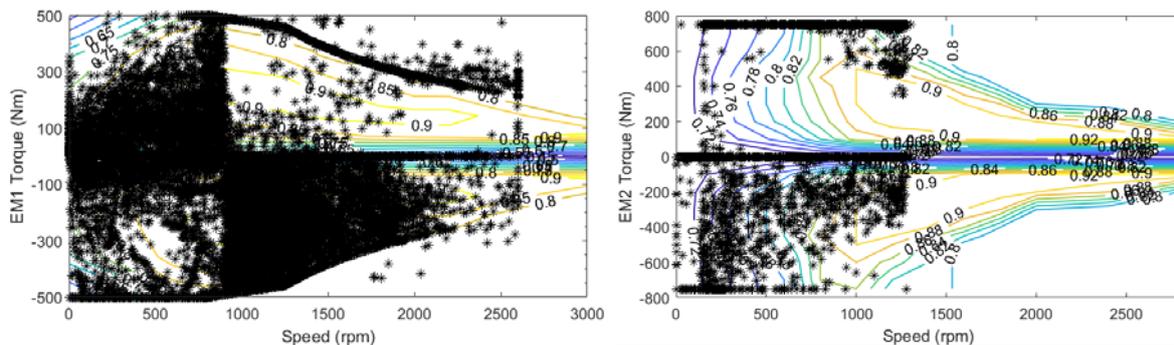
To illustrate the effectiveness of NMPC strategy, a real-world driving curve is taken for testing and the results are described in Figure 5.



**Figure 5.** Validation results of NMPC strategy

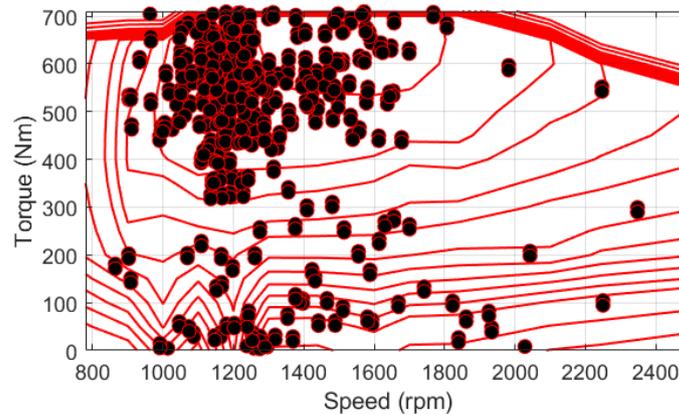
The results demonstrated that the provided driving curve is tracked precisely by PID controller that models the driver's intension. Plus, SOC,  $T_{m1}$  and  $T_{m2}$  are restricted to the boundary. Therefore, constraints of both state variables and control variables are tackled properly.

The two calculated control variables  $T_{m1}$  and  $T_{m2}$  are drawn and their operating points and corresponding efficiency maps are drawn in Figure 6.



**Figure 6.** The operating points of EM1 and EM2, respectively

The torque of engine is calculated by equation (1) and the operating points of engine is obtained and plotted in different color shown as follows.



**Figure 7.** The operating points of engine

According to the distribution of operating points of the engine, EM1 and EM2 above, the three power sources operate mainly in high efficiency region. This means that the cost function of the NMPC is optimized and satisfying optimization performance is guaranteed accordingly.

## 5. Conclusion

This paper focuses on the EMS of HEB based on an optimal control scheme NMPC. The configuration of the HEB powertrain are presented and the modeling of each components of the powertrain is established and lays the foundation for NMPC based optimization strategy. The NMPC problem is then converted into an optimal control framework by PMP and is solved by iterative algorithm with acceptable calculation speed. On this basis, the whole NMPC architecture is realized and the test results show satisfying energy optimization performance and it is demonstrated that NMPC can control a nonlinear process while tackling the constraints of variables. It can be seen from the test results of operating point distribution that engine and EMs are operating in high efficiency region, yet no analysis of coordinated control between engine and EM is done which should be the future work to do.

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