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Local antimagic vertex dynamic coloring of some graphs family

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Abstract. All graphs in this paper are simple and connected graph. A vertex dynamic coloring is a proper vertex k-coloring of graph G such that $|c(N(v_i))| \geq \min\{r, d(v)\}$ and the neighbourhood of vertex u has different colors. A bijection $f : E(G) \to \{1, 2, 3, ..., m\}$ is called a local antimagic dynamic coloring, such that: (1) if $uv \in E(G)$, where $w(u) = \sum_{e \in E(u)} f(e)$ and (2) for each vertex $v \in V(G)$, $|w(N(v_i))| \geq \min\{r, d(v_i)\}$. The local antimagic vertex dynamic chromatic number denoted by χ_r^{la}(G)is the minimum number of colors needed to color G in such a way the graph G to be local antimagic vertex dynamic graph. In this paper, we will study the existence of the local antimagic vertex dynamic chromatic number of some graph classes, namely caterpilar, doublebroom, broom and sun graph.

1. Introduction

In this paper, all graph are simple and connected graph namely caterpilar, doublebroom, broom and sun graph. A graph $G = (V,E)$, V(G) is non-empty vertex set and E(G) is the set that may be empty pairs of u,v where u,$v \in$ V(G). Suppose u,v is vertex of graph G, u is adjacent to v when there has side connecting u and v. N(u) is a notation of all the neighbors u.

The concept r-dynamic coloring of a graph G induces a proper k-coloring of graph G with the result that the neighbors of any vertex v receive at minimum$\{r, d(v)\}$ different colors. Vertex dynamic coloring of graph G is giving different colors to each neighbors of any vertex v receive at minimum $\{r, d(v)\}$. This concept was introduced by Montgomery [7]. Some paper discusses about r-dynamic chromatic numbers, for instance in [1], [2], [5], [6], [7], [8], [9], [10], [11], [13], and [12].

This paper introduce new concept which have combines local antimagic labeling and r-dynamic.

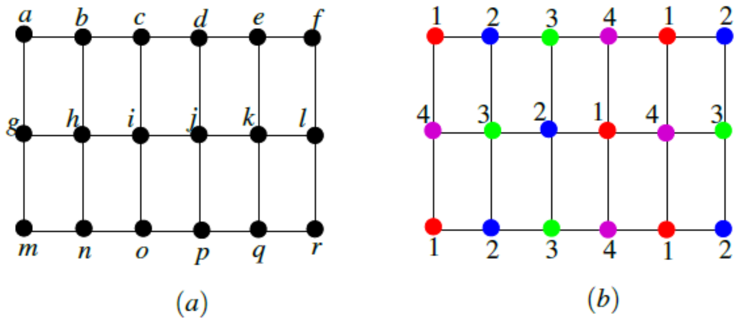

Figure 1. a.) Graph of group ; (b.)Patients in Big Rooms

Definition 1.1. *[3] Let G = (V,E) be a graph of order n and size m having no isolated vertices. A bijection f : $E(G) \rightarrow \{1,2,3,...,\ m\}$ is called local r-dyamic coloring, such that : (1) If uv ϵ E(G), then $w(u) \neq w(v)$, where $w(u) = \Sigma_{e \in E(u)} f(e)$; (2) For each v ϵ V(G), $|w(N(v))| \geq min\{r, d(v)\}$.*

Definition 1.2. *[3] The local antimagic r-dynamic chromatic number, denoted by $\chi_r^{la}(G)$ is the minimum k with the result that graph G has a local antimagic r-dynamic vertex k-coloring induced by local antimagic labelings.*

Potential aplication is in a hospital because there are big rooms. In every big room, there were some beds which were put side by side. There were so many patients in the hospital, therefore, the sick people who are enter to the room must be put maximumly.

The doctors anticipated the condition so that the disease will not spread to other patients. For these anticipations, the doctors separated the patients who have the same blood type, because the patients who have the same blood type will easily infected by the disease. So, there is no patient who is close to other patients who have the same blood type. Patients in big rooms are represented as vertices and big rooms as edge. This can be presented as in a figure. This can be presented as in a figure 1 .

No patient is close to the same type of blood then the big room has a design like figure 1. Red is a patient with blood type A, blue is a patient with blood type B, green is a patient with blood type O, and purple is a patient with blood type AB. That is one example of a vertex dynamic coloring application in life.

Another application is the food menu on the food court. In a food court area, there are 9 places available for several types of food. But there cannot be 2 similar foods close together. This can be represented as in figure 2. The vertex represents the food stand and the edge represents the road. The vertex coloring of the graph is:

There is no food stand that sells similar food items in adjacent places like figure 2. Food type 1 is represented in red, food type 2 is represented in blue and the third type of food is represented in green so that there are maximum 3 types of food that can be sold in the food court.

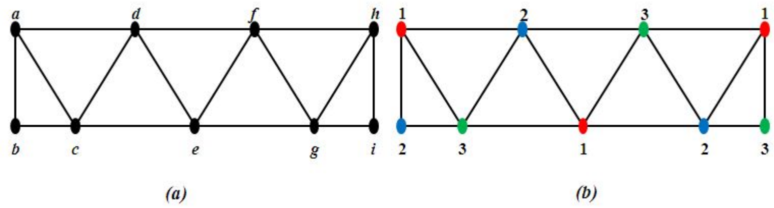

Figure 2. a.) Graph of group ; (b.)The food menu on the food court

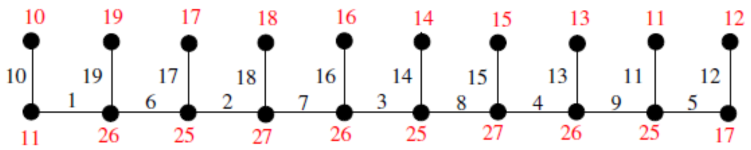

Figure 3. Caterpilar Graph $(C_{10,1})$

2. Main Results

In this section, we we study the existence of local antimagic vertex dynamic coloring of some graphs. We determine the exact values of the local antimagic coloring of sun (M_n), caterpilar $(C_{n,1})$, doublebroom $(DBr_{n,m})$ and broom $(Br_{n,m})$.

Theorem 1. *Let $G \cong (C_{n,1})$ be a caterpilar graph of $(C_{n,1})$ with $n \geq 3$, $\chi_r^{la}(G)$ of $(C_{n,1})$ is:*

$$\chi_r^{la}((C_{n,1})) \leq \begin{cases} n+2; \; n = \; 3 \\ n+3; \; n \geq \; 4 \end{cases}$$

Proof. $(C_{n,1})$ is a connected graph with V$((C_{n,1}))$ = $\{x_i; y_i; 1 \leq i \leq n\}$ and E$((C_{n,1}))$ = $\{x_iy_i; 1 \leq i \leq n\} \bigcup \{x_ix_{i+1}; 1 \leq i \leq n-1\}$. Then $|V((C_{n,1}))| = 2n$ and $|E((C_{n,1}))|$ = $2n$-1.

Define a bijection $f : E((C_{n,1})) \rightarrow$ {1,2,3,... $|E((C_{n,1}))|$} with the following function :

For $n = 3$:

$$f((x_iy_i)) = \begin{cases} n+1 \; ; \; i \; = \; 1 \\ n+2 \; ; \; i \; = \; 2 \\ n+3 \; ; \; i \; = \; n \end{cases}$$

For $n = 4$:

$$f((x_iy_i)) = \begin{cases} n \; ; \; i \; = \; 1 \\ n+3 \; ; \; i \; = \; 2 \\ n+1 \; ; \; i \; = \; 3 \\ n+2 \; ; \; i \; = \; n \end{cases}$$

For $n = 5$:

$$f((x_i y_i)) = \begin{cases} n+1 \; ; \; i \; = \; 1 \\ n+3 \; ; \; i \; = \; 2 \\ n+4 \; ; \; i \; = \; 3 \\ n+2 \; ; \; i \; = \; 4 \\ n \; ; \; i \; = \; n \end{cases}$$

For $n = 6$:

$$f((x_i y_i)) = \begin{cases} n \; ; \; i \; = \; 1 \\ n+4 \; ; \; i \; = \; 2 \\ n+5 \; ; \; i \; = \; 3 \\ n+3 \; ; \; i \; = \; 4 \\ n+1 \; ; \; i \; = \; 5 \\ n+2 \; ; \; i \; = \; n \end{cases}$$

For $n = \text{odd}$:

$$f((x_i y_i)) = \begin{cases} n+1 \; ; \; i \; = \; 1 \\ n \; ; \; i \; = \; n \end{cases}$$

For $i = \text{even}$:

$$f((x_i y_i)) = \begin{cases} n \; ; \; i \; = \; 1 \\ n+1 \; ; \; i \; = \; n-1 \end{cases}$$

For n is even and $n \; \geq \; 7$; $n = 0$ mod 6:

$$f((x_i y_i)) = \begin{cases} 2n-i-1 \; ; \; i \; \equiv \; 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \; \equiv \; 1 \; mod \; 3 \\ 2n-i \; ; \; i \; \equiv \; 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \; \geq \; 7$; $n = 1$ mod 6:

$$f((x_i y_i)) = \begin{cases} 2n-i+2 \; ; \; i \; \equiv \; 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \; \equiv \; 1 \; mod \; 3 \\ 2n-i \; ; \; i \; \equiv \; 2 \; mod \; 3 \end{cases}$$

For n is even and $n \; \geq \; 7$; $n = 2$ mod 6:

$$f((x_i y_i)) = \begin{cases} 2n-i+1 \; ; \; i \; \equiv \; 0 \; mod \; 3 \\ 2n-i \; ; \; i \; \equiv \; 1 \; mod \; 3 \\ 2n-i-1 \; ; \; i \; \equiv \; 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \; \geq \; 7$; $n = 3$ mod 6:

$$f((x_i y_i)) = \begin{cases} 2n-i \; ; \; i \; \equiv \; 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \; \equiv \; 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \; \equiv \; 2 \; mod \; 3 \end{cases}$$

For n is even and $n \; \geq \; 7$; $n = 4$ mod 6:

$$f((x_i y_i)) = \begin{cases} 2n-i \; ; \; i \; \equiv \; 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \; \equiv \; 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \; \equiv \; 2 \; mod \; 3 \end{cases}$$

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For n is odd and $n \geq 7$; $n = 5$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i+2 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n \geq 3$:

$$f((x_ix_{i+1})) = \begin{cases} \frac{i+1}{2} \ ; n = odd \ i \equiv odd \\ \frac{n+i+1}{2} \ ; n = odd \ i \equiv even \\ \frac{i+1}{2} \ ; n = even \ ; \ i \equiv odd \\ \frac{n+i}{2} \ ; n = even \ ; \ i \equiv even \end{cases}$$

Hence f is a local antimagic labelling of $(C_{n,1})$ and we have the vertex weight as follows :

For $n = 3$:

$$W((y_i)) = \begin{cases} n+1 \ ; \ i = 1 \\ n+2 \ ; \ i = 2 \\ n+3 \ ; \ i = n \end{cases}$$

$$W((x_i)) = \begin{cases} n+2 \ ; \ i = 1 \\ 2n \ ; \ i = 2 \\ 2n+2 \ ; \ i = n \end{cases}$$

For $n = 4$:

$$W((y_i)) = \begin{cases} n \ ; \ i = 1 \\ n+3 \ ; \ i = 2 \\ n+1 \ ; \ i = 3 \\ n+2 \ ; \ i = n \end{cases}$$

$$W((x_i)) = \begin{cases} n+i \ ; \ i \equiv 1 \ mod \ 3 \\ 2n+3 \ ; \ i = 2 \\ 2n+2 \ ; \ i = 3 \end{cases}$$

For $n = 5$:

$$W((y_i)) = \begin{cases} n+1 \ ; \ i = 1 \\ n+3 \ ; \ i = 2 \\ n+4 \ ; \ i = 3 \\ n+2 \ ; \ i = 4 \\ n \ ; \ i = n \end{cases}$$

$$W((x_i)) = \begin{cases} n+2i \ ; \ i \equiv 1 \ mod \ 3 \\ 3n-i-1 \ ; \ i \equiv 2 \ mod \ 3 \\ 3n-1 \ ; \ i = 3 \end{cases}$$

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For $n = 6$:

$$W((y_i)) = \begin{cases} n\ ;\ i\ =\ 1 \\ n+4\ ;\ i\ =\ 2 \\ n+5\ ;\ i\ =\ 3 \\ n+3\ ;\ i\ =\ 4 \\ n+1\ ;\ i\ =\ 5 \\ n+2\ ;\ i\ =\ n \end{cases}$$

$$W((x_i)) = \begin{cases} n+1\ ;\ i\ =\ 1 \\ \frac{5n}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \\ \frac{5n+4}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-1\ ;\ i\ =\ n \end{cases}$$

For $n\ \geq\ 7$; $n = 0$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n-2}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 1$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n+3}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+1}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n-1}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 2$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n+2}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n-2}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 3$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n-1}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n-3}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n+1}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 4$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+4}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 5$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n+3}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+1}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n-1}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For n = odd :

$$W(y_i)) = \begin{cases} n+1 \; ; \; i \;=\; 1 \\ n \; ; \; i \;=\; n \end{cases}$$

For n = even :

$$W(y_i)) = \begin{cases} n \; ; \; i \;=\; 1 \\ n+1 \; ; \; i \;=\; n-1 \end{cases}$$

For n is even and $n \;\geq\; 7$; $n = 0$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i-1 \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \;\geq\; 7$; $n = 1$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i+2 \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

For n is even and $n \;\geq\; 7$; $n = 2$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i+1 \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i-1 \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \;\geq\; 7$; $n = 3$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

For n is even and $n \;\geq\; 7$; $n = 4$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \;\geq\; 7$; $n = 5$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i+2 \; ; \; i \;\equiv\; 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \;\equiv\; 1 \; mod \; 3 \\ 2n-i \; ; \; i \;\equiv\; 2 \; mod \; 3 \end{cases}$$

$$W((y_1)) = \begin{cases} n+1 \; ; \; for \; n \;=\; odd \\ 2n-1 \; ; \; for \; n \;= even \end{cases}$$

$$W((y_n)) = \begin{cases} 2n-1 \; ; \; for \; n \;=\; odd \\ \frac{5n-2}{2} \; ; \; for \; n \;=\; even; \; n \;\neq\; 10 \\ \frac{3n+4}{2} \; ; \; n \;=\; 10 \end{cases}$$

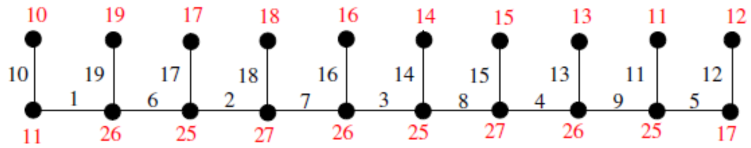

Figure 4. Caterpilar Graph $(C_{10,1})$

It is clear that f is local antimagic vertex dynamic coloring of $(C_{n,1})$. Finally, $\chi_r^{la} \leq n{+}2$ *for* $n = 3$ and $\chi_r^{la} \leq n+3$ *for* $n \geq 4$. The proof is complete.$(C_{n,1})$ is a connected graph with V$((C_{n,1})) = \{x_i; y_i; 1 \leq i \leq n\}$ and E$((C_{n,1})) = \{x_i y_i; 1 \leq i \leq n\}$ $\bigcup \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. Then $|V((C_{n,1}))| = 2n$ and $|E((C_{n,1}))| = 2n$-1.

Define a bijection $f : E((C_{n,1})) \rightarrow \{1,2,3,\ldots\ |E((C_{n,1}))|\}$ with the following function :

For $n = 3$:

$$f((x_i y_i)) = \begin{cases} n+1 \ ; \ i \ = \ 1 \\ n+2 \ ; \ i \ = \ 2 \\ n+3 \ ; \ i \ = \ n \end{cases}$$

For $n = 4$:

$$f((x_i y_i)) = \begin{cases} n \ ; \ i \ = \ 1 \\ n+3 \ ; \ i \ = \ 2 \\ n+1 \ ; \ i \ = \ 3 \\ n+2 \ ; \ i \ = \ n \end{cases}$$

For $n = 5$:

$$f((x_i y_i)) = \begin{cases} n+1 \ ; \ i \ = \ 1 \\ n+3 \ ; \ i \ = \ 2 \\ n+4 \ ; \ i \ = \ 3 \\ n+2 \ ; \ i \ = \ 4 \\ n \ ; \ i \ = \ n \end{cases}$$

For $n = 6$:

$$f((x_i y_i)) = \begin{cases} n \ ; \ i \ = \ 1 \\ n+4 \ ; \ i \ = \ 2 \\ n+5 \ ; \ i \ = \ 3 \\ n+3 \ ; \ i \ = \ 4 \\ n+1 \ ; \ i \ = \ 5 \\ n+2 \ ; \ i \ = \ n \end{cases}$$

For $n =$ odd :

$$f((x_i y_i)) = \begin{cases} n+1 \ ; \ i \ = \ 1 \\ n \ ; \ i \ = \ n \end{cases}$$

For $i =$ even :

$$f((x_i y_i)) = \begin{cases} n \ ; \ i \ = \ 1 \\ n+1 \ ; \ i \ = \ n-1 \end{cases}$$

For n is even and $n \geq 7$; $n = 0$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i-1 \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 1$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i+2 \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For n is even and $n \geq 7$; $n = 2$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i+1 \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i-1 \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 3$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For n is even and $n \geq 7$; $n = 4$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i+2 \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i+1 \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 5$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i+2 \; ; \; i \equiv 0 \; mod \; 3 \\ 2n-i+1 \; ; \; i \equiv 1 \; mod \; 3 \\ 2n-i \; ; \; i \equiv 2 \; mod \; 3 \end{cases}$$

For $n \geq 3$:

$$f((x_ix_{i+1})) = \begin{cases} \frac{i+1}{2} \; ; n = odd \;\; i \equiv odd \\ \frac{n+i+1}{2} \; ; n = odd \;\; i \equiv even \\ \frac{i+1}{2} \; ; n = even \; ; \; i \equiv odd \\ \frac{n+i}{2} \; ; n = even \; ; \; i \equiv even \end{cases}$$

Hence f is a local antimagic labelling of $(C_{n,1})$ and we have the vertex weight as follows :

For $n = 3$:

$$W((y_i)) = \begin{cases} n+1 \; ; \; i = 1 \\ n+2 \; ; \; i = 2 \\ n+3 \; ; \; i = n \end{cases}$$

$$W((x_i)) = \begin{cases} n+2 \; ; \; i = 1 \\ 2n \; ; \; i = 2 \\ 2n+2 \; ; \; i = n \end{cases}$$

For $n=4$:

$$W((y_i))=\begin{cases} n\ ;\ i\ =\ 1 \\ n+3\ ;\ i\ =\ 2 \\ n+1\ ;\ i\ =\ 3 \\ n+2\ ;\ i\ =\ n \end{cases}$$

$$W((x_i))=\begin{cases} n+i\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n+3\ ;\ i\ =\ 2 \\ 2n+2\ ;\ i\ =\ 3 \end{cases}$$

For $n=5$:

$$W((y_i))=\begin{cases} n+1\ ;\ i\ =\ 1 \\ n+3\ ;\ i\ =\ 2 \\ n+4\ ;\ i\ =\ 3 \\ n+2\ ;\ i\ =\ 4 \\ n\ ;\ i\ =\ n \end{cases}$$

$$W((x_i))=\begin{cases} n+2i\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 3n-i-1\ ;\ i\ \equiv\ 2\ mod\ 3 \\ 3n-1\ ;\ i\ =\ 3 \end{cases}$$

For $n=6$:

$$W((y_i))=\begin{cases} n\ ;\ i\ =\ 1 \\ n+4\ ;\ i\ =\ 2 \\ n+5\ ;\ i\ =\ 3 \\ n+3\ ;\ i\ =\ 4 \\ n+1\ ;\ i\ =\ 5 \\ n+2\ ;\ i\ =\ n \end{cases}$$

$$W((x_i))=\begin{cases} n+1\ ;\ i\ =\ 1 \\ \frac{5n}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \\ \frac{5n+4}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-1\ ;\ i\ =\ n \end{cases}$$

For $n\ \geq\ 7$; $n=0$ mod 6 :

$$W((x_i)=\begin{cases} \frac{5n-2}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n=1$ mod 6 :

$$W((x_i)=\begin{cases} \frac{5n+3}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+1}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n-1}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n=2$ mod 6 :

$$W((x_i)=\begin{cases} \frac{5n+2}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \\ \frac{5n+2}{2}\ ;\ i\ \equiv\ 1\ mod\ 3 \\ \frac{5n-2}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

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For $n \geq 7$; $n = 3$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n-1}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n-3}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n+1}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n \geq 7$; $n = 4$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+4}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n+2}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n \geq 7$; $n = 5$ mod 6 :

$$W((x_i) = \begin{cases} \frac{5n+3}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+1}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n-1}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n =$ odd :

$$W(y_i)) = \begin{cases} n+1 \ ; \ i = 1 \\ n \ ; \ i = n \end{cases}$$

For $n =$ even :

$$W(y_i)) = \begin{cases} n \ ; \ i = 1 \\ n+1 \ ; \ i = n-1 \end{cases}$$

For n is even and $n \geq 7$; $n = 0$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i-1 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 1$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i+2 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For n is even and $n \geq 7$; $n = 2$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i+1 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i-1 \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 3$ mod 6:

$$W(y_i)) = \begin{cases} 2n-i \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i+2 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i+1 \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For n is even and $n \geq 7$; $n = 4$ mod 6:

$$W(y_i)) = \begin{cases} 2n - i \ ; \ i \equiv 0 \ mod \ 3 \\ 2n - i + 2 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n - i + 1 \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For n is odd and $n \geq 7$; $n = 5$ mod 6:

$$W(y_i)) = \begin{cases} 2n - i + 2 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n - i + 1 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n - i \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

$$W((y_1)) = \begin{cases} n + 1 \ ; \ for \ n = odd \\ 2n - 1 \ ; \ for \ n = even \end{cases}$$

$$W((y_n)) = \begin{cases} 2n - 1 \ ; \ for \ n = odd \\ \frac{5n-2}{2} \ ; \ for \ n = even; \ n \neq 10 \\ \frac{3n+4}{2} \ ; \ n = 10 \end{cases}$$

It is clear that f is local antimagic vertex dynamic coloring of $(C_{n,1})$. Finally, $\chi_r^{la} \leq n{+}2 \ for \ n = 3$ and $\chi_r^{la} \leq n + 3 \ for \ n \geq 4$. The proof is complete.

Theorem 2. *Let $G \cong M_n$ be a sun graph $\{M_n\}$ with $n \geq 3$, $\chi_r^{la}(G)$ of $\{M_n\}$ is :*

$$\chi_r^{la}(\{M_n\}) \leq \begin{cases} 2n \ ; \ 3 \leq n \leq 4 \\ n + 4 \ ; \ n = \ odd \ ; \ n = 5, 6 \\ n + 5 \ ; \ n = \ even \ ; \ n = 7 \end{cases}$$

Proof. Sun graph is a connected graph with V(M_n) = $\{x_i; y_i; 1 \leq i \leq n\}$ and E(M_n) = $\{x_i y_i; 1 \leq i \leq n\} \bigcup \{x_i x_{i+1}; 1 \leq i \leq n-1\} \bigcup \{x_i x_n; 1\}$. Then $|V(M_n)| = 2n$ and $|E(M_n)| = 2n$.

Define a bijection $f : E(M_n) \rightarrow$ {1,2,3,... $|E(M_n)|$} with the following function :

For $n = 3$:

$$f((x_i x_{i+1})) = \begin{cases} n + 2 \ ; \ i \equiv 1 \\ n + 1; \ i = 2 \\ 2n \ ; \ i = 3 \end{cases}$$

For $n = 4$:

$$f((x_i x_{i+1})) = \begin{cases} \frac{2i-3n-14}{-3} \ ; \ i \equiv 1 \ mod \ 3 \\ n + 3; \ i = 2 \\ n + 1 \ ; \ i = 3 \end{cases}$$

For $n = 5$:

$$f((x_i x_{i+1})) = \begin{cases} \frac{3n+2i+1}{3} \quad i \equiv 1 \ mod \ 3 \\ \frac{3n+2i+2}{3} \quad i \equiv 2 \ mod \ 3 \\ 2n \ ; \ i = 3 \end{cases}$$

For $n = 6$:

$$f((x_i x_{i+1})) = \begin{cases} n - i + 7 \quad i \equiv 1 \ mod \ 3 \\ n - i + 6 \ ; \ i \equiv 2 \ mod \ 3 \\ n - i + 8 \ ; \ i \equiv 0 \ mod \ 3 \end{cases}$$

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For n is odd and $n \geq 7$; $n = 1$ mod 6 :

$$f((x_iy_i)) = \begin{cases} n+i;\ 1 \leq i \leq 2 \\ 2n-i+3\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i+2\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+1\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i-1\ ;\ i\ =\ n \end{cases}$$

For n is odd and $n \geq 7$; $n = 5$ mod 6:

$$f((x_iy_i)) = \begin{cases} n+i;\ 1 \leq i \leq 2 \\ 2n-i+1\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i+3\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+2\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i\ ;\ i\ =\ n \end{cases}$$

For n is odd and $n \geq 7$; $n = 3$ mod 6:

$$f((x_iy_i)) = \begin{cases} n+i;\ 1 \leq i \leq 2 \\ 2n-i+2\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i+1\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+3\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i\ ;\ i\ =\ n \end{cases}$$

For n is odd and $n \geq 7$:

$$f((x_ix_{i+1})) = \begin{cases} \frac{i+1}{2}\ ;\ i\ =\ odd \\ \frac{n+i+1}{2}\ ;\ i\ =\ even \end{cases}$$

For n is even and $n \geq 7$; $n = 0$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i-1\ ;\ i\ =\ 1 \\ 2n-i+1\ ;\ i\ =\ 2 \\ 2n-i\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i-1\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+1\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i\ ;\ i\ =\ n \end{cases}$$

For n is even and $n \geq 7$; $n = 4$ mod 6:

$$f((x_iy_i)) = \begin{cases} 2n-i\ ;\ i\ =\ 1 \\ 2n-i-1\ ;\ i\ =\ 2 \\ 2n-i+1\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i-1\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i\ ;\ i\ =\ n \end{cases}$$

For n is even and $n \geq 7$; $n = 2$ mod 6 :

$$f((x_iy_i)) = \begin{cases} 2n-i \ ; \ 1 \leq i \leq 2 \\ 2n-i-1 \ ; \ i \equiv 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \equiv 1 \ mod \ 3 \\ 2n-i \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n \geq 7$:

$$f((x_ix_{i+1})) = \begin{cases} \frac{i+1}{2} \ ; n = odd \ i \equiv odd \\ \frac{n+i+1}{2} \ ; n = odd \ i \equiv even \\ \frac{i+1}{2} \ ; n = even \ i \equiv odd \\ \frac{n+i}{2} \ ; n = even \ i \equiv even \end{cases}$$

Hence f is a local antimagic labeling of Sun (M_n) and we have the vertex weighting as follows :

For $n = 0$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n-2}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n+2}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n = 1$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n+7}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+5}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n+3}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n = 2$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n-2}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+2}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n = 3$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n+5}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+3}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n+7}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n = 4$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n+2}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n-2}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

For $n = 5$ mod 6 :

$$W(x_i) = \begin{cases} \frac{5n+3}{2} \ ; \ i \equiv 0 \ mod \ 3 \\ \frac{5n+7}{2} \ ; \ i \equiv 1 \ mod \ 3 \\ \frac{5n-5}{2} \ ; \ i \equiv 2 \ mod \ 3 \end{cases}$$

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For n = even :

$$W(x_i) = \begin{cases} 3n-1\ ;\ i\ =\ 1 \\ 3n\ ;\ i\ =\ 2 \\ \frac{5n-5}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \end{cases}$$

For n = odd :

$$W(x_i) = \begin{cases} \frac{3n+5}{2}\ ;\ i\ =\ 1 \\ \frac{3n+9}{2}\ ;\ i\ =\ 2 \end{cases}$$

For $i \equiv n$:

$$W(x_i) = \begin{cases} \frac{7n}{2}\ ;\ n\ =\ even \\ \frac{7n-1}{2}\ ;\ n\ =\ odd \end{cases}$$

For $n\ =\ 3$:

$$W(x_i) = \begin{cases} 3n\quad i\ =\ 1 \\ 2n+1\ ;\ i\ =\ 2 \\ 3n-1\ ;\ i\ =\ 3 \end{cases}$$

For $n\ =\ 4$:

$$W(x_i) = \begin{cases} 3n+1\quad i\ =\ 1 \\ 3n-1\ ;\ i\ =\ 2 \\ 3n-2\ ;\ i\ =\ 3 \\ 3n\ ;\ i\ =\ 4 \end{cases}$$

For $n\ =\ 5$:

$$W(x_i) = \begin{cases} 2n\quad i\ =\ 1 \\ 2n+2\ ;\ i\ =\ 2 \\ 3n+1\ ;\ i\ =\ 3 \\ 3n\ ;\ i\ =\ 4 \\ 2n\ ;\ i\ =\ 5 \end{cases}$$

For $n\ =\ 6$:

$$W(x_i) = \begin{cases} 3n+1\quad i\ =\ 1 \\ \frac{5n}{2}\ ;\ i\ \equiv\ 2\ mod\ 3 \\ \frac{5n+4}{2}\ ;\ i\ \equiv\ 0\ mod\ 3 \end{cases}$$

For $n\ \geq\ 7$; $n = 1$ mod 6 :

$$W(y_j) = \begin{cases} n+i;\ 1 \leq i \leq 2 \\ 2n-i+3\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i+2\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+1\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i-1\ ;\ i\ =\ n \end{cases}$$

For $n\ \geq\ 7$; $n = 5$ mod 6:

$$W(y_j) = \begin{cases} n+i;\ 1 \leq i \leq 2 \\ 2n-i+1\ ;\ i\ \equiv\ 0\ mod\ 3 \\ 2n-i+3\ ;\ i\ \equiv\ 1\ mod\ 3 \\ 2n-i+2\ ;\ i\ \equiv\ 2\ mod\ 3 \\ n+i\ ;\ i\ =\ n \end{cases}$$

For $n \ \geq \ 7$; $n = 3$ mod 6:

$$W(y_j) = \begin{cases} n+i; \ 1 \leq i \leq 2 \\ 2n-i+2 \ ; \ i \ \equiv \ 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \ \equiv \ 1 \ mod \ 3 \\ 2n-i+3 \ ; \ i \ \equiv \ 2 \ mod \ 3 \\ n+i \ ; \ i \ = \ n \end{cases}$$

For n is odd and $n \ \geq \ 7$:

$$W(y_j) = \begin{cases} \frac{i+1}{2} \ ; \ i \ = \ odd \\ \frac{n+i+1}{2} \ ; \ i \ = \ even \end{cases}$$

For $n \ \geq \ 7$; $n = 0$ mod 6:

$$W(y_j) = \begin{cases} 2n-i-1 \ ; \ i \ = \ 1 \\ 2n-i+1 \ ; \ i \ = \ 2 \\ 2n-i \ ; \ i \ \equiv \ 0 \ mod \ 3 \\ 2n-i-1 \ ; \ i \ \equiv \ 1 \ mod \ 3 \\ 2n-i+1 \ ; \ i \ \equiv \ 2 \ mod \ 3 \\ n+i \ ; \ i \ = \ n \end{cases}$$

For $n \ \geq \ 7$; $n = 4$ mod 6:

$$W(y_j) = \begin{cases} 2n-i \ ; \ i \ = \ 1 \\ 2n-i-1 \ ; \ i \ = \ 2 \\ 2n-i+1 \ ; \ i \ \equiv \ 0 \ mod \ 3 \\ 2n-i \ ; \ i \ \equiv \ 1 \ mod \ 3 \\ 2n-i-1 \ ; \ i \ \equiv \ 2 \ mod \ 3 \\ n+i \ ; \ i \ = \ n \end{cases}$$

For $n \ \geq \ 7$; $n = 2$ mod 6:

$$W(y_j) = \begin{cases} 2n-i \ ; \ 1 \leq i \leq 2 \\ 2n-i-1 \ ; \ i \ \equiv \ 0 \ mod \ 3 \\ 2n-i+1 \ ; \ i \ \equiv \ 1 \ mod \ 3 \\ 2n-i \ ; \ i \ \equiv \ 2 \ mod \ 3 \end{cases}$$

For n is odd and $n \ \geq \ 7$:

$$W(y_j) = \begin{cases} \frac{i+1}{2} \ ; \ i \ = \ odd \\ \frac{n+i}{2} \ ; \ i \ = \ even \end{cases}$$

It is clear that f is local antimagic vertex dynamic coloring of (M_n). Finally, $\chi_r^{la} \leq 2n \ for \ 3 \ \leq \ n \ \leq \ 4$, $\chi_r^{la} \leq n{+}4 \ for \ n =$ odd ; $n = 6$ and $\chi_r^{la} \leq n+5 \ for \ n \ \geq \ 7$; n = even. The proof is complete.

Theorem 3. *Let $G \cong DBr_{n,m}$ be a connected graph with $n \geq 3$, $\chi_r^{la}(G)$ of $DBr_{n,m}$ is :*

$$\chi_r^{la}(DBr_{n,m}) \leq \begin{cases} \frac{4m+n+3}{2};\ n =\ odd;\ n \geq 3 \\ \frac{4m+n+2}{2};\ n =\ even;\ n \geq 3 \end{cases}$$

Proof. $DBr_{n,m}$ is a connected graph with V($DBr_{n,m}$) $= \{x_i; 1 \leq i \leq n\} \bigcup \{y_j; 1 \leq j \leq m\} \bigcup \{z_k; 1 \leq k \leq m\}$ and E($DBr_{n,m}$) $= \{x_1y_j; 1 \leq i \leq m; 1 \leq j \leq m\} \bigcup \{x_nz_k; 1 \leq i \leq m; 1 \leq j \leq m\} \bigcup \{x_ix_{i+1}; 1 \leq i \leq n-1\}$. $|V(DBr_{n,m})| = 2m+n$ and $|E(DBr_{n,m})| = 2m+n$-1.
Case 1: Define a bijection $f\ :\ E(DBr_{n,m\ =\ odd}) \rightarrow \{$1,2,3,... $|E(DBr_{n,m})|\}$ with the following function :

For $n =$ even ; $1 \leq i\ \leq \frac{n}{2}$:

$$f((x_ix_{i+1})) = \begin{cases} 2i+6\ ;\ i\ =\ even \\ 2i+9\ ;\ i\ =\ odd \end{cases}$$

For $n = 1$ mod 4 ; $1 \leq i\ \leq \frac{n+3}{2} - 4$:

$$f((x_ix_{i+1})) = \begin{cases} 2i+6\ ;\ i\ =\ even \\ 2i+9\ ;\ i\ =\ odd \end{cases}$$

For $n = 3$ mod 4 ; $1 \leq i\ \leq \frac{n+3}{2} - 3$:

$$f((x_ix_{i+1})) = \begin{cases} 2i+6\ ;\ i\ =\ even \\ 2i+9\ ;\ i\ =\ odd \end{cases}$$

For $n =$ even ; $\frac{n}{2} + 1 \leq n-1$:

$$f((x_ix_{i+1})) = \begin{cases} 2n-2i+8\ ;\ i\ =\ even \\ 2n-2i+7;\ i\ =\ odd \end{cases}$$

For $n = 1$ mod 4 ; $\frac{n+5}{2} \leq n-1$:

$$f((x_ix_{i+1})) = \begin{cases} 2n-2i+7\ ;\ i\ =\ even \\ 2n-2i+8\ ;\ i\ =\ odd \end{cases}$$

For $n = 1$ mod 4 ; $\frac{n+3}{2} - 3 \leq \frac{n+1}{2}$:

$$f((x_ix_{i+1})) = \begin{cases} n+7\ ;\ i\ =\ even \\ \frac{2i-5n-23}{-4}\ ;\ i\ =\ odd \end{cases}$$

For $n = 3$ mod 4 ; $\frac{n-1}{2} \leq \frac{n+3}{2}$:

$$f((x_ix_{i+1})) = \begin{cases} \frac{3n+2i+19}{4}\ ;\ i\ =\ odd \\ n+7\ ;\ i\ =\ even \end{cases}$$

For $m = \text{odd}$; y_j ; $1 \leq j \leq m$:

$$f((x_1 y_j)) = \begin{cases} 2j-1 \; ; \; j \; = \; odd \\ 2j \; ; \; j \; = \; even \\ 2n-1 \; ; \; j \; = \; m \end{cases}$$

For $m = \text{odd}$; z_k ; $1 \leq k \leq m$:

$$f((x_n z_k)) = \begin{cases} 2k \; ; \; k \; = \; odd \\ 2k-1 \; ; \; k \; = \; even \\ 2m \; ; \; k \; = \; m \end{cases}$$

Hence f is a local antimagic labelling of $DBr_{n,m}$ and we have the vertex weight as follows :

$$W((x_i) = \begin{cases} 4i+9 \; ; \; for & \text{n} = \; odd; 3 \; \leq \; i \; \leq \; \frac{n-3}{2} \\ & \text{n} = \; even; 2 \; \leq \; i \; \leq \; \frac{n-2}{2} \end{cases}$$

$$W((x_i) = \begin{cases} 4n-4i+13 \; ; \; for & \text{n} = \; odd; \; i \; \geq \; \frac{n-1}{2}+3 \\ & \text{n} = \; even; \; i \; \geq \; \frac{n-2}{2}+2 \end{cases}$$

$$W((x_i) = \begin{cases} 2n+6 \; ; \; for & \text{n} = \; 1 \; mod \; 4 \; ; \text{n} \neq \; 5 \; ; \; i \; \equiv \; \frac{n+3}{2} \\ & \text{n} = \; 3 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n-1}{2} \\ 2n+8 \; ; \; for & \text{n} = \; 0 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n}{2}+1 \\ & \text{n} = \; 1 \; mod \; 4 \; ; \text{n} \neq \; 5 \; ; \; i \; \equiv \; \frac{n-1}{2} \\ & \text{n} = \; 1 \; mod \; 4 \; ; \text{n} \neq \; 5 \; ; \; i \; \equiv \; \frac{n+3}{2} \\ & \text{n} = \; 2 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n}{2} \\ & \text{n} = \; 3 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n-1}{2} \\ & \text{n} = \; 3 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n+3}{2} \\ 2n+9 \; ; \; for & \text{n} = \; 0 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n}{2} \\ & \text{n} = \; 1 \; mod \; 4 \; ; \text{n} \neq \; 5 \; ; \; i \; \equiv \; n \\ & \text{n} = \; 2 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n+2}{2} \\ & \text{n} = \; 3 \; mod \; 4 \; ; \; i \; \equiv \; \frac{n+1}{2} \end{cases}$$

$$W((x_1)) = \{ \;\; 8 \; + \; \sum f(y_j) \; ; \; for \; \text{n} \geq \; 7 \; ; \; i \; \equiv 1$$

$$W((x_n)) = \{ \;\; 7 \; + \; \sum f(z_k) \; ; \; for \; \text{n} \geq \; 7 \; ; \; i \; \equiv \text{n}$$

$$W((y_j)) = \begin{cases} 2j-1 \; ; \; j \; = \; odd \\ 2j \; ; \; j \; = \; even \\ 2n-1 \; ; \; j \; = \; m \end{cases}$$

$$W((z_k)) = \begin{cases} 2k \; ; \; k \; = \; odd \\ 2k-1 \; ; \; k \; = \; even \\ 2m \; ; \; k \; = \; m \end{cases}$$

It is clear that f is local antimagic vertex dynamic coloring of $DBr_{n,m}$. Finally, $\chi_r^{la} \leq n{+}2$ *for* $n = 3$ and $\chi_r^{la} \leq n+3$ *for* $n \geq 4$.

Case 2: Define a bijection $f : E(DBr_{n,m\ =\ even}) \to \{1,2,3,\ldots\ |E(Br_{n,m\ =\ even})|\}$ with the following function :

For $n =$ even ; $1 \leq i \leq \frac{n}{2}$:

$$f((x_i x_{i+1})) = \begin{cases} 2i+6\ ;\ i = even \\ 2i+9\ ;\ i = odd \end{cases}$$

For $n = 1$ mod 4 ; $1 \leq i \leq \frac{n+3}{2} - 4$:

$$f((x_i x_{i+1})) = \begin{cases} 2i+6\ ;\ i = even \\ 2i+9\ ;\ i = odd \end{cases}$$

For $n = 3$ mod 4 ; $1 \leq i \leq \frac{n+3}{2} - 3$:

$$f((x_i x_{i+1})) = \begin{cases} 2i+6\ ;\ i = even \\ 2i+9\ ;\ i = odd \end{cases}$$

For $n =$ even ; $\frac{n}{2} + 1 \leq n - 1$:

$$f((x_i x_{i+1})) = \begin{cases} 2n-2i+8\ ;\ i = even \\ 2n-2i+7;\ i = odd \end{cases}$$

For $n = 1$ mod 4 ; $\frac{n+5}{2} \leq n - 1$:

$$f((x_i x_{i+1})) = \begin{cases} 2n-2i+7\ ;\ i = even \\ 2n-2i+8\ ;\ i = odd \end{cases}$$

For $n = 1$ mod 4 ; $\frac{n+3}{2} - 3 \leq \frac{n+1}{2}$:

$$f((x_i x_{i+1})) = \begin{cases} n+7\ ;\ i = even \\ \frac{2i-5n-23}{-4}\ ;\ i = odd \end{cases}$$

For $n = 3$ mod 4 ; $\frac{n-1}{2} \leq \frac{n+3}{2}$:

$$f((x_i x_{i+1})) = \begin{cases} \frac{3n+2i+19}{4}\ ;\ i = odd \\ n+7\ ;\ i = even \end{cases}$$

For $m =$ even ; y_j ; $1 \leq j \leq m$:

$$f((x_1 y_j)) = \begin{cases} 2j-1\ ;\ j = odd \\ 2j\ ;\ j = even \end{cases}$$

For $m =$ odd ; z_k ; $1 \leq k \leq m$:

$$f((x_n z_k)) = \begin{cases} 2k\ ;\ k = odd \\ 2k-1\ ;\ k = even \end{cases}$$

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Hence f is a local antimagic labelling of $DBr_{n,m}$ and we have the vertex weight as follows :

For $n = 3$:

$$W((x_i) = \begin{cases} 10 \ + \ \sum f(y_j) \ ; \ i \ = 1; \ 1 \ \leq \ j \ \leq \ m \\ 6n+1; \ i \ = \ 3 \\ 9 \ + \ \sum f(z_k) \ ; \ i \ = \text{n} \ ; \ 1 \ \leq \ k \ \leq \ p \end{cases}$$

For $n = 4$:

$$W((x_i) = \begin{cases} 5n+1; \ i \ = \ 2 \\ 7n+1; \ i \ = \ 3 \end{cases}$$

For $n = 5$:

$$W((x_i) = \begin{cases} 4n+3; \ i \ = \ 2 \\ 4n+2; \ i \ = \ 3 \\ 4n-1; \ i \ = \ 4 \end{cases}$$

For $n > 5$:

$$W((x_i) = \begin{cases} 4i+13; for \text{ n} = \ odd; 1 \ \leq \ i \ \leq \ \frac{n-2}{2} \\ 4i+13; for \text{ n} = \ even; 1 \ \leq \ i \ \leq \ \frac{n}{2}+1 \end{cases}$$

$$W((x_i) = \begin{cases} 4n-4i+17 \ ; \ for \ \frac{n}{2}+2 \ \leq \ n \\ 4n-4i+17 \ ; \ for \ \frac{n+5}{2} \ \leq \ n \end{cases}$$

$$W((x_i) = \begin{cases} 2n+11 \ ; \ for \text{ n} = \ 0 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2}+1 \\ 2n+9 \ ; \ for \text{ n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n-1}{2} \\ \frac{11n-6i+45}{4} \ ; \ for \text{ n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+1}{2} \\ \frac{-9n+2i-53}{-4} \ ; \ for \text{ n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+1}{2} \\ \frac{-9n+2i-51}{-4} \ ; \ for \text{ n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+3}{2} \\ 2n+11; for \text{ n} = \ 2 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2} \\ 4n-4i+17; for \text{ n} = \ 2 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2}+1 \\ \frac{-9n+2i-53}{-4}; for \text{ n} = \ 3 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n-1}{2} \\ \frac{7n+2i+47}{4}; for \text{ n} = \ 3 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+1}{2} \end{cases}$$

$$W((x_i)) = \{ \ 11 \ + \ \sum f(y_j) \ ; \ for \text{ n} \geq \ 4 \ ; \ i \ = 1$$

$$W((x_i)) = \{ \ 9 \ + \ \sum f(z_k) \ ; \ for \text{ n} \geq \ 4 \ ; \ i \ = \text{n}$$

It is clear that f is local antimagic vertex dynamic coloring of $(DBr_{n,m})$. Finally, $\chi_r^{la} \leq \frac{4m+n+3}{2}$ *for* $n =$ odd and $\chi_r^{la} \leq \frac{4m+n+2}{2}$ *for* $n \ = \ even$. The proof is complete.

Theorem 4. *Let* $G \cong Br_{\{n,m\}}$ *be a connected graph with* $n \geq 3$, $\chi_r^{la}(G)$ *of* $Br_{\{n,m\}}$ *is :*

$$\chi(Br_{\{n,m\}}) = \begin{cases} n+m \ ; \ 3 \leq n \leq 4 \\ \frac{2m+n+5}{2} \ ; \ n = \ odd \\ \frac{2m+n+4}{2} \ ; \ n = \ 2 \ mod \ 4 \\ \frac{2m+n+2}{2} \ ; \ n = \ 0 \ mod \ 4 \end{cases}$$

Proof. $Br_{\{n,m\}}$ is a connected graph with V($Br_{\{n,m\}}$) = $\{x_i; 1 \leq i \leq n\} \bigcup \{y_j; 1 \leq i \leq m\}$ and E($Br_{\{n,m\}}$) =$\{x_n y_j; 1 \leq j \leq m\} \bigcup \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. Then $|V(Br_{\{n,m\}})|$ = m+n and $|E(Br_{n,m})|$ = m+n-1.

Define a bijection $f : E(Br_{\{n,m\}}) \rightarrow \{1 , 2, 3,..., |E(Br_{\{n,m\}})|\}$ with the following function :

For $n = 3$:

$$f((x_i x_{i+1})) = \begin{cases} n+2 \ ; \ i \ = \ 1 \\ n+1 \ ; \ i \ = \ 2 \end{cases}$$

For $n = 4$:

$$f((x_i x_{i+1})) = \begin{cases} n+2 \ ; \ i \ = \ 1 \\ n+1 \ ; \ i \ = \ 2 \\ n \ ; \ i \ = \ 3 \end{cases}$$

For $n = 5$:

$$f((x_i x_{i+1})) = \begin{cases} n+2 \ ; \ i \ = \ 1 \\ n \ ; \ i \ = \ 2 \\ n-1 \ ; \ i \ = \ 3 \\ n+1 \ ; \ i \ = \ 4 \end{cases}$$

For $n = 6$:

$$f((x_i x_{i+1})) = \begin{cases} n-1 \ ; \ i \ \equiv \ 2 \\ 10-2i \ ; \ 1 \ \leq \ i \ \leq \ \frac{n}{2} \ ; \ i \ \equiv \ odd \\ n \ ; \ \frac{n}{2}+1 \ \leq \ i \ \leq \ n-1 \ ; \ i \ \equiv \ 4 \\ n+1 \ ; \ \frac{n}{2}+1 \ \leq \ i \ \leq \ n-1 \ ; \ i \ \equiv \ n-1 \end{cases}$$

For $n = 7$:

$$f((x_i x_{i+1})) = \begin{cases} 12-2i \ ; \ 1 \ \leq \ i \ \leq \ \frac{n+1}{2} \ ; \ i \ \equiv \ even \\ 11-2i \ ; \ 1 \ \leq \ i \ \leq \ \frac{n+1}{2} \ ; \ i \ \equiv \ odd \\ n \ ; \ n-2 \ \leq \ i \ \leq \ n-1 \ ; \ i \ = \ 5 \\ n-1 \ ; \ n-2 \ \leq \ i \ \leq \ n-1 \ ; \ i \ = \ 6 \end{cases}$$

For $n = 9$:

$$f((x_i x_{i+1})) = \begin{cases} 14-2i \ ;1 \ \leq \ i \ \leq \ \frac{n-1}{2} \ ; \ i \ \equiv \ even \\ 13-2i \ ;1 \ \leq \ i \ \leq \ \frac{n-1}{2} \ ; \ i \ \equiv \ odd \\ 2i-7 \ ; \frac{n+1}{2} \geq n-1 \ ; \ i \ \equiv \ even \\ 2i-6 \ ; \ \frac{n+1}{2} \geq n-1 \ ; \ i \ \equiv \ odd \end{cases}$$

For $n = 0 \bmod 4$; $n = 2 \bmod 4$; $1 \leq i \leq \frac{n}{2}$:

$$f((x_i x_{i+1})) = \begin{cases} n-2i+4 \ ; \ i \ \equiv \ odd \\ n-21+3 \ ; \ i \ \equiv \ even \end{cases}$$

For $n = 1 \bmod 4$; $n = 3 \bmod 4$; $1 \leq \ i \leq \ \frac{n+1}{2}$; $n \ \neq \ 9$:

$$f((x_i x_{i+1})) = \begin{cases} n-2i+5 \ ; \ i \ \equiv \ even \\ n-21+4 \ ; \ i \ \equiv \ odd \end{cases}$$

For $n = 0 \bmod 4$; $n = 2 \bmod 4$; $\frac{i}{2} + 1 \leq \ i \leq \ n - 1$:

$$f((x_i x_{i+1})) = \begin{cases} n - 2i + 4 \ ; \ i \ \equiv \ even \\ n - 21 + 3 \ ; \ i \ \equiv \ odd \end{cases}$$

For $n = 1 \bmod 4$; $\frac{n-1}{2} \leq \ n - 1 \ \ ; n \ \neq \ 9$:

$$f((x_i x_{i+1})) = \begin{cases} 2i - n + 2 \ ; \ i \ \equiv \ even \\ 21 - n + 3 \ ; \ i \ \equiv \ odd \end{cases}$$

For $n = 3 \bmod 4$; $\frac{n-3}{2} \leq n - 1$; $n \neq 9$:

$$f((x_i x_{i+1})) = \begin{cases} 2i - n + 2 \ ; \ i \ \equiv \ even \\ 21 - n + 3 \ ; \ i \ \equiv \ odd \end{cases}$$

For m ; y_j ; $1 \leq j \leq m$:

$$f((x_{n-1} y_j)) = \{ \ \ i \ ; \ 1 \leq \ i \ \leq \ m$$

Hence f is a local antimagic labelling of $Br_{\{n,m\}}$ and we have the vertex weight as follows :

$$W((x_i)) = \begin{cases} 2n - 4i + 9 \ ; \ for & \text{n} = \ odd; 1 \ \leq \ i \ \frac{n-3}{2} \\ & \text{n} = \ even; 1 \ \leq \ i \ \frac{n-2}{2} + 1 \end{cases}$$

$$W((x_i)) = \begin{cases} 4i - 2n + 5 \ ; \ for & \text{n} = \ odd \ ; \ \frac{n-1}{2} \ \leq \ n \\ & \text{n} = \ even \ ; \ \frac{n}{2} \ \leq \ n \end{cases}$$

$$W((x_i)) =; \ for \begin{cases} 2m + 3 \ ; \ for & \text{n} = \ 0 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2} + 1 \\ & \text{n} \neq \ 13 \ ; \text{n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+1}{2} \\ & \text{n} \geq \ 10 \ ; \text{n} = \ 2 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2} + 1 \\ & \text{n} \leq \ 11 \ ; \text{n} = \ 3 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2} + 1 \\ & \text{n} = \ 13 \ ; \ i \ \equiv \ \frac{n-1}{2} \\ & \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n+1}{2} \\ & \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n+1}{2} + 1 \\ 2m + 4 \ ; & \text{n} = \ 0 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+2}{2} \\ & \text{n} \neq \ 13 \ ; \text{n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n-1}{2} \\ & \text{n} \geq \ 10 \ ; \text{n} = \ 2 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n}{2} \\ & \text{n} \leq \ 11 \ ; \text{n} = \ 3 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n-1}{2} \\ & \text{n} = \ 13 \ ; \ i \ \equiv \ \frac{n+1}{2} \\ & \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n+3}{2} + 1 \\ & \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n+3}{2} \\ 2m + 6 \ ; \ for & \text{n} \neq \ 13 \ ; \text{n} = \ 1 \ mod \ 4 \ ; \ i \ \equiv \ \frac{n+3}{2} \\ & \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n-1}{2} + 1 \\ & 4m + 1; for \ \text{n} = \ 15 \ ; \ i \ \equiv \ \frac{n+5}{2} + 1 \end{cases}$$

$$W((x_i)) = \{ \quad 4m+1; for \text{ n} = \ 15 \ ; \ i \ \equiv \ \frac{n+5}{2}+1$$

$$W((x_i)) = \begin{cases} n+2 \ + \ \sum f(y_j) \ ; \ for \text{ n} = \ even \ ; \ i \ \equiv \text{n} \\ n \ + \ \sum f(y_j) \ ; \ for \text{ n} = \ 1 \ mod \ 4 \ ; \text{n} \geq \ 4 \ ; \ i \ = \text{n} \\ n \ + \ \sum f(y_j) \ ; \ for \text{ n} = \ 3 \ mod \ 4 \ ; \text{n} \geq \ 4 \ ; \ i \ = \text{n} \end{cases}$$

It is clear that f is local antimagic vertex dynamic coloring of $(Br_{\{n,m\}})$. Finally, $\chi_r^{la} \leq n+m \ for \ 3 \leq n \leq 4$, $\chi_r^{la} \leq \frac{2m+n+5}{2}$ for $n =$ odd , $\chi_r^{la} \leq \frac{2m+n+4}{2}$ for $n = 2$ mod 4 , and $\chi_r^{la} \leq \frac{2m+n+2}{2}$ *for* $n = 0$ mod 4. The proof is complete.

3. Concluding Remarks

In this paper, we have found the exact value local antimagic dynamic chromatic number, namely sun graph, caterpilar graph, broom graph and doublebroom graph.

Open Problem 1. *Determine the local antimagic vertex dynamic coloring of operation graphs?*

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