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Local super antimagic total face coloring of planar graphs

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Abstract. We using graph $G = (V(G), E(G), F(G))$ be a nontrivial, finite, connected graph, and a g bijective function mapping total labeling of graph to natural number start form 1 until the sum of vertices, edge, and faces. The sum of vertices, edges, and faces labels in a face f is called the weight of the face $f \in F(G)$. If any adjacent two faces f_1 and f_2 have different weights $w(f_1) \neq w(f_2)$ for $f_1, f_2 \in F(G)$, then g is called a labeling of local antimagic total face. We call labeling of local antimagic total face is super if we add vertices label start from 1 until the sum of vertices, edges label start from the sum of vertices plus 1 until the sum of vertices and edges, and faces label start form the sum of vertices and edges plus one untul the sum of vertices, edges, and faces. The local super antimagic total face labeling that induces a proper faces coloring of G where the faces f is assigned by the color $w(f)$ is called local super antimagic total face coloring. The minimum number of colors in local super antimagic total face coloring is local antimagic total face chromatic number and denoted by $\gamma_{latf}(G)$. In this paper, we used some planar graph such as wheel graph (W_n), jahangir graph ($J(2, n)$), ladder graph (L_n), and circular ladder graph (CL_n). Our results attained the lower bound of local super antimagic total face chromatic number.

1. Introduction

A graph G is nontrivial, finite, and connected graph. A graph G have sets $(V(G), E(G), F(G))$ where $V(G)$ is nonempty set and $E(G), F(G)$ are possibly empty set. The elements of $V(G)$, $E(G)$, and $F(G)$ are vertices, edges, and faces of graph G. The order of graph G is denoted as $|V(G)|$, the size of graph G is denoted as $|E(G)|$, and the quantity of faces in graph G is denoted as $|F(G)|$. We can see [8, 5] for more detail about graphs. Planar graph is defined as a graph without any edges crossing. A planar graph divided into some regions which are called faces [12, 13].

Graph labeling is a bijective function that mapping natural number to elements of graph. Based on the domain of bijective function, the labeling is respectively divided into vertex labeling, edge labeling, face labeling, and total labeling. We can find a general survey of graph labeling in [7].

The concept of antimagic labeling was introduced by Hartsfield and Ringel [9]. A g bijective function mapping total labeling of graph to natural number start form 1 until the sum of vertices, edge, and faces. The sum of vertices, edges, and faces labels in a face f is called the weight of the face $f \in F(G)$. If any adjacent two faces f_1 and f_2 have different weights $w(f_1) \neq w(f_2)$ for $f_1, f_2 \in F(G)$, then g is called a labeling of local antimagic total face. We call labeling of local antimagic total face is super if we add vertices label start from 1 until the sum of vertices, edges label start from the sum of vertices plus 1 until the sum of vertices and edges, and faces label start form the sum of vertices and edges plus one untul the sum of vertices, edges, and faces.

Graph coloring is to assign a color to elements (vertex, edge, and face) of graph such that two adjacent elements has different color. The chromatic number of a graph is the smallest numbers of colors needed in graph coloring. Face coloring of a planar graph assigns a color to each face so that no two faces that share an edge have the same color [11].

The local super antimagic total face labeling that induces a proper faces coloring of G where the faces f is assigned by the color $w(f)$ is called local super antimagic total face coloring. The minimum number of colors in local super antimagic total face coloring is local antimagic total face chromatic number and denoted by $\gamma_{latf}(G)$.

The first person who introduced local antimagic vertex coloring of a graph was Arumugam *et al.* [4]. They get the exact value of local antimagic vertex coloring of path, cycle, complete, friendship, wheel, bipartite, and complete bipartite graph and also a lower and upper bound of joint graph. The research that is related to the labeling is super edge-antimagic total labeling of $mK_{n,n}$ [6].

The different type of local antimagic coloring is called local antimagic edge coloring. It was studied by Agustin *et al.* [2]. Their results are the exact value of path, cycle, friendship, ladder, star, wheel, complete, prism, $C_n \odot mK_1$ and $G \odot mK_1$ and also a lower bound of local edge antimagic chromatic number ($\gamma_{lea} \geq \Delta(G)$). The other results of on super local antimagic total edge coloring of some wheel related graphs [1], and local edge antimagic coloring of comb product of graphs [3].

In this paper, we investigate omit local super antimagic total face coloring of planar graphs such as wheel graph (W_n), jahangir graph ($J(2,n)$), ladder graph (L_n), and circular ladder graph (CL_n). We constructed ladder graph (L_n) from shackle operation of cycles (C_4) as many n copies. We can look at [10] for more details about shackle operation.

2. Results

We present our result local super antimagic total face coloring of planar graphs in this section. Furthermore, we have found chromatic number of local super antimagic total face in wheel graph (W_n), jahangir graph ($J(2,n)$), ladder graph (L_n), and circular ladder graph (CL_n).

Observation 1 *For any G graph, $\gamma_{latf}(G) \geq \chi_f(G)$, where $\chi_f(G)$ is chromatic number of face coloring in graph G.*

We know that faces with an edge in common has different colors, so the chromatic number of face coloring in graph is at least 2, especially planar graphs.

2.1. Chromatic number of local super antimagic total face in wheel graph

Let $n \geq 3$ be a natural number. Wheel graph (W_n) is a graph with $V(W_n) = \{z\} \cup \{x_i; 1 \leq i \leq n\}$, $E(W_n) = \{zx_i; 1 \leq i \leq n\} \cup \{x_ix_{i+1}; 1 \leq i \leq n-1\} \cup \{x_ix_n\}$, and $F(W_n) = \{f_i; 1 \leq i \leq n\}$. Hence $|V(W_n)| = n+1$, $|E(W_n)| = 2n$ and $|F(W_n)| = n$. Then, we determine chromatic number of local super antimagic total face in W_n as follows.

Theorem 1: *Let $n \geq 3$ be a natural number and W_n be a wheel graph. Then,*

$$\gamma_{latf}(W_n) = \begin{cases} 2; & \text{for } n \text{ is even} \\ 3; & \text{for } n \text{ is odd.} \end{cases}$$

Proof. To proof a chromatic number of local super antimagic total face coloring of wheel graph, we divided the proof into two cases.

Case 1.

For n is even, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(W_n) \leq 2$. We define vertex labeling as a g bijective function to natural number until the order of W_n is as follows:

$$g(z) = \{1\}$$

$$g(x_i) = \begin{cases} 2; & \text{for } i = 1 \\ n - i + 3; & \text{for } 2 \leq i \leq n. \end{cases}$$

We define edge labeling as a g bijective function to the order of W_n plus one until the order and edges of W_n is as follows:

$$g(zx_i) = n + i + 1; \text{ for } 1 \leq i \leq n$$

$$g(x_i x_{i+1}) = \begin{cases} 3n + 1; & \text{for } i = 1 \\ 2n + i; & \text{for } 2 \leq i \leq n - 1 \end{cases}$$

$$g(x_1 x_n) = 3n.$$

We define face labeling as a g bijective function to the order and edges of W_n plus one until all the elements of W_n is as follows:

$$g(f_i) = \begin{cases} 4n; & \text{for } i = n \\ 4n - i + 2; & \text{for } i \text{ is odd} \\ 4n - i; & \text{for } i \text{ is even.} \end{cases}$$

It is easy to get $f \in F(G)$ is a local face super antimagic total face labeling of W_n and the face weights are follows:

$$w(f_i) = \begin{cases} 10n + 11; & \text{for } 1 \leq i \leq n, \ i \text{ is odd} \\ 10n + 9; & \text{for } 1 \leq i \leq n, \ i \text{ is even.} \end{cases}$$

The chromatic number of face coloring in wheel graph is at least 2, because wheel graph has at least two adjacent faces. Hence, $\chi_f(W_n) = 2$. Based on Observation 1, we get $\gamma_{latf}(W_n) \geq 2$ and $\gamma_{latf}(W_n) \leq 2$ as a lower bound and an upper bound of local super antimagic total face coloring of W_n. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(W_n) = 2$ for n is even.

Case 2.

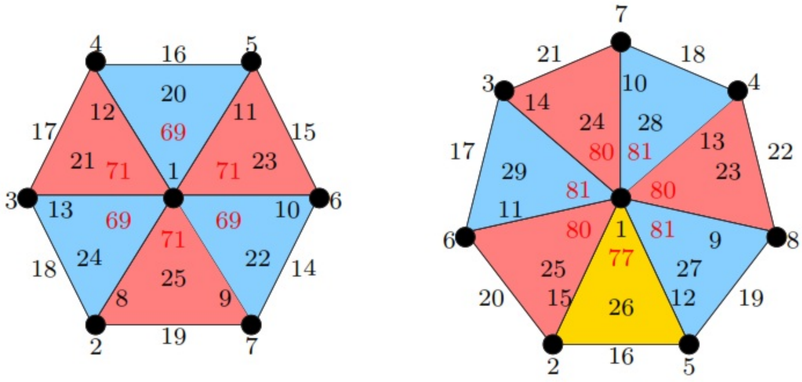
For n is odd, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(W_n) \leq 3$. We define vertex labeling as a g bijective function to natural number until the order of W_n is as follows:

$$g(z) = \{1\}$$

$$g(x_i) = \begin{cases} \frac{n-i+3}{2}; & \text{for } i \text{ is even} \\ \frac{2n-i+3}{2}; & \text{for } i \text{ is odd.} \end{cases}$$

We define edge labeling as a g bijective function to the order of W_n plus one until the order and edges of W_n is as follows:

$$g(zx_i) = \begin{cases} \frac{2n+i+3}{2}; & \text{for } 1 \leq i \leq n, \ i \text{ is odd} \\ \frac{3n+i+3}{2}; & \text{for } 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$



(a) W_6 (b) W_7

Figure 1. Local Super Antimagic Total Face Labeling of (a)W_6 (b)W_7.

$$g(x_i x_{i+1}) = \begin{cases} \frac{6n-i+3}{2}; & \text{for } 1 \leq i \leq n-1,\ i \text{ is odd} \\ \frac{5n-i+3}{2}; & \text{for } 1 \leq i \leq n-1,\ i \text{ is even} \end{cases}$$

$$g(x_1 x_n) = \frac{5n+3}{2}.$$

We define face labeling as a g bijective function to the order and edges of W_n plus one until all the elements of W_n is as follows:

$$g(f_i) = \begin{cases} \frac{5n+2i+5}{2}; & \text{for } i = n-1,\ n \\ \frac{6n+i+3}{2}; & \text{for } i \text{ is odd} \\ \frac{7n+i+5}{2}; & \text{for } i \text{ is even.} \end{cases}$$

It is easy to get $f \in F(G)$ in a local face super antimagic total face labeling of W_n and the face weights are follows:

$$w(f_i) = \begin{cases} 10n+10; & \text{for} 1 \leq i \leq n-2,\ i \text{ is odd} \\ 10n+11; & \text{for} 1 \leq i \leq n-2,\ i \text{ is even} \\ \frac{19n+21}{2}; & \text{for } i = n-1 \\ 10n+11; & \text{for } i = n. \end{cases}$$

The chromatic number of face coloring in wheel graph is at least 2, because wheel graph has at least two adjacent faces. For $n = 3$, $c(f_1) \neq c(f_2)$, $c(f_2) \neq c(f_3)$, and $c(f_3) \neq (f_1)$. We can see that every faces of W_3 must have 3 different colors. Hence, $\chi_f(W_n) = 3$. Based on Observation 1, we get $\gamma_{latf}(W_n) \geq 3$ and $\gamma_{latf}(W_n) \leq 3$ as a lower bound and an upper bound of local super antimagic total face coloring of W_n. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(W_n) = 3$ for n is odd.

2.2. The local super antimagic total face chromatic number of jahangir graph

Let $n \geq 3$ be a natural number. Jahangir graph $(J(2,n))$ is a graph with vertex set $V(J(2,n)) = \{y\} \cup \{x_i; 1 \leq i \leq 2n\}$, edge set $E(J(2,n)) = \{yx_i; 1 \leq i \leq 2n, i$ is odd$\} \cup \{x_i x_{i+1}; 1 \leq i \leq 2n-1\} \cup \{x_i x_2 n\}$, and face set $F(J(2,n)) = \{f_i; 1 \leq i \leq n\}$. Hence $|V(J(2,n))| = 2n+1$, $|E(J(2,n))| = 3n$ and $|F(J(2,n))| = n$. In this subsection we determine the local super antimagic total face chromatic number of jahangir graph as follows.

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Theorem 2: *Let $n \geq 3$ be a natural number and $J(2,n)$ be a jahangir graph. Then,*

$$\gamma_{lfat}(J(2,n)) = \begin{cases} 2; & \text{for } n \text{ is even} \\ 3; & \text{for } n \text{ is odd.} \end{cases}$$

Proof. To proof a chromatic number of local super antimagic total face coloring of jahangir graph, we divided the proof into two cases.

Case 1.

For n is even, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(J(2,n)) \leq 2$. We define vertex labeling as a g bijective function to natural number until the order of $J(2,n)$ is as follows:

$$g(y) = \{1\}$$

$$g(x_i) = \begin{cases} \frac{i+3}{2}; & \text{for } 1 \leq i \leq 2n,\ i \text{ is odd} \\ \frac{4n-i+2}{2}; & \text{for } 1 \leq i \leq 2n-2,\ i \text{ is even} \\ 2n+1; & \text{for } i = 2n. \end{cases}$$

We define edge labeling as a g bijective function to the order of $J(2,n)$ plus one until the order and edges of $J(2,n)$ is as follows:

$$g(yx_i) = \begin{cases} 2n+2; & \text{for } i = 1 \\ \frac{6n-i+5}{2}; & \text{for } 1 \leq i \leq 2n-2,\ i \text{ is odd} \end{cases}$$

$$g(x_i x_{i+1}) = \begin{cases} 4n+1; & \text{for } i = 1 \\ \frac{8n+i}{2}; & \text{for } 1 \leq i \leq 2n-1,\ 0 = i \bmod 4 \\ \frac{14n+i+3}{4}; & \text{for } 1 \leq i \leq 2n-1,\ 1 = i \bmod 4 \\ \frac{8n+i+4}{2}; & \text{for } 1 \leq i \leq 2n-1,\ 2 = i \bmod 4 \\ \frac{12n+i+5}{4}; & \text{for } 1 \leq i \leq 2n-1,\ 3 = i \bmod 4 \end{cases}$$

$$g(x_1 x_{2n}) = 5n.$$

We define face labeling as a g bijective function to the order and edges of $J(2,n)$ plus one until all the elements of $J(2,n)$ is as follows:

$$g(f_i) = \begin{cases} 6n+1; & \text{for } i = 1 \\ \frac{12n-i+3}{2}; & \text{for } 1 \leq i \leq n,\ i \text{ is even} \\ \frac{11n-i+3}{2}; & \text{for } 3 \leq i \leq n-1,\ i \text{ is odd.} \end{cases}$$

It is easy to get $f \in F(G)$ is a local face super antimagic total face labeling of $J(2,n)$ and the face weights are follows:

$$w(f_i) = \begin{cases} 21n+14; & \text{for } 1 \leq i \leq n,\ i \text{ is odd} \\ 21n+12; & \text{for } 1 \leq i \leq n,\ i \text{ is even.} \end{cases}$$

The chromatic number of face coloring in jahangir graph is at least 2, because jahangir graph has at least two adjacent faces. Hence, $\chi_f(J(2,n)) = 2$. Based on Observation 1, we get $\gamma_{latf}(J(2,n)) \geq 2$ and $\gamma_{latf}(J(2,n)) \leq 2$ as a lower bound and an upper bound of local super antimagic total face coloring of jahangir graph. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(J(2,n)) = 2$ for n is even.

Case 2.

For n is odd, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(J(2,n)) \leq$

3. We define vertex labeling as a g bijective function to natural number until the order of $J(2, n)$ is as follows:

$$g(y) = \{1\}$$

$$g(x_i) = \begin{cases} \frac{i+7}{4}; & \text{for } 1 \le i \le 2n,\ 1 = \bmod 4 \\ \frac{2n+i+7}{4}; & \text{for } 1 \le i \le 2n,\ 3 = \bmod 4 \\ \frac{2n+i+2}{2}; & \text{for } 1 \le i \le 2n,\ i \text{ is even.} \end{cases}$$

We define edge labeling as a g bijective function to the order of $J(2, n)$ plus one until the order and edges of $J(2, n)$ is as follows:

$$g(yx_i) = \begin{cases} \frac{12n-i+5}{4}; & \text{for } 1 \le i \le 2n-2,\ 1 = i \bmod 4 \\ \frac{10n-i+5}{4}; & \text{for } 1 \le i \le 2n-2,\ 3 = i \bmod 4 \end{cases}$$

$$g(x_i x_{i+1}) = \begin{cases} \frac{8n-i+3}{2}; & \text{for } 1 \le i \le 2n-1,\ i \text{ is odd} \\ \frac{18n+i+6}{4}; & \text{for } 1 \le i \le 2n-1,\ 0 = i \bmod 4 \\ \frac{16n+i+6}{4}; & \text{for } 1 \le i \le 2n-1,\ 2 = i \bmod 4 \end{cases}$$

$$g(x_1 x_2 n) = \frac{9n+3}{2}.$$

We define face labeling as a g bijective function to the order and edges of $J(2, n)$ plus one until all the elements of $J(2, n)$ is as follows:

$$g(f_i) = \begin{cases} 6n+1; & \text{for } i = n \\ \frac{11n-i+3}{2}; & \text{for } 1 \le i \le n,\ i \text{ is even} \\ \frac{12n-i+1}{2}; & \text{for } 1 \le i \le n-2,\ i \text{ is odd.} \end{cases}$$

It is easy to get $f \in F(G)$ is a local face super antimagic total face labeling of $J(2, n)$ and the face weights are follows:

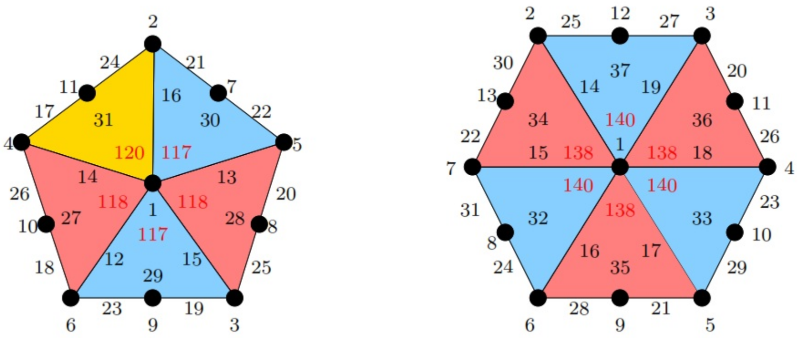
$$w(f_i) = \begin{cases} 21n+12; & \text{for } 1 \le i \le n-2,\ i \text{ is odd} \\ 21n+13; & \text{for } 1 \le i \le n,\ i \text{ is even} \\ \frac{43n+25}{2}; & \text{for } i = n. \end{cases}$$

The chromatic number of face coloring in jahangir graph is at least 2, because jahangir graph has at least two adjacent faces. For $n = 3$, $c(f_1) \neq c(f_2)$, $c(f_2) \neq c(f_3)$, and $c(f_3) \neq (f_1)$. We can see that every faces of $J(2, 3)$ must have 3 different colors. Hence, $\chi_f(J(2, 3)) = 3$. Based on Observation 1, we get $\gamma_{latf}(J(2, n)) \geq 3$ and $\gamma_{latf}(J(2, n)) \leq 3$ as a lower bound and an upper bound of local super antimagic total face coloring of jahangir graph. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(J(2, n)) = 3$ for n is odd.

2.3. The local super antimagic total face chromatic number of ladder graph

Let $n \geq 2$ be a natural number. Ladder graph (L_n) is a graph with vertex set $V(L_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$, edge set $E(L_n) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{y_i y_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_i; 1 \leq i \leq n\}$, and face set $F(L_n) = \{f_i; 1 \leq i \leq n-1\}$. Hence $|V(L_n)| = 2n$, $|E(L_n)| = 3n-2$, and $|F(L_n)| = n-1$. In this subsection we determine the local super antimagic total face chromatic number of ladder graph as follows.

Theorem 3: *Let $n \geq 2$ be a natural number and L_n be a ladder graph. Then, $\gamma_{lfat}(L_n) = 2$.*



(a) $J(2,5)$ (b) $J(2,6)$

Figure 2. Local Super Antimagic Total Face Labeling of (a)$J(2,5)$ (b)$J(2,6)$.

Proof. To proof an upper bound of local super antimagic total face coloring of L_n is $\gamma_{latf}(L_n) \leq 2$, we define vertex labeling as a g bijective function to natural number until the order of L_n is as follows:

$$g(x_i) = \begin{cases} \frac{i+1}{2}; & \text{for } i \text{ is odd} \\ \frac{n+i+1}{2}; & \text{for } i \text{ is even}, n \text{ is odd} \\ \frac{n+i}{2}; & \text{for } i \text{ is even}, n \text{ is even} \end{cases}$$

$$g(y_i) = \begin{cases} \frac{2n+i+1}{2}; & \text{for } i \text{ is odd} \\ \frac{3n+i+1}{2}; & \text{for } i \text{ is even}, n \text{ is odd} \\ \frac{3n+i}{2}; & \text{for } i \text{ is even}, n \text{ is even.} \end{cases}$$

We define edge labeling as a g bijective function to the order of L_n plus one until the order and edges of L_n is as follows:

$$g(x_i y_i) = \quad 3n - i + 1; \text{ for } 1 \leq i \leq n$$

$$g(x_i x_{i+1}) = \begin{cases} \frac{6n+i+1}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is odd} \\ \frac{7n+i-1}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is even}, n \text{ is odd} \\ \frac{7n+i}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is even}, n \text{ is even} \end{cases}$$

$$g(y_i y_{i+1}) = \begin{cases} \frac{8n+i-1}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is odd} \\ \frac{9n+i+3}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is even}, n \text{ is odd} \\ \frac{9n+i-2}{2}; & \text{for } 1 \leq i \leq n-1, \ i \text{ is even}, n \text{ is even.} \end{cases}$$

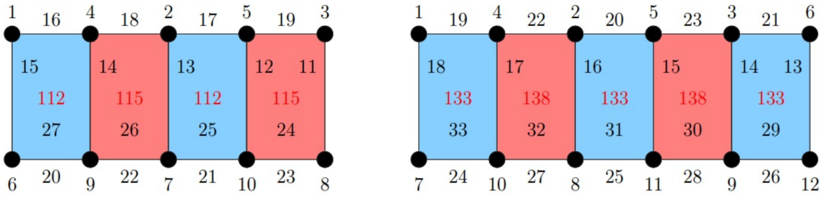
We define face labeling as a g bijective function to the order and edges of L_n plus one until all the elements of L_n is as follows:

$$g(f_i) = \quad 6n - i - 2; \quad \text{for } 1 \leq \mathrm{i} \leq \mathrm{n} - 1.$$

It is easy to get $f \in F(G)$ is a local super antimagic total face labeling of L_n and the face weights for $1 \leq i \leq n-1$ are follows:

$$w(f_i) = \begin{cases} 23n; & \text{for } i \text{ is even} \\ 22n+1; & \text{for } i \text{ is odd}, \ n \text{ is even} \\ 22n+2; & \text{for } i \text{ is odd}, \ n \text{ is odd} \end{cases}$$

The chromatic number of face coloring in ladder graph is at least 2, because ladder graph has at least two adjacent faces. Hence, $\chi_f(L_n) = 2$. Based on Observation 1, we get $\gamma_{latf}(L_n)) \geq 2$



(a) L_5 (b) L_6

Figure 3. Local Super Antimagic Total Face Labeling of (a)L_5 (b)L_6.

and $\gamma_{latf}(L_n) \leq 2$ as a lower bound and an upper bound of local super antimagic total face coloring of ladder graph. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(L_n) = 2$.

2.4. The local super antimagic total face chromatic number of circular ladder graph

Let $n \geq 3$ be a natural number. Circular ladder graph (CL_n) is a graph with vertex set $V(CL_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$, edge set $E(CL_n) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{y_i y_{i+1}; 1 \leq i \leq n-1\} \cup \{y_1 y_n\} \cup \{x_i y_i; 1 \leq i \leq n\}$, and face set $F(CL_n) = \{f_i; 1 \leq i \leq n+1\}$. Hence $|V(CL_n)| = 2n$, $|E(CL_n)| = 3n$, and $|F(CL_n)| = n+1$. In this subsection we determine the local super antimagic total face chromatic number of circular ladder graph as follows.

Theorem 4: *Let $n \geq 3$ be a natural number and CL_n be a circular ladder graph. Then,*

$$\gamma_{lfat}(Cl_n)) = \begin{cases} 3; & \text{for } n \text{ is even} \\ 4; & \text{for } n \text{ is odd.} \end{cases}$$

Proof. To proof a chromatic number of local super antimagic total face coloring of circular ladder graph, we divided the proof into two cases.

Case 1.

For n is even, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(Cl_n) \leq 3$. We define vertex labeling as a g bijective function to natural number until the order of Cl_n is as follows:

$$g(x_i) = \{i;\ 1 \leq i \leq n\}$$

$$g(y_i) = \begin{cases} n+1; & \text{for } i = 1 \\ 2n-i+2; & \text{for } 2 \leq i \leq n \end{cases}$$

We define edge labeling as a g bijective function to the order of Cl_n plus one until the order and edges of Cl_n is as follows:

$$g(x_i y_i) = 3n - i + 1; 1 \leq i \leq n$$

$$g(x_i x_{i+1}) = \begin{cases} 4n; & \text{for } i = 1 \\ 3n+i-1; & \text{for } 2 \leq i \leq n-1 \end{cases}$$

$$g(x_1 x_n) = 4n - 1$$

$$g(y_i y_{i+1}) = \begin{cases} \frac{8n+i+1}{2}; & \text{for } i \text{ is odd} \\ \frac{9n+i+1}{2}; & \text{for } 1 \leq i \leq n-1,\ i \text{ is even, } n \text{ is odd} \end{cases}$$

$$g(y_1 y_n) = 5n; \text{ for } n \text{ is even.}$$

We define face labeling as a g bijective function to the order and edges of Cl_n plus one until all the elements of Cl_n is as follows:

$$g(f_i) = \begin{cases} \frac{10n+i}{2}; & \text{for } 1 \le i \le n,\ i \text{ is even} \\ \frac{11n+i+1}{2}; & \text{for } 1 \le i \le n,\ i \text{ is odd} \end{cases}$$

$$g(f_{n+1}) = 6n + 1.$$

It is easy to get $f \in F(G)$ is a local face super antimagic total face labeling of CL_n and the face weights are follows:

$$w(f_i) = \begin{cases} \frac{45n+10}{2}; & \text{for } 1 \le i \le n,\ i \text{ is odd} \\ \frac{45n+8}{2}; & \text{for } 1 \le i \le n,\ i \text{ is even} \\ 4n^2 + 7n + 1; & i = n + 1. \end{cases}$$

The chromatic number of face coloring in circular ladder graph is at least 2, because circular ladder graph has at least two adjacent faces. Hence, $\chi_f(CL_n) = 2$. Based on Observation 1, we get $\gamma_{latf}(CL_n) \ge 2$ and $\gamma_{latf}(CL_n) \le 2$ as a lower bound and an upper bound of local super antimagic total face coloring of circular ladder graph. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(CL_n) = 2$ for n is even.

Case 2.

For n is odd, we will use the label of elements graph to find chromatic number as $\gamma_{latf}(Cl_n) \le 4$. We define vertex labeling as a g bijective function to natural number until the order of Cl_n is as follows:

$$g(x_i) = \{i;\ 1 \le i \le n\}$$

$$g(y_i) = \begin{cases} n+1; & \text{for } 1 = 1 \\ 2n - i + 2; & \text{for } 2 \le i \le n \end{cases}$$

We define edge labeling as a g bijective function to the order of Cl_n plus one until the order and edges of Cl_n is as follows:

$$g(x_i y_i) = 3n - i + 1; 1 \le i \le n$$

$$g(x_i x_{i+1}) = \begin{cases} 4n; & \text{for } i = 1 \\ 3n + i - 1; & \text{for } 2 \le i \le n-1 \end{cases}$$

$$g(x_1 x_n) = 4n - 1$$

$$g(y_i y_{i+1}) = \begin{cases} \frac{8n+i+1}{2}; & \text{for } i \text{ is odd} \\ \frac{9n+i+1}{2}; & \text{for } 1 \le i \le n-1,\ i \text{ is even},\ n \text{ is odd} \end{cases}$$

$$g(y_1 y_n) = \frac{9n+1}{2}; \text{for } n \text{ is odd}$$

We define face labeling as a g bijective function to the order and edges of Cl_n plus one until all the elements of Cl_n is as follows:

$$g(f_i) = \begin{cases} \frac{10n+i+1}{2}; & \text{for } 1 \le i \le n-2,\ i \text{ is odd} \\ \frac{11n+i-1}{2}; & \text{for } 1 \le i \le n-1,\ i \text{ is even} \\ 6n; & \text{for } i = n \end{cases}$$

$$g(f_{n+1}) = 6n + 1.$$

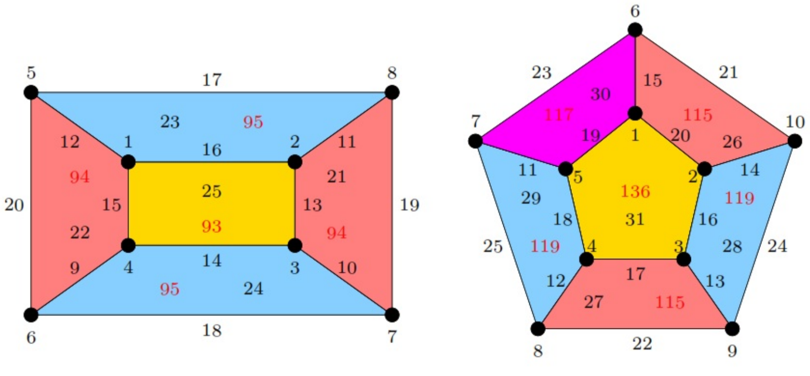


Figure 4. Local Super Antimagic Total Face Labeling of (a)CL_4 (b)CL_5.

It is easy to get $f \in F(G)$ is a local face super antimagic total face labeling of CL_n and the face weights are follows:

$$w(f_i) = \begin{cases} 22n+5; & \text{for } 1 \le i \le n-2,\ i \text{ is odd} \\ 23n+4; & \text{for } 1 \le i \le n-1,\ i \text{ is even} \\ \frac{45n+9}{2}; & \text{for } i = n \\ 4n^2+7n+1; & i = n+1. \end{cases}$$

The chromatic number of face coloring in circular ladder graph is at least 2, because circular ladder graph has at least two adjacent faces. For $n = 3$, $c(f_1) \neq c(f_2)$, $c(f_2) \neq c(f_3)$, and $c(f_3) \neq (f_1)$. We can see that every faces of CL_3 must have 3 different colors. Hence, $\chi_f(CL_n) = 3$. Based on Observation 1, we get $\gamma_{latf}(CL_n) \geq 3$ and $\gamma_{latf}(CL_n) \leq 3$ as a lower bound of local super antimagic total face coloring of circular ladder graph. We combine an upper bound value with a lower bound value. We get $\gamma_{latf}(CL_n) = 3$ for n is odd.

3. Conclusion

In this section, we have given the result of local super antimagic total face coloring of planar graphs. We determine the local super antimagic total face chromatic number of planar graphs such as wheel graph, jahangir graph, ladder graph, and circular ladder graph. We provide an open problem for further researcher as follows.

Open Problem 1 *Local super antimagic total face coloring of planar graph from a convex polyhedron.*

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