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The chromatic number of local antimagic total edge coloring of some related cycle graphs

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Abstract. A graph $G = (V, E)$ be connected, finite, and undirected graph without multiple edges and loops. Graph $G(V, E)$ consists two sets of vertices V and edge E . A graph called related cycle if the subgraph of graph G contains a cycle. Local antimagic total edge labeling is defined a bijection $f : V \cup E \rightarrow \{1, 2, 3, \dots, |V| + |E|\}$, if for any two adjacent edges e_1 and e_2 , $w_t(e_1) \neq w_t(e_2)$, where for $e = ab \in G$, $w_t(e) = f(a) + f(ab) + f(b)$. Thus, the local antimagic total edge labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. The local antimagic total edge chromatic number of G denoted by $\gamma_{late}(G)$, is the minimum of colors needed to coloring the edges of a graph, the number of distinct induced edge labels over all local antimagic total labeling of G . In this paper we study the local antimagic total edge coloring of some related cycle of graphs and determined the chromatic number of some related cycle graphs namely triangular book Bt_r , prism graph P_r , sun graph M_r and kite graph $K_{r,s}$.

1. Introduction

A graph $G = (V, E)$ consists two sets V and E . The elements of V are called vertices V and the elements of E called edges. We consider that graph in this paper contains a cycle and have at least tree vertices and connected finite simple graphs. The detail definition of graph can be seen in [5, 6]. Graph labeling is the assignment of labels to the graph elements such as edges or vertices, or both, from the graph. Every component of graph such as vertex, edge, and face given distinct label by natural number. the labeling is called edge antimagic if all the edge weights of graph G have the distinct value. Concept of antimagic labeling firstly introduced by Hartsfield and Ringel [1]. Dafik *et. al* has studied antimagic labeling. The result can be found in [2, 3].



Graph coloring is assigned colors the component of graph such as vertices, edges and face of a graph, so there are no vertices, edges and face adjacent have the same colors. Local antimagic total edge labeling is an assignment $f : V(G) \cup E(G) \rightarrow \mathcal{C}$ from union of vertex set and edge set $\mathcal{C} = \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$. For every two adjacent edges e_1 and e_2 , $w_t(e_1) \neq w_t(e_2)$, for $e = ab \in G$ and $w_t(e) = f(a) + f(ab) + f(b)$. Thus, the local antimagic total edge labeling induces a proper edge coloring of G if each edge e is assigned by the color $w_t(e)$. The local antimagic total edge chromatic number denoted by χ_{late} is the minimum colors assigned to the edges of graph.

Local antimagic of a graph firstly introduced by Arumugam *et al.* [4]. They gave a lower bound and an exact value of local antimagic coloring. The different type of local antimagic coloring is developed by Agustin *et al.* [7], namely local antimagic edge coloring. They studied the existence of local antimagic edge coloring of some special graphs and given the lower bound denoted by $\gamma_{lea}(G) \geq \Delta(G)$. $\Delta(G)$ is the maximum degree of a vertex in a graph G . The other results about local antimagic of graphs can be seen in [8, 9, 10]

A graph called related cycle if the subgraph of graph G contains a cycle. In this paper we study the local antimagic total edge coloring of some related cycle of graphs and determined the chromatic number of some related cycle graphs, namely triangular book Bt_r , prism graph P_r , sun graph M_r and kite graph $K_{r,s}$.

2. Main Result

We present our result as follows

Theorem 2.1. *Let Bt_r be triangular book graph for $r \geq 3$ and r is even, the local antimagic total edge chromatic number of Bt_r is $\chi_{late}(Bt_r) = r + 1$*

Proof. Let the vertex set and edge set of Bt_r are $\{a, b\} \cup \{a_i : 1 \leq i \leq r\}$ and $\{ab\} \cup \{aa_i : 1 \leq i \leq r\} \cup \{ba_i : 1 \leq i \leq r\}$. The cardinality of vertices and edges of Bt_r are $|V(Bt_r)| = r + 2$ and $|E(Bt_r)| = 2r + 1$. The chromatic number of local antimagic total edge of Bt_r is $\chi_{late}(Bt_r) = r + 1$. To proof $\chi_{late}(Bt_r) = r + 1$ we will show the lower bound and upper bound of the local edge antimagic total coloring of Bt_r . To show the $\gamma_{late}(Bt_r) \geq r + 1$, we will identify $\Delta(Bt_r)$. $\Delta(Bt_r)$ is maximum degrees of Bt_r , the maximum degrees of Bt_r is $\Delta(Bt_r) = r + 1$ so the coloring for Bt_r is $r + 1$ colors. It concludes the lower bound of the local antimagic total edge of Bt_r is $\gamma_{late}(Bt_r) \geq r + 1$

Furthermore, to show the upper bound of the local antimagic total edge coloring of Bt_r is $\gamma_{late}(Bt_r) \leq r + 1$. Define a bijection $f : V(Bt_r) \cup E(Bt_r) \rightarrow \{1, 2, 3, \dots, |V(Bt_r)| + |E(Bt_r)|\}$, the function of vertex and edge labeling are as follows

$$f(v) = \begin{cases} 1, & \text{if } v = a \\ 3, & \text{if } v = b, \text{ when } i \text{ is even and } j \text{ is odd} \\ 3 + i, & \text{if } v = a_i, \text{ when } 1 \leq i \leq r \end{cases}$$

$$f(e) = \begin{cases} 2, & \text{if } e = ab \\ r + 4, & \text{if } e = ba_r \\ r + 2i + 4, & \text{if } e = ba_i, \text{ when } \frac{r}{2} + 1 \leq i \leq r - 1 \\ r + 2i + 3, & \text{if } e = ba_i, \text{ when } 1 \leq i \leq \frac{r}{2} \\ r + 2i + 2, & \text{if } e = aa_i, \text{ when } 2 \leq i \leq \frac{r}{2} + 1 \\ 2r + 5, & \text{if } e = aa_1 \\ r + 2i + 3, & \text{if } e = aa_i, \text{ when } \frac{r}{2} + 2 \leq i \leq r \end{cases}$$

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} 6, & \text{if } e = ab \\ 2r + 10, & \text{if } e = aa_1 \text{ and } e = yx_r \\ r + 3i + 6, & \text{if } e = aa_i, \text{ when } 2 \leq i \leq \frac{r}{2} + 1 \\ r + 3i + 7, & \text{if } e = aa_i, \text{ when } \frac{r}{2} + 2 \leq i \leq r \\ r + 3i + 9, & \text{if } e = ba_i, \text{ when } 1 \leq i \leq \frac{r}{2} \\ r + 3i + 10, & \text{if } e = ba_i, \text{ when } \frac{r}{2} + 1 \leq i \leq r - 1 \end{cases}$$

Hence, from the total edge weights above, it is easy to see that f induces a proper edge coloring of Bt_r and it gives $\gamma_{late}(Bt_r) \leq r + 1$. Based on the lower bound is $\gamma_{late}(Bt_r) \geq \Delta(Bt_r) = r + 1$. It concludes that $\gamma_{late}(Bt_r) = r + 1$ when r is even. $\square$

Figure 1 The Example of local antimagic total edge of Bt_6

Theorem 2.2. Let $K_{r,s}$ be a kite graph for $r, s \geq 3$ and r, s is odd, the local antimagic total edge chromatic number of $K_{r,s}$ is $\chi_{late}(K_{r,s}) = 3$

Proof. Let the vertex set of $K_{r,s}$ be $\{a_i : 1 \leq i \leq r + s - 1\}$ and edge set of $K_{r,s}$ be $\{a_i a_{i+1} : 1 \leq i \leq r + s - 2\} \cup \{a_1 a_r\}$. The cardinality of vertices and edges of $K_{r,s}$ be $|V(K_{r,s})| = r + s - 1$ and $|E(K_{r,s})| = r + s - 1$. To proof $\chi_{late}(K_{r,s}) = 3$ we will show the lower bound and upper bound of the local edge antimagic total coloring of $K_{r,s}$. To show the $\gamma_{late}(K_{r,s}) \geq 3$, we will identify $\Delta(K_{r,s})$. $\Delta(K_{r,s})$ is maximum degrees of $K_{r,s}$, the maximum degrees of $K_{r,s}$ is $\Delta(K_{r,s}) = 3$ so the coloring for $K_{r,s}$ is 3 colors. It concludes that $\gamma_{late}(K_{r,s}) \geq 3$

Furthermore, to show the upper bound of the local antimagic total edge coloring of $K_{r,s}$ is $\gamma_{late}(K_{r,s}) \leq 3$. Define a bijection $f : V(K_{r,s}) \cup E(K_{r,s}) \rightarrow \{1, 2, 3, \dots, |V(K_{r,s})| + |E(K_{r,s})|\}$, the function of vertex and edge labeling are as follows

$$f(v) = \begin{cases} \frac{i+1}{2}, & \text{if } v = a_i, \text{ when } i \text{ odd}, 1 \leq i \leq r + s - 1 \\ \frac{r+s+i+1}{2}, & \text{if } v = a_i, \text{ when } i \text{ even}, 2 \leq i \leq r + s - 1 \end{cases}$$

$$f(e) = \begin{cases} 2(r + s) - i - 2, & \text{if } e = a_i a_{i+1}, \text{ when } i \text{ odd}, \\ & 1 \leq i \leq r + s - 2 \\ , & \\ 2(r + s) - i, & \text{if } e = a_i a_{i+1}, \text{ when } i \text{ even}, \\ & 2 \leq i \leq r + s - 2 \\ , & \\ r + s, & \text{if } e = a_1 a_r \end{cases}$$

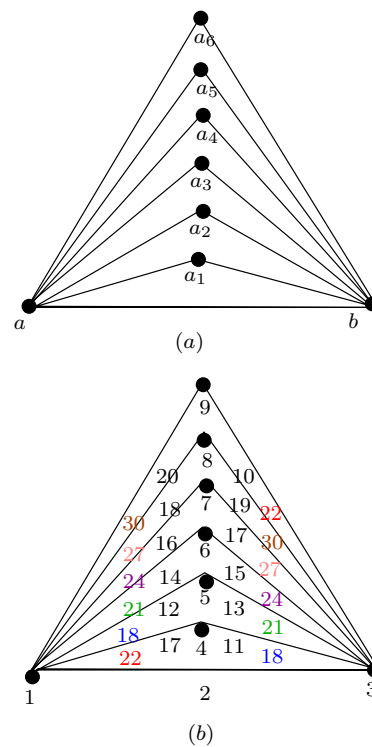


Figure 1. The illustration of (a)graph Bt_6 and (b)local antimagic total edge coloring of Bt_6

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} \frac{5}{2}(r+s) - 1, & \text{if } e = a_i a_{i+1}, \text{ when } i \text{ odd}, 1 \leq i \leq r+s-2 \\ \frac{5}{2}(r+s) + 1, & \text{if } e = a_i a_{i+1}, \text{ when } i \text{ even}, 2 \leq i \leq r+s-2 \\ \frac{3}{2}(r+1) + s, & \text{if } e = a_1 a_r \end{cases}$$

Hence, from the total edge weights above, it is easy to see that f induces a proper edge coloring of $K_{r,s}$ and it gives $\gamma_{late}(K_{r,s}) \leq 3$. Based on the lower bound is $\gamma_{late}(K_{r,s}) \geq \Delta(K_{r,s}) = 3$. It concludes that $\gamma_{late}(K_{r,s}) = 3$ when r, s is odd. $\square$

Figure 2 The Example of local antimagic total edge of $K_{5,7}$

Theorem 2.3. Let M_r be a sun graph for $r \geq 3$ and r is odd, the local antimagic total edge chromatic number of M_r is $3 \leq \chi_{late}(M_r) \leq 4$

Proof. Let the vertex set of M_r be $\{a_i : 1 \leq i \leq r\} \cup \{b_i : 1 \leq i \leq r\}$ and edge set of M_r be $\{a_i a_{i+1} : 1 \leq i \leq r-1\} \cup \{a_1 a_r\} \cup \{a_i b_i : 1 \leq i \leq r\}$. The cardinality of vertices and edges of M_r be $|V(M_r)| = 2r$ and $|E(M_r)| = 2r$. We will show that the lower bound and upper bound of the local edge antimagic total coloring of M_r . To proof the $\gamma_{late}(M_r) \geq 3$, we will identify $\Delta(M_r)$. $\Delta(M_r)$ is maximum degrees of M_r , the

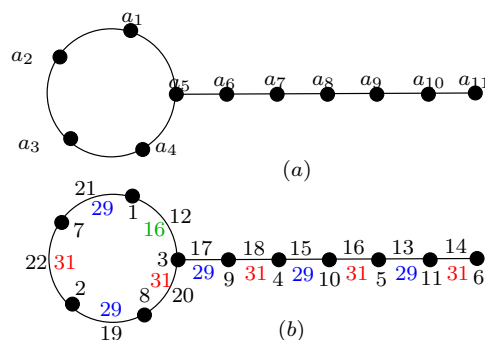


Figure 2. The illustration of (a)graph $K_{5,7}$ and (b)local antimagic total edge coloring of $K_{5,7}$)

maximum degrees of M_r is $\Delta(M_r) = 3$ so the coloring for M_r is 3 colors. It concludes that $\gamma_{late}(M_r) \geq 3$

Furthermore, to show the upper bound of the local antimagic total edge coloring of M_r is $\gamma_{late}(M_r) \leq 4$. Define a bijection $f : V(M_r) \cup E(M_r) \rightarrow \{1, 2, 3, \dots, |V(M_r)| + |E(M_r)|\}$, the function of vertex and edge labeling are as follows

$$f(v) = \begin{cases} \frac{i+1}{2}, & \text{if } v = a_i, \text{ when } i \text{ odd}, 1 \leq i \leq r \\ \frac{r+s+i+1}{2}, & \text{if } v = a_i, \text{ when } i \text{ even}, 2 \leq i \leq r \\ \frac{i+r}{2} + r, & \text{if } v = b_i, \text{ when } i \text{ odd}, 1 \leq i \leq r \\ \frac{i}{2} + r, & \text{if } v = b_i, \text{ when } i \text{ even}, 2 \leq i \leq r \end{cases}$$

$$f(e) = \begin{cases} 2r + 1, & \text{if } e = a_1a_r \\ 3r - i, & \text{if } e = a_ia_{i+1}, \text{ when } i \text{ odd}, 1 \leq i \leq r - 1 \\ 3r - i + 2, & \text{if } e = a_ia_{i+1}, \text{ when } i \text{ even}, 2 \leq i \leq r - 1 \\ 4r - i + 1, & \text{if } e = a_ib_i, 1 \leq i \leq r \end{cases}$$

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} \frac{5}{2}(r+1) - 1, & \text{if } e = a_1a_r \\ \frac{7r+3}{2}, & \text{if } e = a_ia_{i+1}, \text{ when } i \text{ odd}, 1 \leq i \leq r \\ \frac{7r+7}{2}, & \text{if } e = a_ia_{i+1}, \text{ when } i \text{ even}, 2 \leq i \leq r \\ \frac{r+3}{2} + 5r, & \text{if } e = a_ib_i, 1 \leq i \leq r \end{cases}$$

Hence, from the total edge weights above, it is easy to see that f induces a proper edge coloring of M_r and it gives $\gamma_{late}(M_r) \leq 4$. Based on the lower bound is $\gamma_{late}(M_r) \geq \Delta(M_r) = 3$. We obtain that the upper bound of M_r is $\chi_{late}(M_r) \leq 4$ and the lower bound is $\chi_{late}(M_r) \geq 3$. It concludes that the local antimagic total edge chromatic number of M_r is $3 \leq \chi_{late}(M_r) \leq 4$ when r, s is odd. $\square$

Figure 3 shows an example local antimagic total edge coloring of M_7

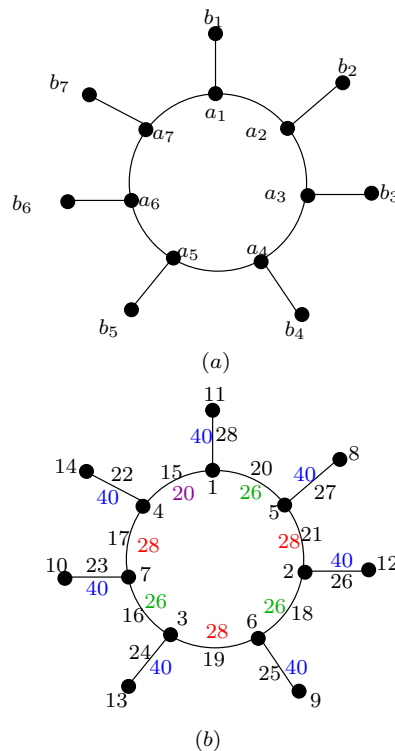


Figure 3. The illustration of (a)graph M_7 and (b)local antimagic total edge coloring of M_7

Theorem 2.4. Let Tl_r be a triangular ladder graph for $r \geq 5$ and r is odd, the local antimagic total edge chromatic number of Tl_r is $4 \leq \chi_{late}(Tl_r) \leq 5$

Proof. Let the vertex set of Tl_r be $\{a_i : 1 \leq i \leq r\} \cup \{b_i : 1 \leq i \leq r\}$ and edge set of Tl_r be $\{a_i a_{i+1} : 1 \leq i \leq r-1\} \cup \{b_i b_{i+1} : 1 \leq i \leq r-1\} \cup \{a_i b_i : 1 \leq i \leq r\} \cup \{a_i b_{i+1} : 1 \leq i \leq r-1\}$. The cardinality of vertices and edges of Tl_r be $|V(Tl_r)| = 2r$ and $|E(Tl_r)| = 4r-3$. We will show that the lower bound and upper bound of the local edge antimagic total coloring of Tl_r . To proof the $\gamma_{late}(Tl_r) \geq 3$, we will identify $\Delta(Tl_r)$. $\Delta(Tl_r)$ is maximum degrees of Tl_r , the maximum degrees of Tl_r is $\Delta(Tl_r) = 4$ so the coloring for Tl_r is 4 colors. It concludes that the lower bound of the local antimagic total edge coloring of Tl_r is $\gamma_{late}(Tl_r) \geq 4$

Furthermore, to show the upper bound of the local antimagic total edge coloring of Tl_r is $\gamma_{late}(Tl_r) \leq 5$. Define a bijection $f : V(Tl_r) \cup E(Tl_r) \rightarrow \{1, 2, 3, \dots, |V(Tl_r)| + |E(Tl_r)|\}$ the function of vertex and edge labeling are as follows.

$$f(v) = \begin{cases} 2i-1, & \text{if } v = a_i, 1 \leq i \leq r \\ 2i, & \text{if } v = b_i, 1 \leq i \leq r \end{cases}$$

We can see that f is a local antimagic labeling of Tl_r and the weights of edge labeling

are as follows

$$w(e) = \begin{cases} 4i, & \text{if } e = a_i a_{i+1}, 1 \leq i \leq r-1 \\ 4i+2, & \text{if } e = b_i b_{i+1}, 1 \leq i \leq r-1 \\ 4i+1, & \text{if } e = a_i b_{i+1}, 1 \leq i \leq r-1 \\ 4i-1, & \text{if } e = a_i b_i, 1 \leq i \leq r \end{cases}$$

For other edges labeling, we use construction to give labels on $e = x_i x_{i+1}$, $1 \leq i \leq r-1$, $e = b_i b_{i+1}$, $1 \leq i \leq r-1$, $e = a_i b_i$, $1 \leq i \leq n$ and $e = x_i y_{i+1}$, $1 \leq i \leq r-1$. We define an algorithm of construction of edges labeling by the following.

- (i) From the vertex labeling, it is easy to find the edge weights of $e = a_i a_{i+1}$, $1 \leq i \leq r-1$, $e = b_i b_{i+1}$, $1 \leq i \leq r-1$, $e = a_i b_i$, $1 \leq i \leq r$ and $e = a_i b_{i+1}$, $1 \leq i \leq r-1$.
- (ii) Arranged the edge weights by ascending order.
- (iii) Arranged the labels of edges $e = a_i a_{i+1}$, $1 \leq i \leq r-1$, $e = b_i b_{i+1}$, $1 \leq i \leq r-1$, $e = a_i b_i$, $1 \leq i \leq r$ and $e = a_i b_{i+1}$, $1 \leq i \leq r-1$ by descending order such that the total edge weights are same.
- (iv) $e = a_i a_{i+1}$, $1 \leq i \leq r-1$, $e = b_i b_{i+1}$, $1 \leq i \leq r-1$, $e = a_i b_i$, $1 \leq i \leq r$ and $e = a_i b_{i+1}$, $1 \leq i \leq r-1$ must have the same weights. Since they are not adjacent.
- (v) Put the other labels of edges randomly such that they have the distinct total edge weights.

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} 6r, & \text{if } e = a_i b_i, 1 \leq i \leq r \\ 6r-4, & \text{if } e = a_i a_{i+1} \text{ dan } a_i a_{i+1}, \text{ when } i \text{ odd}, 1 \leq i \leq r \\ 6r+4, & \text{if } e = a_i a_{i+1} \text{ dan } a_i a_{i+1}, \text{ when } i \text{ even}, 2 \leq i \leq r \\ 6r-8, & \text{if } e = a_i b_{i+1}, 1 \leq i \leq r-3 \\ 10r-12, & \text{if } e = a_i b_{i+1}, r-2 \leq i \leq r-1 \end{cases}$$

Hence, from the total edge weights above, it is easy to see that f induces a proper edge coloring of Tl_r and it gives $\gamma_{late}(Tl_r) \leq 5$. Based on the lower bound is $\gamma_{late}(Tl_r) \geq \Delta(Tl_r) = 4$. We obtain that the upper bound of Tl_r is $\chi_{late}(Tl_r) \leq 5$ and the lower bound is $\chi_{late}(Tl_r) \geq 4$. It concludes that the local antimagic total edge chromatic number of Tl_r is $4 \leq \chi_{late}(Tl_r) \leq 5$ when r is odd. $\square$

Figure 4 shows an example local antimagic total edge coloring of Tl_7

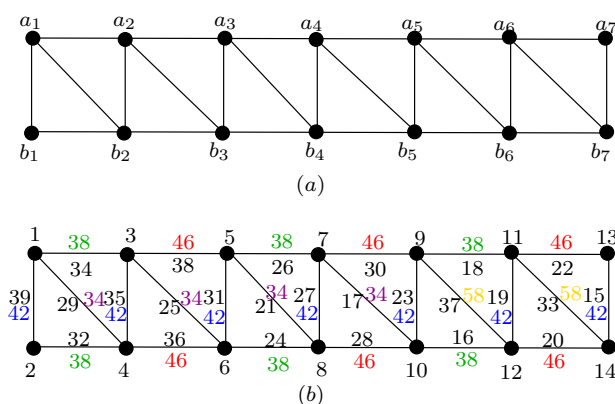


Figure 4. The illustration of (a)graph Tl_7 and (b)local antimagic total edge coloring of Tl_7

3. Conclusion

In this paper we have given some results on local antimagic total edge coloring of some related of cycle graph, namely triangular book Bt_r , prism graph P_r , sun graph M_r and kite graph $K_{r,s}$.

Open Problem 3.1. find the chromatic number of local edge antimagic total coloring of Bt_r for r odd, $K_{r,s}$, Tl_r and M_r for r even.

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References

- [1] Hartsfield N dan Ringel G 1994 *Pearls in Graph Theory* Academic Press. United Kingdom
- [2] Dafik, Miller M, Ryan J, and Baca M 2011 Super edge-antimagic total labelings of $mK_{n,n}$ *Ars Combinatoria* **101** 97-107
- [3] Dafik, M Mirka, Joe Ryan, and Martin Baca 2006 Super edge-antimagicness for a class of disconnected graphs
- [4] Arumugam S, Premalatha K, Baca M and Semanicova-Fenovicikova A 2017 Local Antimagic Vertex Coloring of a Graph, *Graphs and Combinatorics* **33** 275-285
- [5] Gross J L, Yellen J and Zhang P 2014 *Handbook of graph Theory* Second Edition CRC Press Taylor and Francis Group
- [6] Chartrand G and Lesniak L 2000 *Graphs and digraphs 3rd ed* (London: Chapman and Hall)
- [7] Agustin I H, Dafik, Moh. Hasan, Alfarisi R and Prihandini R M 2017 Local edge antimagic coloring of graphs *Far East Journal of Mathematical Sciences*.
- [8] Agustin I H, Dafik, Slamin, Ridho Alfarisi and Kurniawati E Y 2017 The construction of super local edge antimagic total coloring by using an EAVL Technique *accepted*
- [9] Agustin I H, Dafik, Slamin, Ermita E R and Alfarisi R 2017 On the total local edge super antimagicness of special graph and graph with pendant edge *accepted*
- [10] Kurniawati, E. Y., Agustin, I. H., Dafik, and Alfarisi, R. (2018). Super local edge antimagic total coloring of $P_n \triangleright H$, *Journal of Physics: Conference Series*, 1008, p. 1-11.