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Nonlinear System Identification of Structures Subject to Stochastic Loadings

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Abstract. This paper intends to explore the application of the Reverse Multiple Input Multiple Output (R-MISO) technique in the system identification of nonlinear structural behaviour. This paper introduces two concepts in improving the identification estimates of multiple inputs and nonlinear systems, which is the conditioned spectral estimation technique and the inversion of the frequency response function formulas. In this paper, a linear equation of motion with a nonlinear Duffing oscillator is modelled and tested using this technique in order to model the typical nonlinear stiffness characteristics of materials and their design limit. The results indicate a good fit between the predicted physical parameters and the model properties. The nonlinearities were successfully isolated in a non-iterative procedure and the parameters were correctly identified. Multiple coherency was used as a tool to describe the various contributions of the inputs to the system outputs.

1. Introduction

Structural monitoring of dynamically sensitive structures subject to dynamic excitation has been the domain of research in the area of structural identification. System identification provides a non-destructive method in assessing the physical parameters and condition of the structure in a pragmatic monitoring campaign. The method discussed in the paper herein is the Reverse Multiple Input Multiple Output (R-MISO) technique. R-MISO was investigated in 1998 in an attempt to exclusively isolate nonlinear signals from the general response signal. This utilizes MISO formulations in order to accommodate one or more nonlinear functions on top of the base linear function. This enabled the parametric identification of nonlinear and linear functions of the system independent of any leakage effects onto each other. The paper had tested nonlinearities up to fifth order using polynomial functions and were excited using general Gaussian functions [1]. In addition, frequency-dependent nonlinear variables are able to be identified using the R-MISO method. Compared to the prior Reverse Path (RP) method which did not incorporate spectral conditioning of the inputs, an independent review paper [2] had indicated that the R-MISO method was even effective at locations away from the source of nonlinearity. It was also highlighted that the R-MISO method is suited to analysis of MDOF systems, which is far more representative of actual structural systems compared to other more idealized or lumped mass assumptions [3]. An issue pointed out on the method was the requirement for the nonlinearity's characteristic to be identified prior (Richards & Singh, 1998). Nonlinear functions to approximate nonlinear motion especially related to material nonlinearity include cubic



polynomials (Duffing oscillation) [4] to describe nonlinear restoring forces [5-9]. The characteristics of structures being excited to their design limit typically will exhibit nonlinear conditions and is manifested in the plastic performance and response of the materials. Various nonlinear models can be simulated via established models such as the Duffing, Van der Pol, Dead-band and Mathieu systems. The challenge of system identification thus lies largely in the capability of the system identification technique. These nonlinearities are often difficult to be simulated in analytical models and present a large range of assumptions to the analyst. The detailed monitoring of structural components subjected to nonlinear responses can also be prohibitive from a practical standpoint, thus further deterring the identification of exact nonlinear properties. As such, the monitoring of general excitation and response and the subsequent identification using the R-MISO technique is a more practical solution to identifying exact nonlinear system properties as full-scaled measured data can be obtained from excitation levels below the design level limits, thus enabling identification of damage even at operational levels.

2. Reverse MI/SO Theory

The linear relationships between single inputs and outputs of structural systems through the identification of frequency response functions (FRF) of the multiple degrees of freedom (DOF). This methodology assumes that the structural system behaves in a constant-parameter linear fashion. It further assumes that the total output response, $y(t)$ is due to the exclusive linear dependency on the output $v_i(t)$ arising from a single input component $x_i(t)$. This assumption becomes a limitation when there is a contribution of more than one input, $x_i(t)$ on the predicted response in a particular degree of freedom on the structure. Take for example, the torsional response of a structure can be correlated with the bending responses in both the weak and strong plane of the structure. In multiple input / single output (MISO) the output response, $y(t)$ is the sum of linear outputs, $v_i(t)$, $i = 1, 2, \dots, q$, in addition to any noise and unaccounted input responses, $n(t)$ as shown in Figure 1. Whereby the linear outputs, $v_i(t)$ are as a result of q measureable inputs, $x_i(t)$, $i=1, 2, \dots, q$ being operated through a linear system of $H_i(f)$, $i=1, 2, \dots, q$. Due to the linear nature of analysis in the chapter herein, it is assumed that the noise or unaccounted term $n(t)$ is due to random noise as well as unaccounted nonlinear phenomena in the system. The Fourier transforms of the model that is illustrated in Figure 1 can be described with Eq. 1 and Eq. 2 where $Y(f,T)$, $V_i(f,T)$, $N(f,T)$ and $X_i(f,T)$ are Fourier transforms of $y(t)$, $v_i(t)$, $n(t)$ and $x_i(t)$ respectively.

$$Y(f, T) = \sum_{i=1}^q V_i(f, T) + N(f, T) \quad \text{Eq. 1}$$

$$V_i(f, T) = H_i(f) X_i(f, T) \quad i = 1, 2, \dots, q \quad \text{Eq. 2}$$

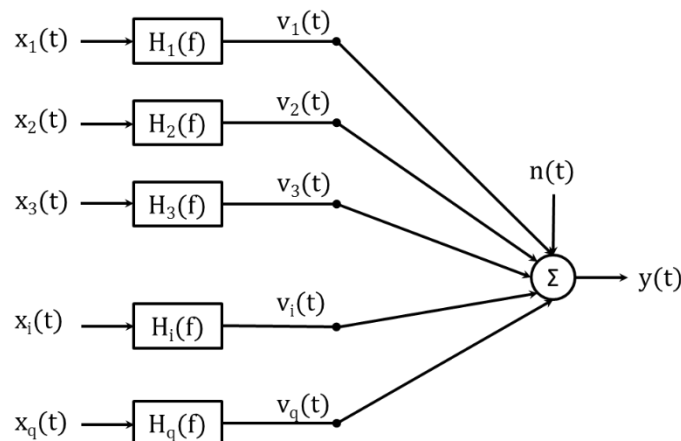


Figure 1: Illustration of multiple input / single output model with unknown noise inputs

In a MI/SO system, the correlation between various inputs $x(t)$ and the output term $y(t)$ in the frequency domain can be represented as the cross-spectral terms between $x(t)$ and $y(t)$ represented by S_{xy} and of two different input records of $x(t)$, represented by $S_{x_i x_j}$ for a record length of L divided into n_d segments.

$$S_{x_i x_j}(f) = \frac{2}{n_d L} \sum_{m=1}^{n_d} X_{im}^*(f) X_{jm}(f) \quad \text{Eq.3}$$

$$S_{xy}(f) = \frac{2}{n_d L} \sum_{m=1}^{n_d} X_m^*(f) Y_m(f) \quad \text{Eq.4}$$

Under the following (2) assumptions that $n(t)$ is uncorrelated with the inputs of $x_i(t)$ and that the inputs $x(t)$ are not correlated with each other, we find that the following equations Eq.5 to hold true.

$$S_{yy}(f) = \sum_{i=1}^q |H_i(f)|^2 S_{x_i x_i}(f) + G_{nn}(f) \quad \text{Eq.5}$$

Such an ideal SI//SO situation can only exist when an input $x(t)$ exactly describes the output $y(t)$ passing through the frequency response function (FRF) H_i since it has no relationship with other inputs $x_i(t)$ thus causing the frequency response function of $H_{ij}(f) = 0$ for all cases where $i \neq j$. The uncorrelated Fourier transform of noise is represented as G_{nn} . However, such an ideal situation does not occur in a structural system since the responses are correlated to each other through a process of both linear and nonlinear coupling. This will cause several problems in the system identification process through the spectral estimates derived since no meaningful physical interpretation can be made. As such, the process of conditioning the auto-spectral and cross-spectral estimates of all inputs $x_i(t)$ will allow the linear correlated effects to be effectively removed, thus resulting in cross linear function, $H_{ij}(f) = 0$; and allowing $S_{yy}(f)$ estimates to be a function of the true summation of all inputs $x_i(t)$. In a conditioned model, the FRF function H_i is replaced with the optimal linear functions of L_{iy} , where the subscript i represents the one or multiple uncorrelated inputs to the system up to q number of total of inputs. This is illustrated diagrammatically in Figure 2.

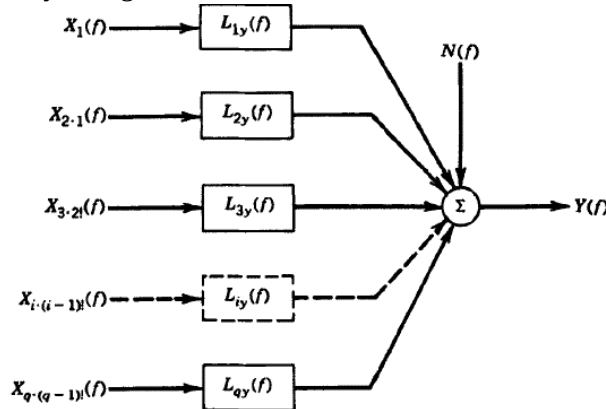


Figure 2: Conditioned MI/MO system model consisting of MI/MO optimal linear functions

Figure 2 illustrates a q number of conditioned inputs that go through optimal functions of L_{iy} to obtain the total output signal. The inputs in Figure 2 can be summarized as $X_{i,(i-1)}$ where it indicates that the input X_i is a conditioned input of i with the linear effects of $x_1(t)$ up to $x_{i-1}(t)$ removed from X_i . For example $X_{3,2}$ represents the conditioned input of $x_3(t)$ with the linear effects of $x_1(t)$ and $x_2(t)$ removed. The consequent residual inputs are uncorrelated for all arbitrary conditions. The mathematical expression of Figure 2 can be represented in Eq.6.

$$Y = \sum_{i=1}^q L_{iy} X_{i.(i-1)} + N \quad \text{Eq.6}$$

Optimum linear functions form an integral part of the analysis of MI/MO system models and can be defined as such based on Eq.7 in general mathematical terms for all inputs $i = 1, 2, \dots, q$ whereby the inputs of the system are conditioned such that they are uncorrelated and have a cross-spectral density value of zero across all frequencies of measure. It is defined as the ratio of the cross-spectral density between the output signal and the input signal and the auto-spectral density of the input variable.

$$L_{iy}(f) = \frac{S_{iy.(i-1)}(f)}{S_{ii.(i-1)}(f)} \quad \text{Eq.7}$$

In order to derive the solutions to a conditioned output as a function of the input functions, we first take a look at Eq.6 whereby the output $Y(t)$ is considered an extension of the input terms $X(t)$ by an increased index of $q+1$, yielding the term $X_{(q+1)}$. All noise terms are treated as a conditioned input of the output term $X_{(q+1)}$ with linear effect previous input terms removed, yielding $X_{(q+1).q!}$ as in Eq.8.

$$X_{(q+1)} = \sum_{i=1}^q L_{i(q+1)} X_{i.(i-1)} + X_{(q+1).q!} \quad \text{Eq.8}$$

We now assign the term r to preceding inputs in the system to allow for conditioning of any order of inputs prior to the term $q+1$ whereby $r \leq q$. This yields the following Eq.9. We subsequently substitute $q+1$ terms with a more general j term to yield Eq.10. Consequently, this equation represents for any j th record in the system rather than specifically the output term $q+1$.

$$X_{(q+1)} = \sum_{i=1}^r L_{i(q+1)} X_{i.(i-1)} + X_{(q+1).r!} \quad \text{Eq.9}$$

$$X_j = \sum_{i=1}^r L_{ij} X_{i.(i-1)} + X_{j.r!} \quad \text{Eq.10}$$

We can consequently conclude that the conditioned output for any term $X_{j.r!}$ is a product of the term X_j less the preceding terms of $L_{ij} X_{i(i-1).r}$. To modify this equation for any subset of X_j , we consider the following Eq.11. Converting the formula into the frequency domain by multiplying the complex conjugate X_i^* , expected value and scale factor to yield Eq.12.

$$X_{j.r!} = X_{j.(r-1)!} - L_{rj} X_{r.(r-1)!} \quad \text{Eq.11}$$

$$S_{ij.r!} = S_{ij.(r-1)!} - L_{rj} S_{ir.(r-1)!} \quad \text{Eq.12}$$

Eq.12 has the condition that the subscripts $i > r$ and $j > r$ must be satisfied to account for all possible subsets that can occur for any given number of q arbitrary inputs. The equation can be explained as such, whereby $\{S_{ij.r!}\}$ can be represented by the precedent terms of $\{S_{ij.(r-1)!}\}$ and precedent optimal linear function of $\{L_{rj}\}$. This loop will iterate itself for all instances of $r = 1, 2, 3 \dots q$. For example, the term $S_{ij.1}$ is a function of $S_{ij.1}$ and L_{1j} when $r=1$. When $r=2$, $S_{ij.2}$ now becomes a function of $S_{ij.1}$ and L_{2j} . When the condition of $i = j$, taking the conditions from Eq.7 and Eq.12, we yield the following formula Eq.13.

$$S_{ii.r!} = S_{ii.(r-1)!} - |L_{ri}|^2 S_{rr.(r-1)!} \quad \text{Eq.13}$$

The matrix representation of the above Eq.12 can be illustrated as a 3-dimensional matrix whereby the first and second dimension are represented by the subscripts i and j while the third dimension represents the iteration of system across all frequencies of measure or concern, typically up to $f_{N/2}$ as limited by the Nyquist frequency theorem. The conditioning process is thus complete with the resolution of the equations Eq.12 and Eq.7 for all inputs and outputs of the system.

3. System Coherency

Coherency are a set of functions to measure the linear dependency of an output as a function of its inputs. As opposed to adopting ordinary coherence functions of a SI/SO, the output of $y(t)$ due to $x_i(t)$ is not exclusively due to the contributions of the $x_i(t)$ input alone, and has to pass through the system

frequency response function of $H_2(f)$ as well. This condition occurs when the inputs of $x_1(t)$ and $x_2(t)$ are correlated thus resulting in the quantity of S_{12} and S_{21} to not equal to zero. Consequently, $\gamma^2_{12}(f)$ is also a nonzero for this condition. This condition also holds true for the output of $y(t)$ not due to the exclusive input of $x_2(t)$ alone. As discussed in the section on conditioned spectral estimates, correlated inputs will consequently result in $\gamma^2_{xy}(f) > 1$ for the summation of all q input coherencies, $\sum_{k=1}^q \gamma^2_{ky}(f)$ which yields no useful and physically interpretable results. A partial coherence function can thus be defined as a function of its' conditioned inputs to the system, for all inputs $i=1,2,3...q$ and iterations of $r=1,2,3...q$, whereby $r=i-1$ as in Eq. 14

$$\gamma^2_{iy.r!}(f) = \frac{|S_{iy.r!}(f)|^2}{S_{ii.r!}(f)S_{yy}(f)} \quad \text{Eq. 14}$$

The concept of multiple coherencies is simply the percentile of the mean square value of the multiple inputs $x(t)$ describing the output $y(t)$ for at any frequency f . Since the concept of conditioned spectral estimates has been discussed extensively, we shall consider the multiple coherence function as a specific condition whereby the inputs are uncorrelated and independent of each other (conditioned inputs). Eq. 15 illustrates the general form of a multiple coherence function for q number of ordinary coherence functions of uncorrelated arbitrary inputs.

$$\gamma^2_{y:q}(f) = \sum_{i=1}^q \gamma^2_{iy.r!}(f) \quad \text{Eq. 15}$$

Whereby $0 \leq \gamma^2_{y:q}(f) \leq 1$, for all ranges of frequency, f . The partial coherence function is consequently defined as the coherence functions of the input that has been conditioned for the linear effects of other inputs in the system to be removed. The ordering of inputs can thus be decided based upon the permutation which the first input yields the highest ordinary coherence function and so on and so forth. It is of further note that during the analysis of the ideal upper and lower bound conditions, the permutation of choice shall be determined based on the frequency response range of concern.

4. Equations of Motion and System Identification

Physical system identification is the process of utilizing statistical and mathematical models to describe the dynamically time-varying and occasionally frequency dependent physical systems. The linear FRFs of H_{iy} are described using a set of equations of motions with unidentified parameters of mass, damping and stiffness induced by some forcing function. In the R-MISO technique, the roles of system input and outputs are reversed; as such the imposed force in a particular DOF is the input of the system $x(t)$ while the response of the structure is the output of the system $y(t)$. The identification of the physical parameters is now discussed based on the linear FRF of H_{iy} . The physical parameters are based on the linear equation of motion for a single-DOF system for any one dynamic system as shown in Eq. 16

$$(m)\ddot{y}(t) + c\dot{y}(t) + ky(t) = x(t) \quad \text{Eq. 16}$$

m = mass matrix
 c = linear damping matrix
 k = linear spring stiffness matrix
 $x(t)$ = time-varying excitation input (force)
 $y(t)$ = excitation output (y and $\ddot{y}$ represents its differenced value)

In such a condition the term H_{iy} in the R-MISO theory can be identified based on the following Eq. 17. This is an inverse of the conventional system identification method and is also known as the system's dynamic stiffness function.

$$H_{iy}(f) = k_{yi} - (2\pi f)^2(m) + j(2\pi f)c_{yi} \quad \text{Eq. 17}$$

In order to identify the various unknown physical parameters in the system, the following mathematical relationships and asymptotic condition are defined in Eq. 18 to Eq. 22.

$$\text{Re}[H_{iy}(f)] = [k_{yi} - (2\pi f)^2(m)] \quad \text{Eq. 18}$$

$$\text{Im}[H_{iy}(f)] = [(2\pi f)c_{yi}] \quad \text{Eq. 19}$$

$$\text{Re}[H_{iy}(f)] = k_{yi} \quad \text{when } f = 0 \text{ (static condition)} \quad \text{Eq. 20}$$

$$- \text{Re}[H_{iy}(f)] / (2\pi f)^2 = m \quad \text{for all values of } f \quad \text{Eq. 21}$$

$$\text{Im}[H_{iy}(f)] / (2\pi f) = c \quad \text{for all values of } f \quad \text{Eq. 22}$$

$$\zeta = c / (2\sqrt{km}) \quad \text{Eq. 23}$$

In order to account for nonlinear phenomena within the structure, Eq. 16 is expanded to include a nonlinear term (Bendat, Palo, & Coppolino, 1992) (Bendat et al., 1992). In this case, a nonlinear cubic terms is selected, representing the Duffing oscillator properties of a structural material under design level conditions when plasticity is achieved; this is otherwise known as strain hardening. This is represented as the nonlinear term, $Q(t)$ in Eq. 24 or otherwise known as non-linear frequency-dependent stiffness. A cubic polynomial terms is used to describe the characteristics of the Duffing oscillator.

$$(m)\ddot{y}(t) + c\dot{y}(t) + ky(t) + Qy^3(t) = x(t) \quad \text{Eq. 24}$$

$$Y(f) = X_L A_1 + X_{NL} A_2 \quad \text{Eq. 25}$$

X_L = linear input term

X_{NL} = nonlinear input term

In the R-MISO technique, the outputs and inputs are arranged as in Figure 3 whereby two parallel conditioned inputs are summed to produce the output term. This is reflected in Eq. 25 as a parallel linear and nonlinear equations of motion. In addition, we can define the amplitude of the system as in Eq. 26 and subsequently the damping ratio as a function of amplitude as in Eq. 27.

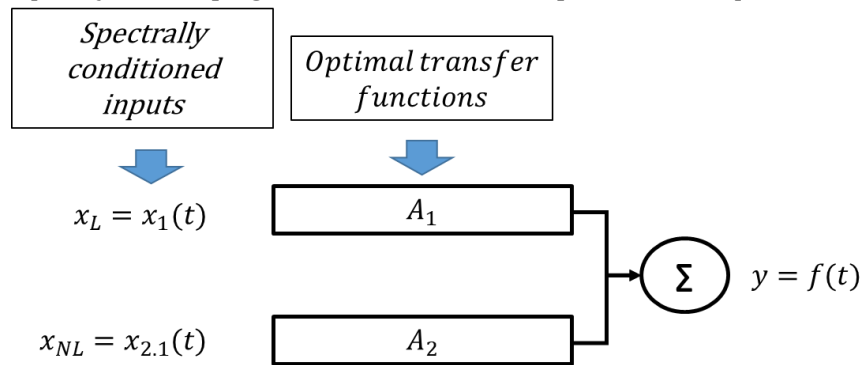


Figure 3: Two input single output model (with linear and nonlinear equations of motion)

$$|A_{iy}(f_n)| = \sqrt{(k - (2\pi f)^2 m)^2 + (2\pi f c)^2} \quad \text{Eq. 26}$$

$$\zeta = |H_{iy}(f_n)| / 2k \quad \text{Eq. 27}$$

For a R-MI/SO nonlinear model with the term A_2 as in Eq. 25, the system identification is simply defined as such in Eq. 28.

$$A_{NL}(f) = k_{NL}(f) \quad \text{Eq. 28}$$

$$k_{NL}(f) = \text{frequency-dependent nonlinear stiffness}$$

5. Time Domain Simulation and Results

The following single DOF oscillator is adopted in putting the R-MISO theory to test, this is illustrated in Figure 4a and Figure 4b whereby an initial control test is made on a linear SDOF system while a

subsequent test is performed on a nonlinear oscillator exhibiting Duffing oscillation properties which is modelled with a $y^3(t)$ term as described in Eq. 24. The latter nonlinear test requires the deployment of both the spectral conditioning method to remove cross-correlations between the linear and nonlinear inputs. The parameters of both systems are described in Table 1. The only difference between both Case 1 and Case 2 is the presence of a nonlinear spring which has a stiffness of 5 N/m^3 .

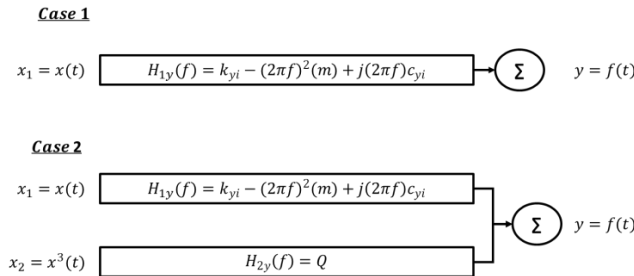


Figure 4: a) Linear SDOF oscillator, b) Duffing type nonlinear SDOF oscillator

Figure 5 illustrates the autospectral plot of the forces imposed on the SDOF system which forms the input while the output displacement response forms the outputs. A Gaussian excitation force is applied on to the system, yielding a resonant peak at approximately 7.54 rad/s . This peak is the eigenvalue of the system given there is no forcing harmonic function on the SDOF system which may excite other modes or result in a forced oscillation. The signal processing for the results herein are executed using the Welch periodogram method with a Hanning window function applied. A 50% data overlap across 8 data segments is applied to the overall data length.

Table 1: System properties of linear and nonlinear SDOF oscillator

System parameters	Case 1 (Linear)	Case 2 (Nonlinear)
linear stiffness, k	454.8 N/m	454.8 N/m
linear damping, c	6.03 Ns/m ($\zeta = 5.0\%$)	6.03 Ns/m ($\zeta = 5.0\%$)
mass, m	8 kg	8 kg
system eigenvalue	1.2 Hz (7.54 rad/s)	1.2 Hz (7.54 rad/s)
nonlinear stiffness, Q	0 N/m^3	5 N/m^3

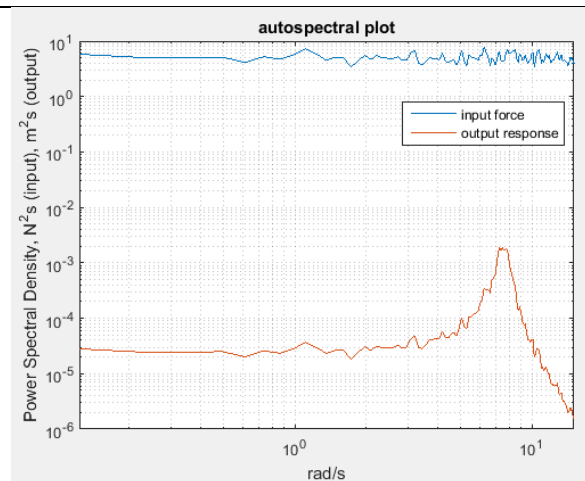


Figure 5: Autospectral densities of the system inputs and outputs

The results of the R-MISO linear identification method are illustrated in **Error! Reference source not found.** shows the dynamic stiffness plot, H_{1y} which represents the linear frequency response function of the SDOF system. At the frequency of $f = 0$, the system is assumed to be static and the dynamic stiffness value represents the system's static stiffness. The R-MISO identified value 454.75 N/m shows a 0.01% error from the analytical model. The system minimum also correctly indicates the eigenvalue of the system at 7.54 rad/s although this is only applicable due to Gaussian excitation input. The phase angle plot in **Error! Reference source not found.b** confirms the presence of the eigenvalue with a phase shift at the corresponding frequency. Applying Eq. 21 and Eq. 22 on to the real and complex components of H_{1y} respectively yields the linear damping coefficient and mass curve plots respectively seen in **Error! Reference source not found.c** and **Error! Reference source not found.d**. The damping coefficient exhibits a frequency independence which is typical in linear systems operating in negligible viscous effects condition. An average value of 6.11 Ns/m was identified by the R-MISO method, resulting in a 1.33% error. It is noted that there is a drift of the damping plots from at the frequencies of 10 rad/s onwards. The mass curves also show a good agreement with theoretical values with an identified value of 8.16 kg, yielding an error of 2.00%. Figure 7 is the linear identification portion of the equations of motion described in Figure 3. It has been spectrally conditioned to remove the linear leakage between the system inputs $x(t)$ and $x^3(t)$ with the latter forming the input to the nonlinear equations of motion. Results in Figure 7 show that the linear component of the nonlinear system in Case 2 has been successfully conditioned and isolated from the nonlinear effects, thus producing almost identical results as in **Error! Reference source not found.**

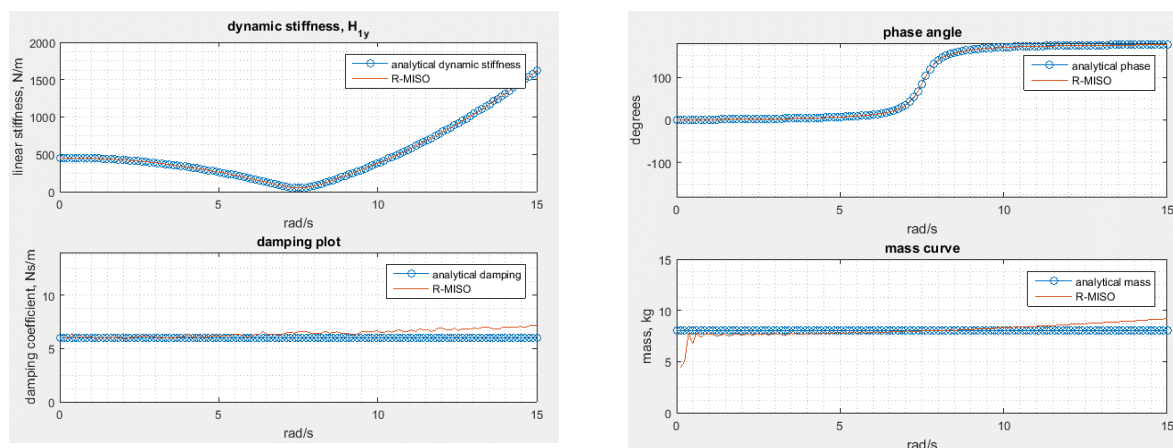


Figure 6: a) dynamic stiffness H_{1y} (Case 1), b) phase angle plot, c) damping plot, d) mass curve

Figure 8 shows that the identification of the Duffing-type nonlinearity is relatively accurate with an average of approximately 5.12 N/m^3 , yielding an error of 2.4%. There is however a noticeable variance of the frequency-independent nonlinear plot which is attributed to the amplification of the magnitude of errors introduced when converting a $x^3(t)$ output response signal to the frequency domain. Figure 9 shows the multiple coherency plot of the Case 2 equations of motion whereby the conditioned linear equations of motion shows a high degree of correlation between the input and outputs with the exception of the range below 0.2 rad/s whereby there is some influence of nonlinear equations of motions in describing the system output motion. We can describe that from the frequency range up to 0.2 rad/s the excitation of the structure linearly describes approximately more than 90% of the responses of the total structural system.

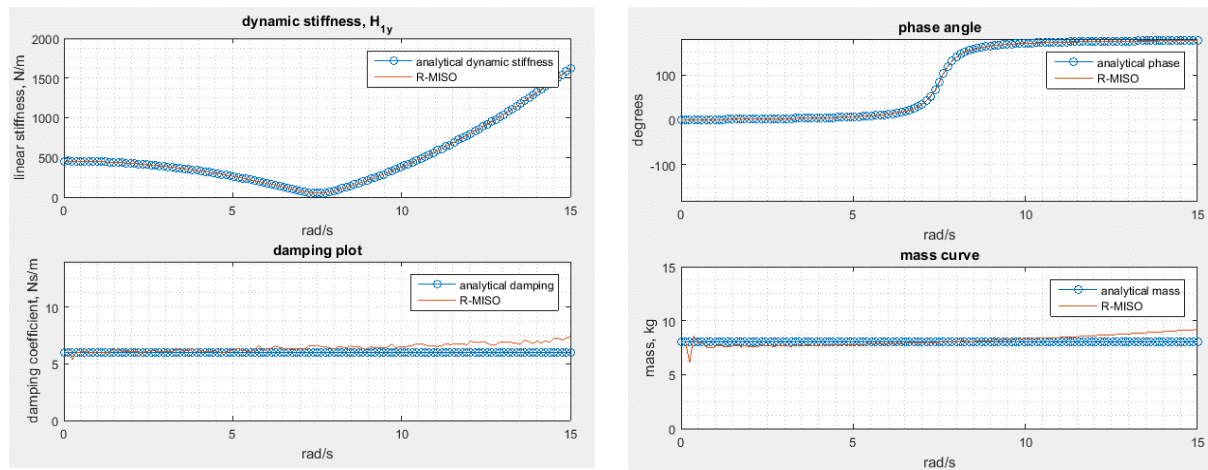


Figure 7: a) dynamic stiffness H_{1y} of a conditioned spectrum in Case 2, b) phase angle plot, c) damping plot, d) mass curve

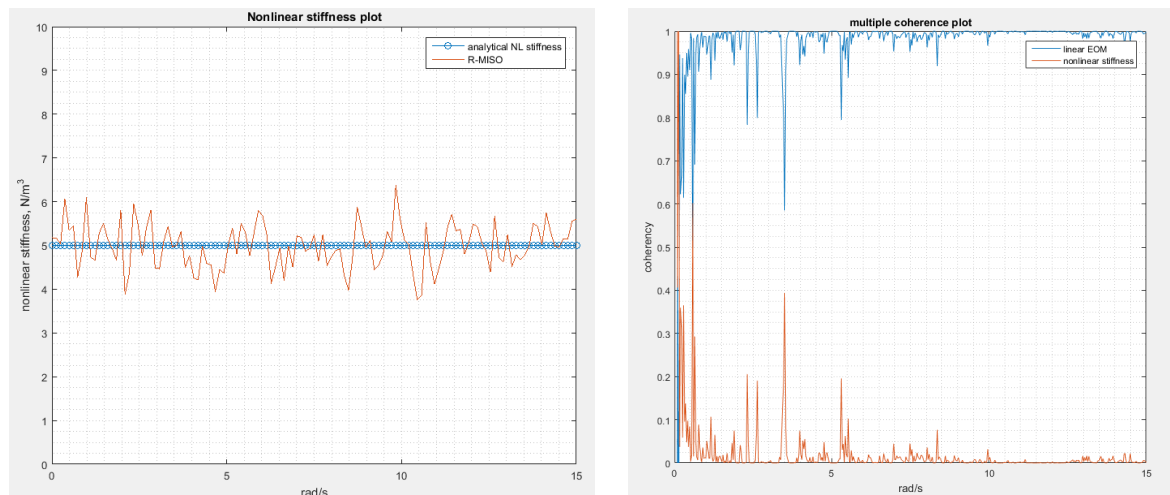


Figure 8: Nonlinear stiffness plot from R-MISO identification of nonlinear system, Case 2

Figure 9: Multiple coherence plots of nonlinear system, Case 2

6. Conclusions

It is observed that the R-MISO technique exhibits high accuracy in detecting and parameterizing nonlinear components through a parallel conditioned spectral modelling of the structure's equations of motions. Multiple coherence is also an efficient tool in describing the contributions of both linear and nonlinear equations of motions on the overall motion of the structure over a prescribed frequency spectrum. This allows us to identify highly nonlinear behaviour that occurs over a specific frequency band. This paper is intended to be an exploratory and theoretical test of the R-MISO's capabilities and is intended to be followed up with a full-scale structural modelling, analysis and identification using the R-MISO technique subject to varying loadings including base excitations and forced harmonic oscillations.

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