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Robust Controller for Planar Magnetic Levitation System Based on Interval Matrix

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Abstract. In this paper, the positioning system for precision machining table has been realized through Planar Magnetic Levitation System. The Planar Magnetic Levitation System has nonlinearity and structural uncertainties, so the interval matrix minimum upper bound method is to design robust control. The simulation result shows that with the controller the Planar Magnetic Levitation closed-loop system has the strong robustness, the speed of response is quick and machining precision can be effectively improved.

1. Introduction

Micromachining is needed in microelectronics packaging industry. Ultra-precision workbench required for processing is realized by an ultra-precise motion control system with nanometer precision. Such systems must overcome the effects of friction and have sufficient travel [1]. It is especially difficult to achieve for ultra-precision workbench with large stroke and multiple degrees of freedom.

Using electrostrictive ceramics can avoid friction and achieve high motion accuracy [2]. However, the stroke is generally less than one millimeter, so the currently used equipment consists of two parts: highly accurate motion by electrostrictive ceramics, and large stroke coarse positioning linear movement mechanism. Such ultra-precision workbench, the rough positioning system has friction and wear, the structure and control method of the worktable are complicated, and the installation and debugging are difficult, which is not conducive to further improvement of the motion precision and long-term maintenance. Due to the mechanical connection, the accuracy of the lithography process is affected, and the electrostrictive ceramics also have nonlinear hysteresis. In addition, the air suspension guide can also eliminate friction. However, the disturbance of the compressed air during release causes uncontrolled high-frequency jitter on the workbench, and the system of this structure cannot be used in a vacuum environment. Ultra-precise motion control system based on direct drive of magnetic levitation and linear motor can completely eliminate Coulomb friction and avoid the shortcomings of air suspension. The accuracy index of the motion system depends in principle only on the quality of the displacement sensor. The highest precision of precise positioning using magnetic levitation technology has reached 3nm [3]. The advantage of using magnetic levitation is not only in terms of accuracy. In the realization of multi-degree-of-freedom movement, if the conventional guide rail and bearing are used as the support and guidance of translation and rotation respectively, the mechanical structure of the system becomes very complicated with the increase of the degree of freedom, and the rigidity of the system decreases. When the magnetic levitation method is adopted, the mechanical structure of the system becomes simple, and the components with precision requirements are also greatly reduced.



This paper studies the use of electromagnets to form a planar magnetic levitation system with two degrees of freedom (2 DOF) and a large operating range in one plane, and avoids the above limitations. As shown in the system of Figure 1, three electromagnets are evenly spaced on the plane (A minimum of three electromagnets is required to achieve two degrees of freedom). We want to control the position of the ferromagnetic material disc placed in the area surrounded by it. The disc is suspended on the plane by the repulsive force of the independently controllable fourth electromagnet, and the vertical air gap between them can be adjusted. In addition to eliminating the friction, it has little relationship with our positioning system. We are concerned with the positioning subsystem in the horizontal direction.

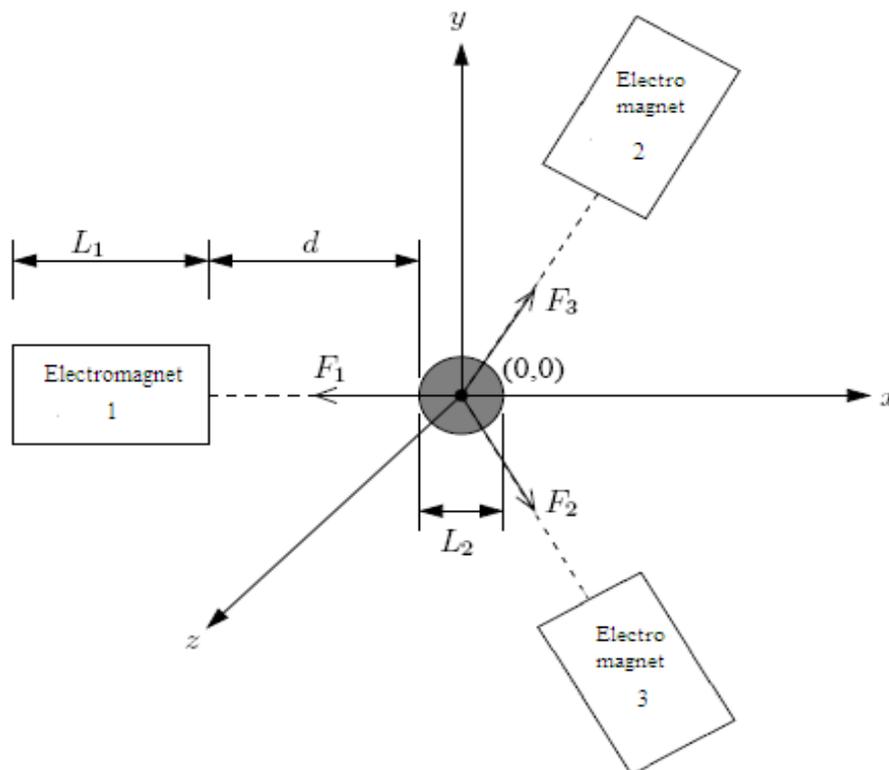


Figure 1. Planar Magnetic Levitation System

2. Mathematical model of plane magnetic levitation precise positioning system

In the system shown in Figure 1, the force analysis of the disc is described in detail in [4] and [5]. The force of the disc in the x, y direction is:

$$\begin{cases} F_x = -\frac{1}{2\mu_0 A_1} \left[\varphi_1(x, y)(x+d)I_1^2 + \varphi_2(x, y)\left(x - \frac{d}{2}\right)I_2^2 + \varphi_3(x, y)\left(x - \frac{d}{2}\right)I_3^2 \right] + m\delta_x(x, \dot{x}, y) \\ F_y = -\frac{1}{2\mu_0 A_1} \left[\varphi_1(x, y)(-x)I_1^2 + \varphi_2(x, y)\left(x + \frac{\sqrt{3}}{2}d\right)I_2^2 + \varphi_3(x, y)\left(x - \frac{\sqrt{3}}{2}d\right)I_3^2 \right] + m\delta_y(x, y, \dot{y}) \end{cases} \quad (1)$$

In the formula:

$$\varphi_1(x, y) = \frac{N_1^2}{\left(\frac{\sqrt{(x+d)^2 + y^2}}{\mu_0 A_1} + \frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} \right)^2 \sqrt{(x+d)^2 + y^2}}$$

$$\varphi_2(x, y) = \frac{N_2^2}{\left(\frac{\sqrt{(x-d/2)^2 + (y+\sqrt{3}d/2)^2}}{\mu_0 A_1} + \frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} \right)^2 \sqrt{(x-d/2)^2 + (y+\sqrt{3}d/2)^2}}$$

$$\varphi_3(x, y) = \frac{N_3^2}{\left(\frac{\sqrt{(x-d/2)^2 + (y-\sqrt{3}d/2)^2}}{\mu_0 A_1} + \frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} \right)^2 \sqrt{(x-d/2)^2 + (y-\sqrt{3}d/2)^2}}$$

F_x and F_y are the electromagnetic forces of the disk in the x and y directions, respectively, x and y are the displacement of the disk in the x and y directions, the control inputs I_1, I_2 and I_3 are the currents of the respective electromagnet coils, and m is the disk quality.

$\delta_x(x, \dot{x}, y)$ and $\delta_y(x, y, \dot{y})$ indicate the influence of other factors on the system (such as the magnetic line edge effect), which indicates the uncertainty of the system, L_1 : electromagnet core length, L_2 : disc diameter, N_1, N_2, N_3 are electromagnet windings Number of turns, A_1 is the cross-sectional area of the electromagnet core, A_2 is the cross-sectional area of the disc, μ_0 is the vacuum permeability, $\mu_1 = \mu_0 \mu_r$ is the magnetic permeability of the core material, $\mu_2 = \mu_0 \mu_r$ is the magnetic permeability of the disc material, d is The distance between the end face of the electromagnet and the edge of the disc when the disc is at the origin (as shown in Figure 1).

Define state variables:

$$\mathbf{X} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \equiv [x \quad \dot{x} \quad y \quad \dot{y}]^T \quad (2)$$

Disc motion equation:

$$\ddot{x} = \frac{F_x}{m}, \quad \ddot{y} = \frac{F_y}{m} \quad (3)$$

Available from Equation (1) (2) (3):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{2\mu_0 m A_1} \left[\varphi_1(x, y)(x+d)I_1^2 + \varphi_2(x, y)(x-\frac{d}{2})I_2^2 + \varphi_3(x, y)(x-\frac{d}{2})I_3^2 \right] + \delta_x(x_1, x_2, x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\frac{1}{2\mu_0 m A_1} \left[\varphi_1(x, y)(-x)I_1^2 + \varphi_2(x, y)(x+\frac{\sqrt{3}}{2}d)I_2^2 + \varphi_3(x, y)(x-\frac{\sqrt{3}}{2}d)I_3^2 \right] + \delta_y(x_1, x_3, x_4) \end{cases} \quad (4)$$

3. Precision Linearization System Design LQR Controller

The nonlinear system described in equation (4) can be converted to the following Bruno sky standard type by a special feedback transformation $[I_1^2 \ I_2^2 \ I_3^2]^T = \mathbf{T}(\mathbf{X}, \mathbf{U})$ when considering the ideal case of $\delta_x(x_1, x_2, x_3) = 0$ and $\delta_y(x_1, x_3, x_4) = 0$, see [6].

$$\dot{X} = AX + BU \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad U = [u_1, u_2]^T, \quad X = [x_1, x_2, x_3, x_4]^T \quad (5)$$

$$[I_1^2 I_2^2 I_3^2]^T = T(X, U) = \begin{bmatrix} \frac{2m\mu_0 A_1}{\varphi_1(x_1, x_3)(x_1 + x_3 + d)} \eta_1(x_1, x_3, u_1, u_3) \\ \frac{2m\mu_0 A_1}{\varphi_2(x_1, x_3)(x_1 - x_3 - \frac{\sqrt{3} + 1}{2}d)} \eta_2(x_1, x_3, u_1, u_3) \\ \frac{2m\mu_0 A_1}{\varphi_3(x_1, x_3)(x_1 - x_3 + \frac{\sqrt{3} - 1}{2}d)} \eta_3(x_1, x_3, u_1, u_3) \end{bmatrix} \quad (6)$$

Controller $U = [u_1, u_2]^T = -KX$ is designed using a linear quadratic regulator (LQR) method for a real symmetric weighting matrix R, Q for a given amount of control and state quantities. There is a feedback gain vector $K = -R^{-1}B^T P$, in which P is the solution that satisfies the following Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (7)$$

4. Robust controller based on interval matrix design

Lemma [8] Let the n -th order Hermite matrix $A_1 = A_0 + E$, ∂_i, η_i , ($i = 1, 2, \dots, n$) be the eigenvalues of the matrix A_1, A_0 in descending order, and $\varepsilon_{\max}, \varepsilon_{\min}$ is the maximum and minimum eigenvalues of the matrix E , then

$$\eta_i + \varepsilon_i \leq \partial_i \leq \eta_i + \varepsilon_i \quad (i = 1, 2, \dots, n) \quad (8)$$

Let ΔA be the interval matrix to be determined (to describe the system uncertainty), then the following theorem for system $\dot{X} = (A + \Delta A)X + BU$:

Theorem: Let P be the solution of the following Riccati algebraic equation

$$M^T P + PM - PBR^{-1}B^T P + Q = 0 \quad (R, Q \text{ is a real symmetric matrix}) \quad (9)$$

And make the Hermite matrix $P^2 BR^{-1}B^T + BR^{-1}B^T P^2 \geq 0$. In which $M = [(A + A^T)/2] + E_{\Delta A}$, $E_{\Delta A}$ is the least upper bound of the matrix set $\Psi = \{(\Delta A_1 + \Delta A_1^T)/2, (\Delta A_2 + \Delta A_2^T)/2, \dots, (\Delta A_d + \Delta A_d^T)/2\}$, d is the number of vertices of the interval matrix ΔA , and ΔA_i is the i th ($i = 1, 2, \dots, d$) vertex of ΔA , then the feedback control law $U = -R^{-1}B^T P X$ makes the closed-loop asymptotic stability of the system.

Prove:

Let $\hat{M} = M - BR^{-1}B^T P$, by the theorem, M is a real symmetric matrix, so $M = M^T$, then:

$$\begin{aligned}
 & P\left(\frac{\widehat{M} + \widehat{M}^T}{2}\right) + \left(\frac{\widehat{M} + \widehat{M}^T}{2}\right)P + Q \\
 &= P\left(\frac{M + M^T}{2} - \frac{BR^{-1}B^T P + PBR^{-1}B^T}{2}\right) + \left(\frac{M + M^T}{2} - \frac{BR^{-1}B^T P + PBR^{-1}B^T}{2}\right)P + Q \\
 &= PM + M^T P - PBR^{-1}B^T P - \frac{P^2 BR^{-1}B^T + BR^{-1}B^T P^2}{2} + Q \\
 &\leq PM + M^T P - PBR^{-1}B^T P + Q = 0
 \end{aligned}$$

So the matrix $\frac{\widehat{M} + \widehat{M}^T}{2}$ is stable, i.e. $\text{Re } \lambda\left(\frac{\widehat{M} + \widehat{M}^T}{2}\right) < 0$

Set $M_0 = A + \Delta A$, $i = 1, 2, \dots, n$, and $\partial_i = \lambda_i\left(\frac{\widehat{M} + \widehat{M}^T}{2}\right)$, $\eta_i = \lambda_i\left(\frac{M_0 + M_0^T}{2}\right)$, in descending order.

$$\varepsilon_{\max} = \max\left(\lambda_i\left(E_{\Delta A} - \frac{\Delta A + \Delta A^T}{2}\right)\right), \quad \varepsilon_{\min} = \min\left(\lambda_i\left(E_{\Delta A} - \frac{\Delta A + \Delta A^T}{2}\right)\right)$$

Because: $\widehat{M} = M_0 + (E_{\Delta A} - \Delta A)$, so $\frac{\widehat{M} + \widehat{M}^T}{2} = \frac{M_0 + M_0^T}{2} + (E_{\Delta A} - \frac{\Delta A + \Delta A^T}{2})$

Available from the lemma: $\eta_i + \varepsilon_{\max} \leq \partial_i \leq \eta_i + \varepsilon_{\min}$

Because of $\partial_i < 0$, and each element of $(E_{\Delta A} - \frac{\Delta A + \Delta A^T}{2})$ is non-negative, then

$0 \leq \varepsilon_{\min} \leq \varepsilon_{\max}$, so $\eta_i < 0$, and then $\text{Re } \lambda\left(\frac{M + M^T}{2}\right) < 0$, so the system stability. q. e. d.

The definition of the minimum upper bound of the set of matrices in the theorem and the algorithm for solving the least upper bound of the set of matrices can be found in the literature [7]. If the uncertainty parameter in ΔA is N, then the number of vertices of ΔA is 2^N . And it is noted that in the processing of the uncertainty term $(\Delta A_i + \Delta A_i^T)/2$, the minimum upper bound matrix $E_{\Delta A}$ is adopted, thereby improving the possibility that the robust control has a solution, and also effectively reducing the size of the P -array.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} U + \begin{bmatrix} 0 \\ \delta_x(x_1, x_2, x_3) \\ 0 \\ \delta_y(x_1, x_3, x_4) \end{bmatrix} \tag{10}$$

Assume: $\delta_x(x_1, x_2, x_3) = \Delta_x(x_1, x_3) - \theta_x x_2$, $|\Delta_x(x_1, x_3)| \leq \beta_{x1}|x_1| + \beta_{x2}|x_3|$

$\delta_y(x_1, x_3, x_4) = \Delta_y(x_1, x_3) - \theta_y x_4$, $|\Delta_y(x_1, x_3)| \leq \beta_{y1}|x_1| + \beta_{y2}|x_3|$

In which: θ_x, θ_y take values in the interval $[-\theta, +\theta] \in R$, $\beta_{x1}, \beta_{x2}, \beta_{y1}, \beta_{y2}$ take values in the interval $[-\beta, +\beta] \in R$.

$$\dot{X} = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \beta_{x1} & -\theta_x & \beta_{x2} & 0 \\ 0 & 0 & 0 & 0 \\ \beta_{y1} & 0 & \beta_{y2} & -\theta_y \\ 0 & 0 & 0 & 0 \end{bmatrix} \right) X + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} U$$

According to the theorem:

$$E_{\Delta A} = \begin{bmatrix} \beta & -\frac{\theta}{2} & \frac{\beta}{2} & 0 \\ -\frac{\theta}{2} & 0 & 0 & 0 \\ \frac{\beta}{2} & 0 & \beta & -\frac{\theta}{2} \\ 0 & 0 & -\frac{\theta}{2} & 0 \end{bmatrix} \quad M = [(A + A^T) / 2] + E_{\Delta A} = \begin{bmatrix} \beta & \frac{1-\theta}{2} & \frac{\beta}{2} & 0 \\ \frac{1-\theta}{2} & 0 & 0 & 0 \\ \frac{\beta}{2} & 0 & \beta & \frac{1-\theta}{2} \\ 0 & 0 & \frac{1-\theta}{2} & 0 \end{bmatrix}$$

Select the real symmetric matrix R, Q to solve the equation (9), and set the solution to P_0 , then the robust feedback control law of the system is $U = -R^{-1} B^T P_0 X$

5. Simulation studies

Set system parameter $\mu_0 = 4\pi \times 10^{-7}, \mu_r = 700, L_1 = 0.1000 \text{ m}, L_2 = 0.0167 \text{ m}, d = 0.0500 \text{ m}, m = 0.5000 \text{ kg}, h = 0.0083 \text{ m}, N = 100, A_1 = 0.01 \text{ m}^2$. Assume that the system uncertainty is $\delta_x(x_1, x_2, x_3) \leq \beta|x_1| + \beta|x_3| - \theta x_2, \delta_y(x_1, x_3, x_4) \leq \beta|x_1| + \beta|x_3| - \theta x_4, \beta = 1.1, \theta = 0.01$.

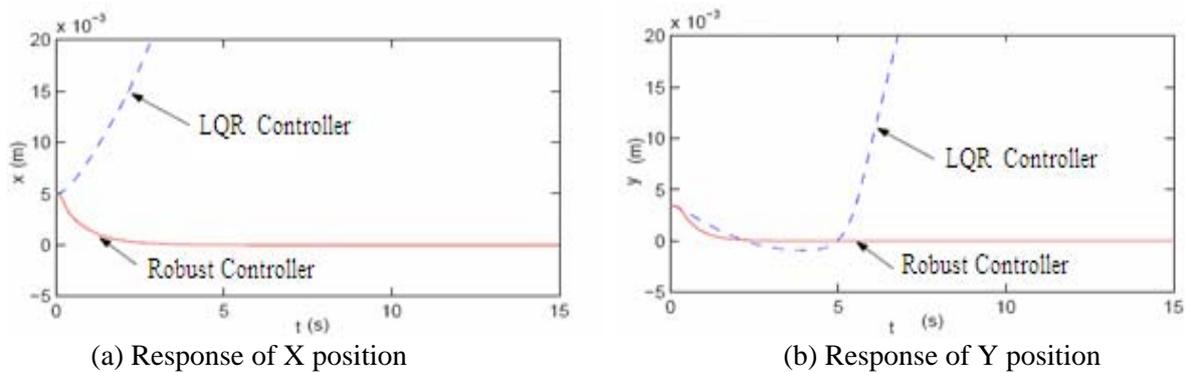


Figure 2. Simulation results

When simulating the two controllers designed above, the sampling period is 0.5ms, and the same weighting matrix is selected:

$$Q = \text{diag}\{5000, 100, 700, 2000\}, R = \begin{bmatrix} 5000 & 100 \\ 100 & 5000 \end{bmatrix}$$

The simulation results are shown in Fig. 2. It can be seen that the precise linearized LQR controller of the above design is unstable, and the robust controller is not only stable but the steady-state error is very small.

6. Conclusion

In this paper, a planar magnetic levitation system is proposed and a mathematical model is established. A robust controller is designed for this nonlinear uncertain system. The simulation results show that the robust controller based on interval matrix design has strong stability robustness and performance robustness to system structural uncertainty, and has fast tracking performance.

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