Profile of students' mathematical representation ability in solving geometry problems

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Abstract. Mathematical representation is students' ability in expressing mathematical ideas in the form of drawings, mathematical equations, and words/written text. Mathematical representation is needed in solving mathematical problems. This research aims to find out the profile of students' mathematical representation in solving geometry problems. The research method uses descriptive with subjects of 33 students of class VIII Junior High School Muhammadiyah 1 Surakarta. Data is collected by a mathematical representation test. Data analysis show that there are 21.21% of students with high representation ability, 33.33% students with moderate representation ability, and 45.46% of students with low representation ability. Based on the results, it is known that most of the students still have low mathematical representation ability. Students find it difficult to understand problems and write equations correctly. This is because students are not used to solving problems in the form of visual, verbal, and symbolic representations. Therefore, the teacher must train the students' mathematical representation skills by applying multiple representation learning models.

1. Introduction

Mathematics has an important role in supporting the development of science, technology, and communication. Mathematics is a compulsory subject that must be taught to elementary and even middle school level so that the ability and skills of students can be well enhanced [1]. By learning mathematics, information can be easily processed. Not all students can process the information obtained well because the students' ability differs from one to another. Moreover, every student has a different perception and view in learning mathematics. Many students have difficulty learning mathematics. Students assume mathematics is a frightening and boring subject [2]. The mentioned aspect is one of the reasons behind the low quality of student mathematics learning. The percentage of the students' material mastery in Junior high school Muhammadiyah 1 Surakarta in the national exam is still low. The percentage in the least sizable geometry material com- pared with other tested capability is 55.06% at the regency level, 45.78% at the provincial level, and 48.57% at the national level [3]. This fact reveals that there are still students who have not fully mastered the geometry material. Geometry is one of the branches of mathematics that deals with the nature and relationship of lines, angles, curves, and shapes, etc. [4]. Geometry is included in the school curriculum around the world due to its benefits and applications in real life [5]. Studying and learning geom etry is not an easy journey, most of the students fail to develop an understanding of geometric concepts, geometric reasoning, and skills in solving geometric problems [6]. Thus, it is important for them to develop skills in solving geometrical problems [7].

Skills in solving with geometry problems can be developed through students' mathematical abilities. NCTM reveals that there are 5 mathematical abilities, one of which is representation ability [8].

Brenner states that successful problem-solving processes depend on skills in representing problems such as constructing and using mathematical representations in symbol words, graphics, tables, equations, solutions, and symbol manipulation [9]. One of the indicators of achieving the competence based on *Permendikbud* is presenting concepts in various forms of mathematical representation such as forms of tables, graphs, diagrams, drawings, sketches, mathematical models, or other means [10]. Representation is one of the abilities that students should possess. The ability of representation facilitates students to understand, communicate, and connect mathematical concepts [11][12]. Representations are employed by students as a tool for understanding mathematics by building abstract ideas into concrete ideas using logical thinking [12]. Meaning that the ability to represent is one of the abilities that must be possessed by students to be developed. Representation ability is the center of mathematics learning so that students can develop and deepen the concept of mathematical understanding and its relationship with making, comparing, and using representations in various forms [8]. In mathematics learn- ing, teachers often use many representations to facilitate teaching mathematical concepts [13]. The use of representation forms facilitates students to solve geometry problems. The representation forms used include visual representations (images), symbolic representations (equations or mathematical expressions), and verbal representation (words or written texts) [8]. The use of multiple representations can be a more effective tool in demonstrating different aspects of mathematical concepts [14]. Hence, the current study aims to determine the profile of students' mathematical representation in solving geometry problems.

2. Method

The method employed in this research is the descriptive method. In the descriptive method, descriptive statistics are used to process data obtained within the research process. Descriptive statistics are statistics pertaining to how to describe or decipher data to elaborate it [15]. The subjects of this research are the students of class VIIIA Junior High School Muhammadiyah 1 Surakarta. The pur- posive sampling technique is used. Purposive sampling is characterized by the selection of research subjects by using certain considerations held by the researchers [16]. Data collection is done through the test of mathematical representation. Data obtained from the results of mathematical representation tests are then scored based on scoring criteria, calculated using the formula be- low [17]:

$$N_i = \frac{x_i}{s_i} \times 100 \qquad (1)$$

Expansions:

N_i: the score of students' representation ability
X_i: the total score obtained by the students
S_i: the total of the maximum score

To determine the category of the students' representation ability level, their representation ability score is converted to qualitative form by considering the categorization guidance as shown in Table 1 below.

Table 1. Category of mathematical representation ability. [18]

Interval	Category
$x_i > \bar{x} + 0.5s$	High
$\bar{x} - 0.5s \leq x_i \leq \bar{x} + 0.5s$	Medium
$x_i < \bar{x} - 0.5s$	Low

Expansions:

x_i: score of the students' ability in each subject
$\bar{x}$: the average score of students in the
s: subject standard deviation of the entire subject

The following is a problem to measure the students' mathematical representation abilities.

The floor of Imam's house is rectangular with the size of 9 m x 6 m. He will buy a brown tile-shaped tile for the floor of his house.

He bought a tile in the shop named "*Enggal Murah*" with a diagonal of 36 cm x 48 cm. Calculate: How many tiles does Imam have to buy and if 1 box contains 5 pieces and the price is Rp.52.500,00/ box, how much does it cost to purchase the tiles!

The next following problem is Rizal will make a kite. He has two pieces of bamboo that will be used as the diagonal of the kite that the wants to make with a length of 48 cm and 50 cm respectively. The short and long diagonals intersect at a certain point, where the mentioned point lies at 18 cm from the longest diagonal's end. He also needs a kite paper to cover the kite shape she has created. The kite paper is extended at its edges 4 cm to cover the kite skeleton. Draw a sketch of the kite shape that Rizal will make and determine the extent of the paper needed by Rizal to make the kite!

The next following problem is trapezoid PQRS bracket at point P. PQ // RS. PQ = 17cm, RS = 11cm and SP = 8cm. Draw a trapezoid sketch of PQRS. Calculate the area and circumference of the trapezoida PQRS

3. Result and Discussion

The data obtained from the results of the students' mathematical representation ability test are based on aspects of students' mathematical representation (visual representation, symbol representation, verbal representation). The data obtained from the field are analyzed and presented in the form of description as an overview of the research findings. The mentioned data as a whole are presented in Table 2 below.

Table 2. Descriptive statistics of mathematical representation ability.

Interval	Category
Total	2136
Mean	64.72
Standard Deviation	13.44
Maximum	95
Minimum	42

As shown in Table 2. above, the average of students' mathematical representation ability is 64.72, which means that their average of mathematical representation ability is included in the low category. This is due to the fact that students have not received special treatment in the form of learning that trained their mathematical representation skills. The results of this research are similar to the results of other relevant studies which state that the representation abilities of students are included in the low category [19][20]. Students with low representational skills show a lack in skills of generating ideas, asking questions and responding to questions or opinions of others [12].

The mathematics' learning process in the school still emphasizes students' understanding without providing them with the opportunity of exploring representational skills in understanding a mathematical concept. To add, the questions given are made slightly different from the examples usually given by the teacher, therefore, the students are fixated and dependant to the given example. As a result, students are not able to represent their mathematical ideas properly. Students have difficulty in solving problems of mathematical representation [21]. This is because the learning process in the classroom is teacher centered. Teachers often emphasize only aspects of the mathematical process compared to their applications in everyday life [22]. Muhamad & Hashim state that the learning process must involve teachers and students [23]. Therefore, to improve the ability of mathematical representation, students must participate and interact during the learning process [24].

The ability to solve problems depends on students' mathematical representation abilities [25]. Solving geometry related problems require good representation skills. The application of geometric material can be a problem faced daily, requiring good mathematical representation in solving a given geometry problem. The teacher must emphasize geometric material in everyday life [26]. In fact, students encounter difficulties in representing problems into mathematical models. This is relevant to the results of Yanti's research which claims that students find difficulties in solving mathematical representation problems [25]. This is because representation has not been used as a tool for thinking and solving problems. Suryowati claims that students did not understand how to illustrate real world problems into representative mathematical problems [27].
The student's representation ability profile is categorized using Table 1. Meanwhile, its results are then presented in Table 3 below.

Table 3. Category of student' representation ability level.

Category	Frequency	Percentage
High	7	21.21
Moderate	1	33.33
Low	15	45.46

As Table 3 shows, there are 7 students in the category of high representation ability level (21.21%), 11 students in moderate representation ability level (33.33%), and 15 students in low representation ability level (45.46%). There are more students at low representation category than those at the high and moderate category. This is because every problem given in this study contains three indicators of mathematical representation at once. Students are not familiar yet with using these three indicators when working on the given problem. Students usually use more the ways taught by their teachers in solving the same problems [28]. This is because during the learning process students are rarely given the opportunity to express their own representations [25]. Students only listen and see their teacher explain the material. Students rarely ask their teacher about the material they do not understand. Students only do the questions as exemplified by the teacher [19]. Some examples of students' answer to mathematical representation ability test can be seen in Figures 1, 2, and 3.

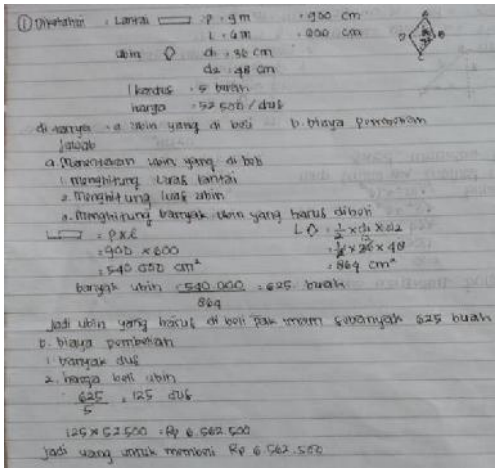
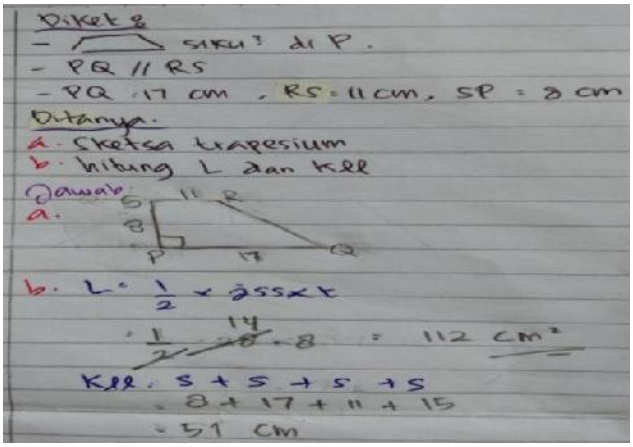


Figure 1. Students' answer in the category of high representation ability.



F**igure 2**. Students' answer in the category of moderate representation ability.

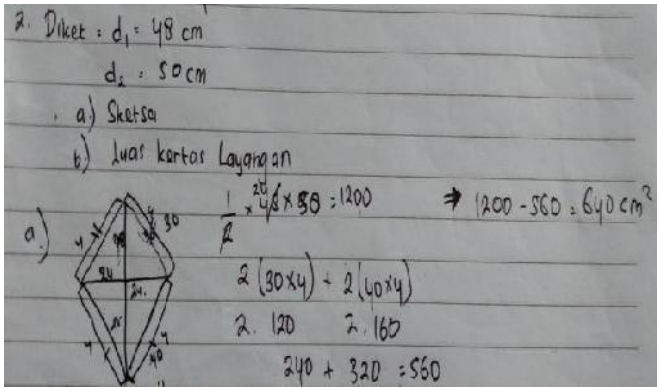


Figure 3. Students' answer in the category of low representation ability.

Figure 1 illustrates students in the category of high representation ability level where students are able to make mathematical ideas in drawings, able to make mathematical equations, and write solving steps by words/written text. The use of various representations can facilitate students in understanding the problem [29]. Sahendra's research result claims that students with high representation ability use more than one form of representation to solve problems [30]. Students can use different representations in solving problems [31]. Figure 2 illustrates the students in the moderate representation level category where students are able to make mathematical ideas in drawings, able to make mathematical equations but unable to write solving steps by words/written text. This result is relevant to the results of Rahmawati's research which states that students who have moderate representation ability are more able to represent problems in symbolic rather than verbal forms [32]. Students who have difficulty representing problems in verbal form will find difficulties in solving problems [31].

Figure 3 illustrates students in the category of low representation ability level where students are able to make mathematical ideas in drawings but unable to make mathematical equations and write solving steps by words/written text. This result is relevant to the results of Sahendra's research which states that students with low representation ability only use one form of representation based on learning experience [30]. The ability of students' mathematical representation is low because students find it difficult to bridge representation and change from one representation to another, especially in the form of words [32]. Low representation ability makes it difficult for students to solve various algebra, geometry, and linear equations because students find it difficult to change

from one type of representation to another [19].

In the research, the students mostly encounter difficulty in answer- ing the problem number 2. They can represent the problem in the form of images, but they find it difficult to solve it, especially in representing it in the form of words. When students use verbal representations, they tend to have difficulty in constructing words [28]. Students do not understand the initial steps that must be written when dealing with the problem. This is because they only memorize the formula, so most of them cannot solve the problem when given a different problem from the sample of the questions they learned. Students' difficulties in understanding mathematics not only come from the students themselves but also from the environment and the teacher. Therefore, teachers can avoid this problem by changing students' perceptions towards mathematics [12]. Integrating representations in learning can lead teachers to enhance their teaching skills.

The mathematical representation ability is important for students in learning mathematics because the way in conveying ideas dif- fers from a student to another. This is relevant to Suryana's research which states that representation skills can help students to understand and communicate the concepts they have learned [11]. Representing information visually is perceived as an efficient representation process in mathematics education, especially in problem solving [8]. According to Owens & Clements in Guler & Ciltas visualization has a significant role in understanding problems, guiding problem-solving methods and affecting mental structure significantly [28]. Teaching students how to use mathe- matical representations in terms of equations and picture representations can facilitate students to describe the explanations written in the form of words [13].

4. Conclusion

The results of this research indicate that the students' representa- tion ability in the high category is 21.21%, moderate category 33.33%, and low category 45.46%. Students with high representation ability are able to create mathematical ideas in drawings, able to make mathematical equations and write solving steps by words/written text. Students with moderate representation ability are able to make mathematical ideas in drawings, able to make mathematical equations but unable to write solving steps by words/written text. Students with low representation ability are able to make mathematical ideas in drawings but unable to make mathematical equations and write solving steps by words/written text. Based on the results, it is known that most of the students still have low mathematical representation ability. Students find it difficult to solve problems in the form of mathematical representations. This research has an implication that the teacher must train students' mathematical representation skills by applying multiple representation learning models. In addition, the teacher must familiarize students with questions in the form of visual, verbal, and symbolic representations.

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References

[1] Depdiknas 2003 *Undang-Undang Republik Indonesia Nomor 20 tahun 2003 tentang Sistem Pendidikan Nasional* (Jakarta: Sinar Grafika)

[2] Ramdhani M R , Usodo B and Subanti S 2017 Student's mathematical understanding ability based on self-efficacy *Journal of Physics: Conference Series,* doi: 10.1088/1742-6596/909/1/012065

[3] BSNP 2017 *Laporan Hasil Ujian Nasional Tahun Pelajaran* 2016/2017

[4] Bayrak N, Yuce S and Yuce M K 2014 The Investigation of the View- point of Academic Staff and Graduate Students in Teaching Geometry in Elementary School *Procedia Social and Behavioral Sciences* **116** p 2115-2119

[5] Arici S and Tutak A F 2015 The Effect of Origami-Based Instruction on Spatial Visualization, Geometry Achievement, and Geometric Reasoning *International Journal of Science an*

Mathematics Educa- tion **13** p 179-200
[6] Saha R A, Ayub A F M and Tarmizi R A *The Effects of GeoGebra on Mathematics Achievement:Enlightening Coordinate*
[7] Ozerem A 2012 Misconceptions in Geometry and Suggested Solution for Seventh Grade Students *Procedia Social and Behavioral Sciences* **55** p 720-729
[8] NCTM 2000 *Principles and standards for school mathematics* (Reston: VA NCTM)
[9] Neria D and Amit M 2004 Students Preference of Non-Algebraic Repre- sentations In Mathematical Communication *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* **3** p 409-416
[10] Kemdikbud 2014 *Permendikbud nomor 58 Tahun 2014 tentang Kurikulum 2013 SMP/MTs Bagian Pedoman Mata Pelajaran (PMP) Matematika* (Jakarta: Kemdikbud)
[11] Suryana A 2014 Improving Mathematical Representation Skill By Using Pace Model *Proceeding of International Conference On Research, Implementation And Education Of Mathematics And Sciences*, Yogyakarta State University p 79 84
[12] Widakdo W A 2017 Mathematical Representation Ability by Using Project-Based Learning on the Topic of Statistics *Journal of Physics: Conference Series*
[13] Hebert M A and Powell S R 2016 Examining Fourth-Grade Mathematics Writing: Features of Organization, Mathematics Vocabulary, and Mathematical Representations *Reading and Writing-Springer* **29** p 1511-1537
[14] Stylianou D A 2011 An Examination of Middle School Students' Representation Practices in Mathematical Problem Solving Through The Lens of Expert Work: Towards An Organizing Scheme *Educational Studies in Mathematics* **76** p 265-280
[15] Siregar S 2010 *Statistika Deskriptif untuk Penelitian* (Jakarta: Rajawali Pers)
[16] Budiyono 2017 *Pengantar Metodologi Penelitian Pendidikan* (Surakarta: UNS Pers)
[17] Djaali and Muljono P 2008 *Pengukuran dalam Bidang Pendidikan* (Jakarta: Grasindo)
[18] Budiyono 2015 *Pengantar Penilaian Hasil Belajar* (Surakarta: UNS Press)
[19] Minarni A, Napitupulu E E and Husein R 2016 Mathematical Understanding and Representation Ability of Public Junior High School in North Sumatra *Journal on Mathematics Education* **7 (1)** p 43-56
[20] Kowiyah and Mulyawati I 2018 An Analysis of Primary School Students' Representational Ability in Mathematics Based on Gender Perspective *Journal of Physics: Conference Series*
[21] Dewi S V P and Sopiany H N 2017 Analisis Kemampuan Representasi Matematis Siswa SMP Kelas VII Pada Penerapan Open-Ended *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika (SESIOMADIKA)* p 680-688
[22] Duval R 1999 Representation, Vision, and Visualization: Cognitive Functions In Mathematical Thinking. Basic Issue for Learning *Proceeding Of The Annual Meeting Of The North American Chapter of The International Group for The Psychology of Mathematics Education* p 3-26
[23] Muhamad S and Hashim A S 2018 Online Education: The Academic Impact And Learning effects *International Journal of Engineering & Technology* **7 (3.7)** p 80-82
[24] Isip B C A 2018 Utilization of ICT Tools in the Teaching-Learning Process by State Universities and Colleges in Caraga Region: Issues, Problems and Concerns *International Journal of Engineering & Technology* **7 (3.7)** p 83-86
[25] Yanti Y R, Amin S M and Sulaiman R 2018 Representation of Students in Solving Simultaneous Linear Equation Problems Based on Multiple Intelligence *Journal of Physics: Conference Series*, doi:10.1088/1742-6596/947/1/012038
[26] Hoogland K, Pepin B, Bakker A, de Koning J and Gravemeijer K 2016 Representing contextual mathematical problems in descriptive or depictive form: Design of an instrument and validation of its uses *Studies in Educational Evaluation* **50** p 22-32
[27] Suryowati E 2015 Kesalahan siswa sekolah dasar dalam merepresenta- sikan pecahan pada garis bilangan *Aksioma* **4 (1)** p 38-52
[28] Guler G and Ciltas A 2011 The Visual Representation Usage Levels Of Mathematics Teachers

And Students In Solving Verbal Problems *International Journal of Humanities and Social Science* **1 (11)** p 145-154

[29] Taher M, Hamidah I and Suwarma I R Profile of Students' Mental Model Change on Law Concepts Archimedes as Impact of Multi- Representation Approach *Journal of Physics:Conference Series*

[30] Sahendra A, Budiarto M T and Fuad Y 2018 Students' Representation in Mathematical Word Problem Solving: Exploring Students' Self efficacy *Journal of Physics: Conference Series*

[31] Sukmaningthias N and Hadi A R 2016 Improve Analytical Thinking Skill and Mathematical Representation of The Students Through Math Problem Solving *Proceeding of 3rd international conference on research, implementation and education of mathematics and science* p 449-454

[32] Rahmawati D, et al. 2017 Process of Mathematical Representation Translation from Verbal into Graphic *IEJME* **12 (4)** p 367-381