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# Research on Route Planning of Electric Buses

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**Abstract.** Electric bus has the problems of limited driving distance and long charging time, which cannot fully adapt to the traditional bus lines. In the traditional bus route planning, the optimization of factors such as the number of stations and the location, the frequency of departure, the size of the fleet, and the fare are not considered for the charging waiting time limit. To this end, this paper studies the electric bus route planning, comprehensively considers the design of bus line operation characteristic parameters and the configuration of charging station and charging pile. Constrained by charging waiting time and capacity, a social welfare maximization model is established. According to the number of charging stations, the influence of the configuration of the charging station on one end and both ends of the line is discussed. Based on two different charging strategies, the influence of the charging waiting time on the queue in the charging station is discussed. A heuristic solution algorithm is proposed based on Lagrange method. Numerical examples show that the model can not only realize the design of the number of charging piles and the line operation characteristic parameters that can minimize the social welfare under the constraint of charging waiting time, but also compare and analyze the influence of different charging strategies on the number of charging stations and the configuration of charging piles.

## 1. Introduction

Replacing traditional fuel vehicles with electric vehicles has become an inevitable trend for the sustainable development of the automotive industry. It is also an effective way to solve the problem of energy consumption and environmental pollution. However, the rapid popularization of electric vehicles is limited by the short range, long charging time and poor charging facilities. Meanwhile, the bus has a fixed route, a fixed departure time and a fixed parking lot, can better adapt to its characteristics, making the electric bus as the first step in the promotion of electric vehicles in China market.

With the strong support and promotion of the country, China's electric bus industry has entered a period of rapid development. In 2015, the Ministry of Communications issued the "Opinions on Accelerating the Promotion and Application of New Energy Vehicles in Transportation Industry", pointing out that by 2020, the number of new energy city buses will reach 200,000 [1]. But judging from the construction of charging facilities of electric buses in different cities at present, due to the lack of experience and theoretical guidance, the number of charging piles varies greatly. Some are arranged according to "one vehicle, one pile", some are arranged according to "four vehicles, one pile" and some are arranged according to "six vehicles, one pile", which results in unreasonable



configuration of charging piles [2]. Therefore, the development of electric bus, the first thing to do is to study the planning and construction of its charging infrastructure.

Significant progress has been made in electric bus infrastructure planning. Chen Wei et al. [3] calculate the number of charging piles needed according to the capacity of charging piles in the charging station and the parameters and operation mode of the electric bus, but do not consider the impact of bus charging waiting time on bus operation. Cai Zi long et al. [4] aiming at minimizing the number of charging piles, set up a bus charging queuing system model in a bus terminal, and studied the optimal allocation of charging facilities in peak and peak periods under DC fast charging mode. However, the model does not consider bus operating costs. In order to analyze the operational cost issues in detail, Zou et al. [5] studied the configuration optimization of electric public exchange power station, established a model with the objective of minimizing operation cost, optimized the charging pile in the station, and carried out a case study. Wang et al. [6] set a model to optimize the charging schedule of electric buses with the goal of minimum annual total operating cost. The model can provide corresponding decision support for the location, quantity and operation of charging stations. Li et al. [7] established the vehicle timetable model based on the maximum mileage of public transportation vehicles, analyzed the total operating costs of electric, fuel and hybrid buses, and proposed a column generation algorithm to solve the timetable problem. The research results show that electric buses with battery swapping will not significantly increase the number of fleet vehicles. Zheng et al. [8] established an operational model for electric bus, and gave a multi-objective operation plan to maximize the profit of the power station and minimize the impact on the power grid. The above literature focuses on the study of the number of charging pile configurations, but also there are also many studies on charging strategies and government policies. Qin et al. [9] simulated the electric bus operation and charging demand for one day in Tallahassee, Florida, and determined the optimal charging strategy with the goal of reducing charging demand. Wolbertus et al. [10] analyzed the impact of daytime parking and free parking policies on charging behavior in quantitative form, and pointed out that the interaction between charging behavior and policy should be considered when designing policies. Yang et al. [11] and Sarker et al. [12] gave corresponding optimization operation plans and charging strategies for the fluctuation of electricity price.

The above literature optimizes the charging facility configuration or charging strategy based on the determined operating conditions from different angles, but none of them consider the impact of the bus line itself and the operation mode. Meanwhile, there has been a deep research on the design and operation optimization of public transportation lines. Wirasinghe et al. [13] determined the optimal bus service characteristics with the goal of minimizing the sum of bus operation cost and passenger time cost, and analyzed operational parameters such as station spacing, feeder bus area boundary and headway time interval, and formulated it in the form of a formula. Daganzo et al. [14] analyzed the principles and optimization methods of public transportation operations in two-point systems, traffic corridor systems, and two-dimensional planar network systems under ideal and realistic conditions. Li et al. [15] established an optimization model with the goal of maximizing profit and maximizing social welfare, and giving the optimal headway, fare, the number of stations and locations.

This paper studies the electric bus route planning, comprehensively considers the design of bus line operation characteristic parameters and the configuration of charging station and charging pile. Constrained by charging waiting time and capacity, a social welfare maximization model is established in which the number and location of the stations, headway, fleet size, fare, and the number of charging piles are jointly optimized. The paper also discusses the charging station setup problem and charging strategy.

The rest of the paper is organized as follows: In the next section, some basic model assumptions are described and a social welfare maximization model is established; Section 3 discusses the charging station setup problem and charging strategy; Section 4, a heuristic solution algorithm is developed for jointly solving the design variables of the electric bus line. In Section 5, an example is used to illustrate the application of the proposed models and solution algorithm. Finally, conclusions are given in Section 6 together with recommendations for further studies.

## 2. Model formulation

### 2.1. Assumptions

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper.

A1 A linear urban transportation corridor is considered and the length of the bus line is equal to the length of the corridor.

A2 A flat fare structure is applied to the bus system, implying that all passengers in the bus system are charged the same fare regardless of the length of their trips.

A3 A linear elastic demand density function is used to represent the responses of passengers to the quality of the transit line service.

A4 Passengers are assumed to board buses at the nearest bus station in terms of access time.

A5 A cycle operation starts from the terminal station.

A6 The battery charge is proportional to the charging time and the battery consumption is proportional to the mileage.

A7 We represent the population density at distance  $x$  from the terminal station as  $g(x), \forall x \in [0, B]$ .

### 2.2. Passenger demand for each bus station

Set up an electric bus line on the linear transportation corridor of length  $B$ , as shown in Figure 1. In this figure,  $N$  denotes the total number of stations on the line.  $D_s$  denotes the distance of station  $s$  from the terminal station. Let  $l_s$  be the passenger watershed line between stations  $s$  and  $s + 1$ , and  $L_s$  be the distance of the passenger watershed line  $l_s$  from the terminal station. Based on A4, the watershed line  $l_s$  is located at the middle point of the line segment  $(s, s + 1)$ , which implies

$$L_s = \frac{D_s + D_{s+1}}{2}, \forall s = 1, 2, \dots, N-1 \quad (1)$$

Where  $D_1 = 0, L_N = D_N$ , to convenient to express the formula without affecting the result, let  $L_0 = 0$ .

Let  $q(x, s)$  denote the density of passenger demand for station  $s$  at location  $x$ . we define a linear elastic demand density function according to A3 as follows.

$$q(x, s) = g(x)(1 - e_a u_s(x) - e_w w_s - e_f f - e_v V_b), \quad \forall x \in [0, B], \forall s = 1, 2, \dots, N \quad (2)$$

Where  $u_s(x)$  is the passenger access time to station  $s$  from location  $x$ ,  $w_s$  is the average passenger wait time at station  $s$ ,  $f$  is the fare,  $V_b$  is the difference between the maximum speed ( $V_{b_{max}}$ ) and the actual average speed ( $V_{b1}$ ) of a bus.  $e_a, e_w, e_f$  and  $e_v$  are the sensitivity parameters for the access time, wait time, fare and speed difference, respectively.

To ensure the non-negativity of the passenger demand, the following condition should be satisfied

$$0 \leq 1 - e_a u_s(x) - e_w w_s - e_f f - e_v V_b \leq 1, \forall x \in [L_{s-1}, L_s], s = 1, 2, \dots, N \quad (3)$$

We now define the components that are included in the linear demand function (2). The passenger access time  $u_s(x)$  depends on the walking distance between location  $x$  and station  $s$  and the walking speed of passengers,  $V_a$ . It is expressed as

$$u_s(x) = \begin{cases} (D_s - x)/V_a, & \forall x \leq D_s \\ (x - D_s)/V_a, & \forall x > D_s \end{cases}, \forall x \in [0, B], s = 1, 2, \dots, N \quad (4)$$

The average passenger wait time at station  $s$ ,  $w_s$ , can be calculated by

$$w_s = \alpha H, \quad \forall s = 1, 2, \dots, N \quad (5)$$

Where  $H$  is the headway of the bus service, and  $\alpha$  is a calibration parameter that depends on the distributions of bus headway and passenger arrival. The value  $\alpha = 0.5$  is commonly used to suggest a constant headway between buses and a uniform random passenger arrival distribution.

The actual speed of the electric bus is related to the number of stations. Assuming that the stops are fixed,  $\beta$ , The maximum speed of electric buses on roads is  $V_{b\max}$ , thus the actual average driving speed of electric buses can be expressed as

$$V_{bl} = \frac{BV_{b\max}}{B + \beta NV_{b\max}} \quad (6)$$

Therefore, the speed difference expressed as

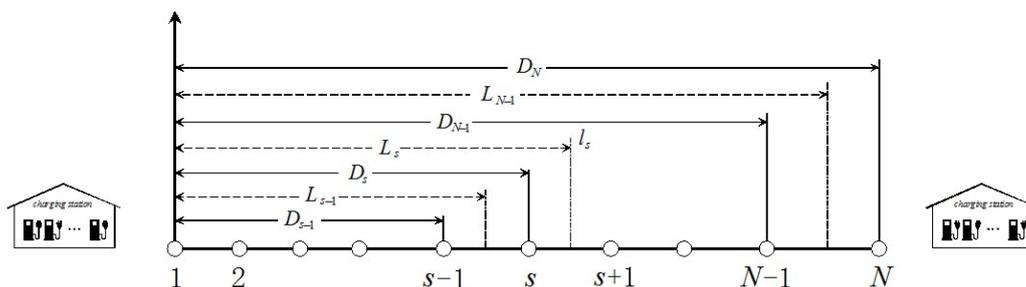
$$V_b = \frac{\beta NV_{b\max}^2}{B + \beta NV_{b\max}} \quad (7)$$

The number of passengers at each station  $Q_s$  is

$$Q_s = \int_{L_{s-1}}^{L_s} q(x, s) dx, \quad \forall s = 1, 2, \dots, N \quad (8)$$

Substituting Eqs. (2) – (7) into Eq. (8),  $Q_s$  can then be rewritten as

$$Q_s = (1 - e_w \alpha H - e_f f - e_v V_b) \int_{L_{s-1}}^{L_s} g(x) dx - \frac{e_a}{V_a} \left( \int_{L_{s-1}}^{D_s} g(x) (D_s - x) dx + \int_{D_s}^{L_s} g(x) (x - D_s) dx \right), \forall s = 1, 2, \dots, N \quad (9)$$



**Figure 1.** The bus line configuration along a linear transportation corridor.

### 2.3. Social welfare

Because the bus is a kind of public transport, it is more reasonable to take the maximization of social welfare as the decision-making objective. It is defined as the sum of the consumer surplus (represented by  $G$ ) and the net profit of the bus operator (represented by  $\pi$ ), that is,

$$W = G + \pi \quad (10)$$

In the following, we define the consumer surplus  $G$  and the operator's net profit  $\pi$ , respectively.

**2.3.1. Consumer surplus.** To determine the consumer surplus  $G$ , we first define the surplus of the consumers boarding at station  $s$ , represented as  $G_s$ . To do so, we convert the linear demand density function shown in Equation (2) to find fare as a function of demand density. The total social benefit can then be obtained by integrating the inverted function over the demand. The consumer surplus can thus be derived as the price that the users are willing to pay minus the price that the users actually pay. Along this vein, the inverse of the demand density function in Equation (2) can be expressed as

$$q^{-1}(x, s) = \frac{1}{e_f} (1 - e_a u_s(x) - e_w w_s - e_v V_b) - \frac{q(x, s)}{e_f g(x)}, \forall x \in [0, B], s = 1, 2, \dots, N \quad (11)$$

The surplus of consumers originating at location  $x$  and boarding at station  $s$ , which is represented by  $G(x, s)$ , can thus be expressed as

$$\begin{aligned} G(x, s) &= \int_0^{q(x, s)} \left[ \frac{1}{e_f} (1 - e_a u_s(x) - e_w w_s - e_v V_b) - \frac{z}{e_f g(x)} \right] dz - q(x, s) f \\ &= \frac{g(x)}{2e_f} (1 - e_a u_s(x) - e_w w_s - e_v V_b)^2, \forall x \in [0, B], s = 1, 2, \dots, N \end{aligned} \quad (12)$$

Thus, the consumer surplus  $G_s$  for station  $s$  can be given by

$$G_s = \int_{L_{s-1}}^{L_s} G(x, s) dx, \forall x \in [L_{s-1}, L_s], s = 1, 2, \dots, N \quad (13)$$

The total consumer surplus of the bus system can thus be calculated by

$$G = \sum_{s=1}^N G_s \quad (14)$$

**2.3.2. Net profit of the operator.** We now define the operator's net profit  $\pi$ . It equals the total operating revenue,  $R$ , minus the total cost,  $C$ , that is,

$$\pi = R - C \quad (15)$$

The total operating revenue  $R$  is the sum of the number of passengers boarding at each station multiplied by the corresponding fare, that is,

$$R = \sum_{s=1}^N f Q_s \quad (16)$$

Where the passenger demand for station  $s$ ,  $Q_s$ , is given by Equation (8).

The total cost  $C$  includes the vehicle cost  $C_b$ , operating cost  $C_o$ , configuration cost of charging station and charging pile  $C_e$ , and charging waiting time cost  $C_w$ , that is,

$$C = C_b + C_o + C_e + C_w \quad (17)$$

The vehicle cost  $C_b$  is calculated by multiplying the price of a bus by the total number of vehicles, which is expressed as

$$C_b = c_b \cdot F \quad (18)$$

The operating cost  $C_o$  is incurred by the electricity consumed by the electric vehicle, which is available on A6

$$C_o = \frac{2\mu_e D_N T_{oper}}{H} \quad (19)$$

Where  $T_{oper}$  is the line operation time,  $\mu_e$  is the cost of electricity consumption per kilometer.

Charging station and charging pile configuration costs  $C_e$  include fixed cost and variable cost, which is expressed as

$$C_e = \Delta_0 + c\Delta \quad (20)$$

Where  $\Delta_0$  is the fixed cost of charging station system construction,  $\Delta$  is the cost of a charging pile and  $c$  is the number of charging piles.

Charging waiting time cost  $C_w$  is equal to total charge waiting time multiplied by unit waiting time cost ( $\eta$ ) while the total charge waiting time is equal to the average charge waiting time per vehicle multiplied by the total number of vehicles, and is represented as

$$C_w = \eta \overline{W}_s \cdot F \cdot T_{oper} \quad (21)$$

Where  $\overline{W}_s$  is the average charge waiting time per car per hour.

The vehicle arrival rate is the number of charging times per vehicle per day divided by the total operation time, that is,

$$\lambda = \frac{2D_N}{SHF} \quad (22)$$

Where  $S$  is the electric bus cruising range.

Average charging waiting time of the charging vehicle in the charging station  $W_s$  has a relationship with  $\overline{W}_s$ , expressed as

$$\overline{W}_s = \frac{\lambda W_s}{F} = W_s \cdot \frac{2D_N}{SHF^2} \quad (23)$$

Where  $W_s$  can be obtained by finite source customer queuing model ( $M/M/c/\infty/F$ ). The derivation formula is as follows

$$P_0 = \frac{1}{F! \left( \sum_{k=0}^c \frac{1}{k!(F-k)!} \left( \frac{c\rho}{F} \right)^k + \frac{c^c}{c!} \sum_{k=c+1}^F \frac{1}{(F-k)!} \left( \frac{\rho}{F} \right)^k \right)} \quad (24)$$

$$P_n = \begin{cases} \frac{F!}{(F-n)!n!} \left(\frac{\lambda}{\mu}\right)^n P_0, 1 \leq n \leq c \\ \frac{F!}{(F-n)!c!c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n P_0, c \leq n \leq F \end{cases} \quad (25)$$

$$W_s = \frac{\sum_{n=1}^F nP_n}{\lambda \left(F - \sum_{n=1}^F nP_n\right)} \quad (26)$$

Where  $\rho = \lambda F / c\mu$ ,  $\lambda$  is the vehicle arrival rate,  $\mu$  is the charging service rate,  $F$  is the number of the buses,  $c$  is the number of the charging piles.

On the basis of Equations (16–26), the total cost  $C$  can be represented as

$$C = \frac{c_b \cdot F + \Delta_0 + c\Delta}{T_{life}} + \frac{2\mu_e D_N T_{oper}}{H} + \eta W_s \cdot \lambda \cdot T_{oper} \quad (27)$$

Substituting Equations (14), (15), (16), and (27) into Equation (10), one obtains the social welfare of the bus system as follows:

$$W(D_s, H, f) = \sum_{s=1}^N G_s + \sum_{s=1}^N f \cdot Q_s - \frac{c_b \cdot F + \Delta_0 + c\Delta}{T_{life}} - \frac{2\mu_e D_N T_{oper}}{H} - \eta W_s \cdot \lambda \cdot T_{oper} \quad (28)$$

#### 2.4. Social welfare maximization model

Based on the above analysis, the social welfare maximization model is

$$\max W(D_s, H, f) = \sum_{s=1}^N G_s + \sum_{s=1}^N f \cdot Q_s - \frac{c_b \cdot F + \Delta_0 + c\Delta}{T_{life}} - \frac{2\mu_e D_N T_{oper}}{H} - \frac{2D_N \eta}{SHF} W_s \cdot T_{oper} \quad (29)$$

$$\text{s.t. } \sum_{s=1}^N Q_s \leq \frac{T_{oper}}{H} \cdot Q_0 \quad (30)$$

$$W_s \leq SF \left( \frac{F}{2HD_N} - \frac{1}{V_{b1}} \right) \quad (31)$$

In order to ensure that the supply of the rail transit service satisfies the passenger demand, the capacity constraint is presented represented as formula (30), where  $Q_0$  is the maximum number of passengers allowed in a bus, both seated and standing. Formula (31) represents charge waiting time constraint. The derivation process is as follows.

The total cycle time of each vehicle  $T$  is equal to the sum of the average charging waiting time  $W'_s$ , line running time and idle time  $t_0$ . The formula is expressed as

$$T = W'_s + \frac{2D_N}{V_{b1}} + t_0 \quad (32)$$

The average charge waiting time  $W'_s$  for cyclic operation can be converted according to the average charging time per hour  $\bar{W}_s$ , that is,

$$F\bar{W}_s T_{oper} = W'_s \frac{T_{oper}}{H} \quad (33)$$

So there is

$$W'_s = FH\bar{W}_s \quad (34)$$

The formula for calculating the size of the fleet is

$$F = \frac{T}{H} \quad (35)$$

Substituting Equations (23), (32), (33), and (34) into Equation (35), there is

$$F = \frac{1}{H} \left( W'_s \cdot \frac{2D_N}{SF} + \frac{2D_N}{V_{bl}} + t_0 \right) \quad (36)$$

Because of  $t_0 \geq 0$ , there is

$$W'_s \leq SF \left( \frac{FH}{2D_N} - \frac{1}{V_{bl}} \right) \quad (37)$$

### 3. Charging station setup and charging strategy

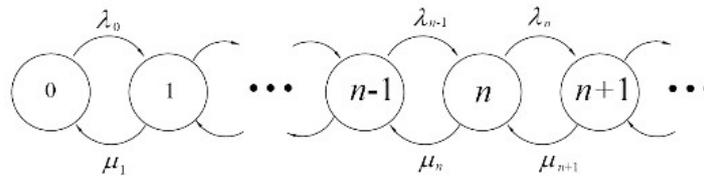
The charging strategies of electric buses in different cities are different, and can basically be divided into two methods. One is that the bus won't charge until the battery is below the threshold and the other is that when the bus is finished a cycle, it will be charged as long as there is an idle charging pile. Based on the above two charging strategies, and considering whether the charging station is set at both ends, the following four cases are analyzed.

Case 1: The charging piles are only set at one end of the line and the electric bus charging rule is to charge when the remaining power cannot support the next cycle. According to finite source customer queuing model ( $M/M/c/\infty/F$ ), the formula (24)-(26), the charging waiting time  $W'_s$  can be obtained,

where the vehicle arrival rate  $\lambda = \frac{2D_N}{SHF}$ .

Case 2: The charging piles are set at one end of the line. The charging rule is that when the bus arrived at the charging station at the end of a cycle, if there is a spare charging pile, it will be charged immediately and if there is no spare charging pile, whether the bus is waiting for charging depends on whether the remaining battery capacity can satisfy the next cycle.

Thinking of the charging station as a queuing system,  $n$  is the number of bus in the system,  $\lambda_n$  is the arrival rate and  $\mu_n$  is the exit rate when there are  $n$  buses in the system.  $p_n$  indicates the steady state probability. Figure 2 is a diagram showing the state transition of the vehicle queue in the charging station.



**Figure 2.** The state transition of charging buses.

Under steady state conditions, the expected rate of the inflow and outflow states must be equal, and the state  $n$  can only become the state of  $n - 1$  or  $n + 1$ , so the equilibrium equation for the state  $n$  can be obtained.

$$\lambda_{n-1}p_{n-1} + \mu_{n+1}p_{n+1} = (\lambda_n + \mu_n) p_n \tag{38}$$

For  $n = 0$ , the equilibrium equation is  $\lambda_0 p_0 = \mu_1 p_1$ , so there is  $p_1 = \left(\frac{\lambda_0}{\mu_1}\right) p_0$ .

Generally, it can be obtained by induction

$$p_n = \left(\frac{\lambda_{n-1}\lambda_{n-2}\dots\lambda_0}{\mu_n\mu_{n-1}\dots\mu_1}\right) p_0, n = 1, 2, \dots \tag{39}$$

The value of  $p_0$  can be obtained from the equation  $\sum_{n=0}^{\infty} p_n = 1$ .

When the number of bus in the charging station  $n$  is less than the number of charging piles  $c$ , The arrival rate of the vehicle in the charging station is equal to the ending rate of the vehicle running from the line, that is  $\lambda_n = \frac{1}{H}$ . The rule of charging vehicle leaving is that the charging power meets the next

cycle. So According to A6, there is  $\mu_n = \frac{S}{2D_N} \mu$ .

When  $n \geq c$ , the bus will only charge when the power cannot satisfy the next cycle. So  $\lambda_n = \frac{2D_N}{SHF}$ , and the bus leaving rate is  $\mu_n = \frac{S}{2D_N} \mu$ .

In summary, the arrival and leaving rate of the bus in the charging station is expressed as

$$\begin{cases} \lambda_n = \frac{1}{H}, \mu_n = \frac{S}{2D_N} \mu, n < c \\ \lambda_n = \frac{2D_N}{SHF}, \mu_n = \frac{S}{2D_N} \mu, c \leq n < F \end{cases} \tag{40}$$

Substituting Equations (40) into Equation (39), the value of the state probability  $p_n$  can be obtained. According to Little formula, there is

$$\begin{cases} L_s = \sum_{n=1}^{\infty} np_n \\ L_s = \lambda_{eff} W_s \end{cases} \tag{41}$$

Therefore, it can calculate the waiting time of the vehicle  $W_s$  in the charging station. Where effective arrival rate  $\lambda_{eff}$  is

$$\lambda_{eff} = \frac{1}{H} - \left( \frac{1}{H} - \frac{2D_N}{SHF} \right) \sum_{n=c}^{F-1} p_n \tag{42}$$

Case 3: The charging piles are set at both ends of the line, and the number of charging piles at both ends is  $c_1, c_2$  respectively, and the electric bus charging rule is to charge when the remaining electric quantity cannot support the next cycle. The bus arrival rate vehicle  $\lambda = \frac{2D_N}{SHF}$ , Assume that the proportion of the number of times the bus is charged at both ends of the line is  $\xi_1, \xi_2$ , and there is  $\xi_1 + \xi_2 = 1$ . According to the calculation formula of the finite source customer queuing model, the charging waiting time of both ends can be respectively obtained as  $W_{s1}, W_{s2}$ . The arrival rate of the bus at both ends is  $\lambda_1 = \xi_1 \frac{2D_N}{SHF}, \lambda_2 = \xi_2 \frac{2D_N}{SHF}$ . Therefore, according to the derivation process of formulas (32)-(37), the charging waiting time constraint can be obtained, formulated as,

$$W_{s1} + W_{s2} \leq \frac{SHF^2}{2D_N} - \frac{SF}{V_{bl}} \tag{43}$$

Case 4: The charging piles are set at both ends of the line, and the number of charging piles at both ends is  $c_1, c_2$  respectively. The charging rule is that when the bus arrived at the charging station at the end of a cycle, if there is a spare charging pile, it will be charged immediately and if there is no spare charging pile, whether the bus is waiting for charging depends on whether the remaining battery capacity can satisfy the next cycle. The analysis of the charging waiting time at either end in Case 4 is the same as the analysis of Case 2 above.

#### 4. Solution algorithm

For the optimization problem under inequality constraints, this paper uses the method of constructing Lagrangian function to find the KT point of the programming problem, uses KT conditions and constraints to establish the equations, and discusses the value of the multiplier to determine the optimal condition. Deviate the variables and multipliers separately, and the result is as follows

$$\begin{cases} \sum_{i=s-1}^{s+1} \frac{\partial G_i}{\partial D_s} + (f - \lambda_1) \sum_{i=s-1}^{s+1} \frac{\partial Q_i}{\partial D_s} - \delta_s \left( \frac{2\mu_e T_{oper}}{H} + \frac{2\eta T_{oper} W_s}{SHF} + \frac{\lambda_2 SHF^2}{2D_N^2} \right) = 0, \\ \frac{\alpha e_w}{e_f} \sum_{s=1}^N Q_s + \frac{T_{oper}}{H^2} \left( \lambda_1 Q_0 - 2\mu_e D_N - \frac{2D_N \eta W_s}{SF} \right) + \left( \frac{2D_N \eta T_{oper}}{SHF} + \lambda_2 \right) \frac{\partial W_s}{\partial H} - \frac{\lambda_2 SF^2}{2D_N} = 0, \\ f - \lambda_1 = 0, \\ \lambda_1 \left( \frac{T_{oper}}{H} \cdot Q_0 - \sum_{s=1}^N Q_s \right) = 0, \\ \lambda_2 \left( \frac{SHF^2}{2D_N} - \frac{SF}{V_{bl}} - W_s \right) = 0, \\ \lambda_1 \geq 0, \lambda_2 \geq 0 \end{cases} \tag{44}$$

Where if  $s = N$ ,  $\delta_s = 1$ , else  $\delta_s = 0$ .  $\lambda_1, \lambda_2$  are Lagrangian multipliers.

The solution algorithm developed below is directly based on Formula (44). For the number of charging piles  $c$ , the total size of the fleet  $F$ , and the number of stations  $N$ , the three integer variables are solved by the traversal method. The step-by-step procedure is given as follows.

Step 1. Initialization. Set initial value for each design variable of the bus line, including station locations  $D_s^{(0)}$  ( $s = 1, 2, \dots, N$ ), headway  $H^{(0)}$ , and fare  $f^{(0)}$ . Determine the corresponding passenger demand  $Q_s^{(0)}$  ( $s = 1, 2, \dots, N$ ) and consumer surplus  $G_s^{(0)}$  ( $s = 1, 2, \dots, N$ ) for each station on the bus line by Eqs. (9), (13). For the given  $H^{(0)}$ , the traversal method is used to find the minimum number of charging piles and the minimum number of fleets that satisfy the formulas (24)-(26). Solving the model objective function value  $W^{(0)}$  according to formula (29). Set iteration counter  $j=1$ .

Step 2. Updating of the design variables. Sequentially update the values of the headway, fare and station location (or spacing) according to the formula (44).

Step 2.1. Update  $H^{(j)}$  with fixed  $D_s^{(j-1)}$  ( $s = 1, 2, \dots, N$ ) and  $f^{(j-1)}$ . Check whether the resultant headway  $H^{(j)}$  satisfies the capacity constraint (30), Charging waiting time constraint (31) and the non-negative passenger demand constraint (3). If it exceeds some constraint bound, then it is set at the corresponding bound.

Step 2.2. Update  $f^{(j)}$  with fixed  $D_s^{(j-1)}$  ( $s = 1, 2, \dots, N$ ) and  $H^{(j)}$ . Check the non-negative passenger demand constraint (3) for the resultant fare  $f^{(j)}$ . If it exceeds the constraint bound, then it is set at the corresponding bound.

Step 2.3. Update  $D_s^{(j)}$  ( $s = 1, 2, \dots, N$ ) with fixed  $H^{(j)}$  and  $f^{(j)}$ . Check the non-negative passenger demand constraint (3) and charging waiting time constraint (31). If it exceeds some constraint bound, then it is set at the corresponding bound.

Step 3. Updating of the objective function. Update the passenger demand  $Q_s^{(j)}$  ( $s = 1, 2, \dots, N$ ) and consumer surplus  $G_s^{(j)}$  ( $s = 1, 2, \dots, N$ ) based on the formulas (9), (13), and update the number of charging piles  $c^{(j)}$  and the total fleet size  $F^{(j)}$  by traversing method. The resultant value of the objective function  $W^{(j)}$  can be obtained.

Step 4. Termination check. If the resultant objective function values for successive iterations are sufficiently close, then terminate the algorithm and output the optimal solution  $\{\mathbf{D}^*, H^*, f^*, c^*, F^*\}$  and the corresponding objective function value  $W^*$ . Otherwise, set  $j = j + 1$ , and go to Step 2.

It should be mentioned that the above solution procedure is based on a fixed number of stations. Note that the number of stations is an integer variable, which makes it difficult to solve the resultant mixed integer programming problem. Fortunately, the number of stations on a rail transit line is a finite number. Therefore, a simple approach for finding the optimal number of stations is to compare the resultant objective function values with different numbers of stations.

## 5. Numerical studies

### 5.1. Parameters setting

In this section, A numerical example is used to verify the validity of the proposed model and the algorithm. Take the 931 pure electric bus line in Suzhou as a reference case, as shown in Figure 3.



**Figure 3.** Map of 931 bus routes in Suzhou.

The Suzhou 931 bus serves between the Xin zhuang interchange hub and the South Ring Bridge. The bus line is about 12.394 kilometers long and has 23 stations.

The length of the example bus line  $B$  is set to 12.4 kilometers, and the population density refers to the average population density of 1,184 people/km<sup>2</sup> at the end of 2016 in Suzhou. The remaining parameters are shown in Table 1.

**Table 1.** Parameter value of example.

Symbol	Definition	Value
$B$	Length of the line (km)	12.4
$g(x)$	Population density (persons/km <sup>2</sup> )	1184
$e_a$	Sensitivity parameter for access time (1/h)	0.98
$e_w$	Sensitivity parameter for wait time (1/h)	0.98
$e_f$	Sensitivity parameter for fare (1/¥)	0.098
$V_a$	Average walking speed of passengers (km/h)	4
$\alpha$	Ratio of passenger waiting time to headway	0.5
$\beta$	Average bus dwell time at a bus station (s)	30
$c_b$	Average price of an electric bus (¥/veh)	2000000
$T_{oper}$	Bus operation time (h/d)	16
$\mu_e$	Electric bus power consumption cost (¥/km)	2
$\Delta_0$	Charging station system construction cost (¥)	5000000
$\Delta$	Charging pile configuration cost (¥)	100000
$\eta$	Unit waiting time cost (¥/h)	20
$S$	Electric bus cruising range (km)	100
$\mu$	charging service rate (veh/h)	0.5
$T_{life}$	Average life of electric buses (d)	3650
$Q_0$	Capacity of an electric bus (person/veh)	70
$V_{b\max}$	Maximum speed of a bus (km/h)	40

## 5.2. Social welfare analysis

**5.2.1. Model applications.** The above example is solved according to the solving algorithm in Section 3. Figure 4 shows the optimal welfare value for the number of different stations. The line shows the fitting curve for the points in the graph. It can be observed that with the increase of stations, social

welfare increases first and then decreases. As can be seen from the figure, when 27 stations are set, the social welfare has reached the maximum value of 22,078 yuan/day. At this time, the fare is 0.6304 yuan, the headway is 4.3389 minutes, the fleet size is 27, and the number of charging piles is 7.

Figure 5 shows the optimal fare, headway, fleet size, and number of charging piles for the corresponding number of stations. It can be seen that, as the number of stations increases, the fleet size and the number of charging piles gradually increase, while the headway is reduced. The increase in the number of fleets is much faster than the increase in the number of charging piles. For the fare, according to formula (44), the optimal value is equal to the Lagrange multiplier with capacity constraints. Therefore, when capacity constraint is inactive, the optimal transit fare is zero, as showing in Figure. However, the fare becomes positive if capacity constraint becomes active.

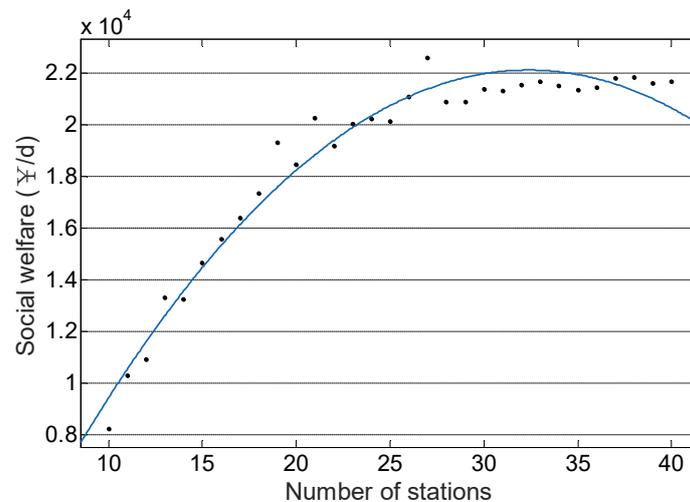


Figure 4. Optimal social welfare against number of stations.

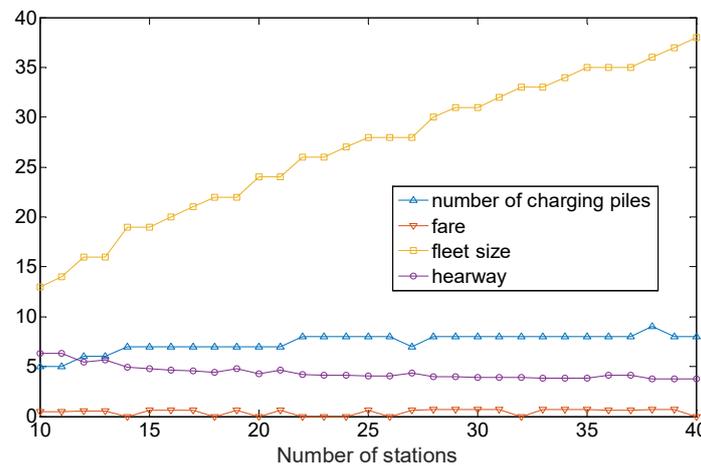


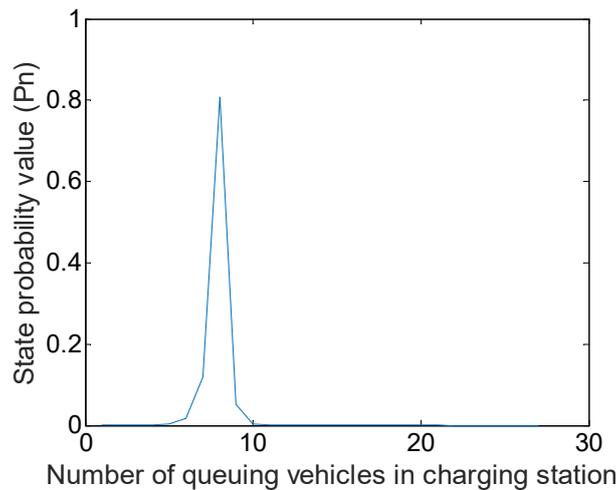
Figure 5. Optimal values of variables against number of stations.

5.2.2. *Charging strategy discussion.* When the charging pile is only set at one end of the line, according to the social welfare maximization model (Equation 29), the social welfare is only related to vehicle charging waiting time cost in the case of fixed fares, headway, fleet size, the site location and number of charging piles. Taking the above numerical as an example, in the case where the number of stations is 27, the fare is 0.6304 yuan, the headway is 4.3389 minutes, the fleet size is 27, and the number of charging piles is 7, According to the finite source customer queuing model ( $M/M/7/\infty/27$ ),  $W_s = 2.2246$ .

In case 2, as we substitute the known variables into the formula (40), we have

$$\begin{cases} \lambda_n = 13.831, \mu_n = 2.016, n < 7 \\ \lambda_n = 0.127, \mu_n = 2.016, 7 \leq n < 27 \end{cases}$$

Therefore, according to formula (39), the steady state probability  $p_n$  can be calculated. The values of  $p_n$  are shown in Figure 6 and table 2 below lists the main values of  $p_n$  and the number of buses in charging system correspondingly. Calculate  $L_s = 6.8966$ ,  $W_s = 3.4209$  according to formulas (41) and (42). Comparing the value of  $W_s$ , it can be found that in this example, case 1 is better than case 2.



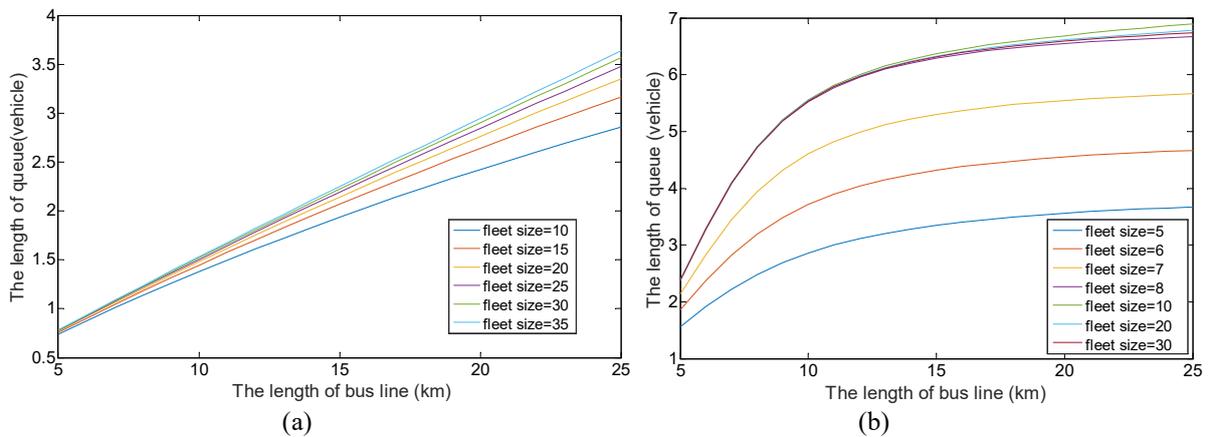
**Figure 6.** Queueing probability of vehicle in charging station.

**Table 2.** Main probability value.

$n$	4	5	6	7	8	9
$p_n$	0.0025	0.0172	0.1178	0.8078	0.0509	0.0032

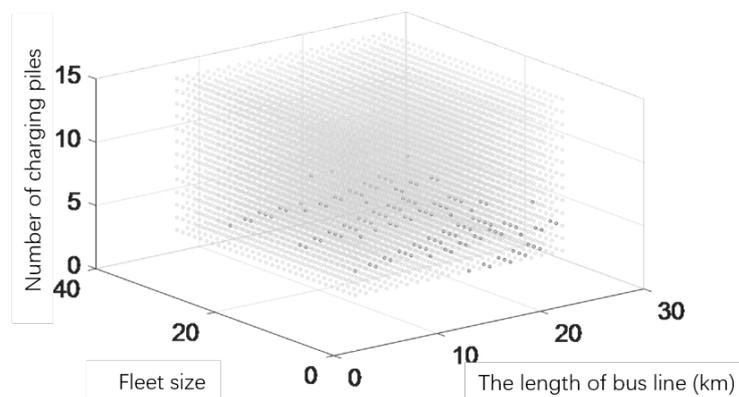
When the charging piles are only set at one end of the line, the comparison between case 1 and case 2 depends only on the vehicle charging waiting time cost, and the charging waiting time is related to the headway, the line length, the fleet size, and the number of charging piles. Since the queuing theory is complicated and it is difficult to simplify, we discuss the relationship between the charging waiting time (charging queue length) and the headway, line length, fleet size, and number of charging piles in two cases with the numerical analysis method.

Fig.7 (a), (b) show the relationship between the vehicle charging queue length and the line length and the fleet size in Cases 1 and 2, respectively, in the case where the headway is 10 minutes and the number of charging piles is 7. As can be seen from Fig. 7 (a), in case 1, the length of the line has a long influence on the length of the queue, and as the length of the line becomes larger, the influence also becomes larger. The size of fleet has a certain influence on the length of the queue. In case 2, when  $F \leq c+1$ , the size of the fleet has a great influence on the queue length of the charging pile shown in fig.7 (b). This is due to the excessive number of charging piles. While  $F > c+1$ , the impact is very small. As for the influence of line length on queue length, it can be seen from Fig. 7(b) that in case 2, when the line length is relatively short (within 10 km in the numerical example), the line length has a great influence on the queue length, and when the line is too long, the effect of the line length on the queue length becomes low.



**Figure 7.** The relationship between line length, fleet size and charging queue length.

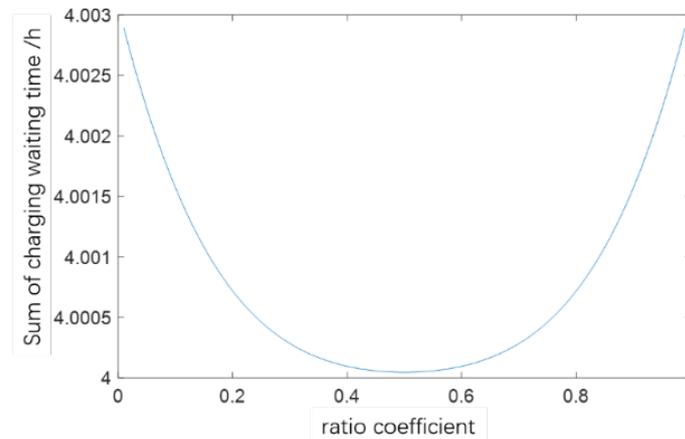
In order to further explore the relationship between the length of the charging queue and the length of the line, the size of the fleet, the number of charging piles, using the numerical method, we calculate the charge queue length at the time of that the lengths of line are 5-25 km, the fleet sizes are 5-35, and the charging piles are 3-15. Fig.8 shows the difference in queue length between Case 1 and Case 2. The light color value in the figure indicates that the case 1 queue length value is smaller than the case 2, that is, the charging strategy of case 1 is better than case 2. And the dark color indicates that the charging strategy of case 2 is better than case 1. Therefore, it can be seen from the color distribution in Fig. 8 that in the case where the length of the line is long and the number of charging piles is low, the charging strategy of the case 2 is superior.



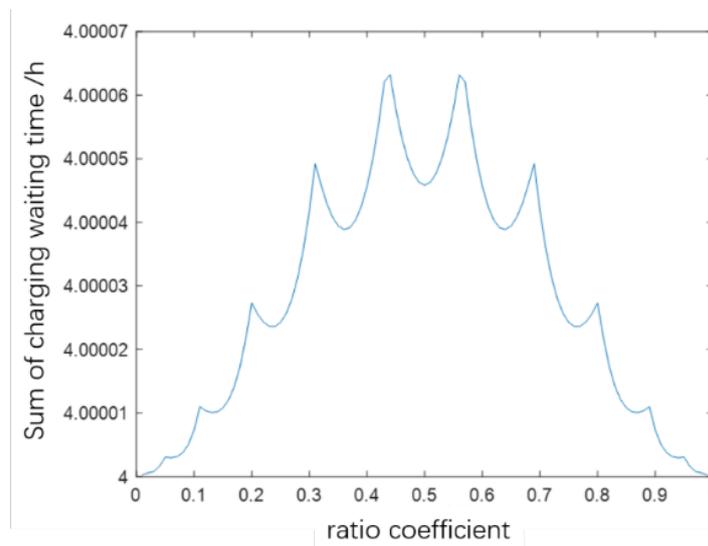
**Figure 8.** Comparison of charging queue length.

**5.2.3. Discussion on different settings of charging station.** In this section, we compared the Case 1 and Case 3 at the time of the same number of stations, headway, fare, fleet size and number of charging piles. For case 3, fig.9 shows the relationship between the charging ratio at one end of the line and the total waiting time at both ends as the number of charging piles at both ends of the line is equal ( $c_1=c_2$ ). From fig.9, we found when the charging ratio coefficient is 0.5, that is, when the charging ratios of both ends are the same, and the total charging time is the smallest. When  $c_1 \neq c_2, c_1 + c_2 = c$ , for a determined charging ratio coefficient, there is a unique minimum for the charging waiting time at both ends with the change of the number of charging piles at both ends. Fig.10 shows the relationship between the minimum charging waiting time and the charging ratio coefficient. As can be seen from Fig. 10, in case 3, the minimum charging time sum acquired the

minimum value as the charging ratio coefficient tends to 0 or 1, that is, when all the buses are charged to one side, the charging waiting time is the smallest just like case 1. Considering that there are two charging stations in case 3, so the case 1 is better. However, this paper is aimed at a bus line. For multiple lines, further research is needed.



**Figure 9.** Relationship between charging time and charging ratio coefficient.



**Figure 10.** Relationship between minimum charging time and charging ratio coefficient.

## 6. Conclusion and further studies

In this paper, a social welfare maximization model was proposed for optimizing the design variables of an electric bus line in a transportation corridor. In the proposed model, the number of stations, headway, fleet size, fare and the number of charging piles were jointly optimized. A heuristic solution algorithm for determining the design variables was developed based on the Lagrangian method. The effects of two different charging strategies are analyzed, the influence of whether the charging station is set at both ends is considered, and thus four cases are proposed for comparing in the paper. This model can provide decision-making basis for electric bus route planning.

Based on the proposed model of this paper, we can further expand from the following aspects. Firstly, our model did not consider multiple bus lines in terminal stations, Therefore, the problem of charging pile configuration in the bus network can be further studied. Secondly, the paper does not consider the impact of population density on the design of line parameters. The population density of

different cities varies greatly, and the population density will change greatly with time at one city. Therefore, it is of practical significance to analyze the impact of population density. Thirdly, the impact of vehicle mass production, technological advancement, and changes in government subsidy policies on electric bus prices and the speed deserve further study.

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