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Supporting students' reasoning and argumentation skills through mathematical literacy problem on relation and function topic

L I Hermawan¹, N D S Lestari¹, A F Rahmawati², Suwarno³

¹ Mathematics Education Department, Universitas Jember, Indonesia

² SMP Negeri 3 Jember, Indonesia

³ Mathematics Education Department, IAIN Jember, Indonesia

Email: Lendiikehermawan@gmail.com

Abstract. Reasoning and argumentation skills are one of the fundamental mathematical abilities used in mathematical literacy. This ability can be taught to students through mathematics teaching. It involves a process of logical and profound thinking through a process that connects the problem elements to make a conclusion. This research aimed to describe how to support students' reasoning and argumentation through mathematical literacy problem on function and relation. The research was conducted through the observation of mathematic class of Junior High School on function and relation topic by scientific approach, and the assignment of mathematical literacy problem solving through student worksheet. Then the data were analyzed by descriptive qualitatively to describe the teaching process and students' reasoning and argumentation. The data of this research indicate that mathematical literacy problems presented through scientific approach, discovery learning and problem based learning can support students' reasoning and argumentation on function and relation topic on Junior High School.

1. Introduction

PISA (Program for International Student Assessment) is an international study organized by the OECD (Organization for Economic Cooperation and Development) which examines students' literacy ability [1]. The mathematical literacy in the PISA Mathematics 2012 framework is defined as the individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using concepts, procedures, facts, as a tools to describe, explain and predict a phenomenon [2]. Literacy Problems on tests conducted by PISA focus to measure reasoning ability, problem solving, argument and communicate [3]. Reasoning has an important role for individual's thinking process. Reasoning and argumentation skills are the capability to involves logically rooted thought processes that explore and link elements problem, so as to make inferences from them, check justification of statements or solutions to problems [4]. Therefor, reasoning and argumentation skills become one aspect on the 2013 Curriculum. Relation and function topic are one of the topics taught in the 2013 curriculum. These topics are a prerequisite for the mastery of the concepts of calculus, so mastery of concepts on this topic is very important. Reasoning and argument ability is necessary to find relations and functions concept. In this topic the thinking process of students can determine the concept that they build. Hence it will require reasoning ability



and arguments to learn it. Based on the above description, this research will show how to support students' reasoning and argument ability through mathematical literacy problems on relation and function topic.

2. Literature Review

Mathematic literacy in PISA Mathematics 2012 framework is defined as individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens [2]. While Ojose defines mathematics literacy as the knowledge to know and use basic math in everyday life [5]. In other words, mathematical literacy is the ability to know what mathematical knowledge should be used in solving everyday problems [6]. From the definition mathematics literacy can be defined as a individual's capacity to formulate, employ (concepts, prosedures, facts, and symbols) and interpret mathematics in a variety of context problems in everyday life.

OECD describe the domain categories of mathematics into three parts which is used to assess literacy, namely content, process, and context [4]. To solve the problem, three components of the nature of thought and action mathematical are needed [7]. The first component is content mathematics classified as shape and space, change and relationships, quantity, and uncertainty and data. The second component is the process of solving problems classified as formulate, employ and interpret. The last component is fundamental mathematics capability classified as mathematisation, devising strategies, representation, reasoning, symbols and communication. While Personal, societal, occupational and scientific are labelled for the mathematical literacy context categories [8]. One of the math content that can be used to teach mathematic literacy is a relation and a function topic. Relation and function topic is the mathematics content taught in grade VIII Junior High School. In everyday life, unconsciously we often encountered the problems associated with a relation and function topic.

This research focused on one of fundamental mathematical capabilities namely reasoning and argument. This capability involves logically rooted thought processes that explore and link problem elements so as to make inferences from them, check a justification that is given, or provide a justification of statements or solutions to problems [4]. Reasoning ability includes: (1) reasoning commonly associated with ability to find a resolution or problem solving; (2) the ability associated with drawing conclusions, as in the syllogism, and which is associated with ability to assess the implications of a argument; and (3) the ability to see relationships, not only relationship between objects but also the relationship between the ideas, and then use that connection to obtain object or other ideas [9]. While, Ontario Ministry Resource states that "the reasoning process supports a deeper understanding of mathematics by enabling students to make sense of the mathematics they are learning. The process involves exploring phenomena, developing ideas, making mathematical conjectures, and justifying results" [10]. From the opinions above reasoning and argument capability is the ability which involves a logical thinking process that explores and connects the elements of the problem to find a resolution or troubleshooting and drawing conclusions.

PISA, is an international program to assess students Mathematical literacy administered once every three years. The test subject in this program are students aged about 15 years old or age of compulsory education. It is intended to see how well students prepared for the world of work or post-school environment [6]. The achieved result for Indonesian students is far from satisfactory. On PISA 2015 test, Indonesia was only able to reach position 62 of the 70 participating countries with a score of 386 [11]. Achievement level of Indonesian students literacy skills between 2000 to 2009 if the review of the achieved score can only reach a value under 400 with the cognitive abilities of most average height could only reach level 3 and 4 [7]. If seen from the result, Indonesian students is very far behind in terms of the literacy skills of mathematics. Whereas in everyday life, students will be faced with problems related to personal, societal, occupational and scientific. From those problems many of them related to the application of mathematics. Mastery of mathematics that will help students solve

the problem. One of PISA goals is to assess mathematical knowledge of students in solving problems in everyday life. That is why we use the term mathematics literacy in PISA mathematics for not only viewed as a discipline of knowledge, but also how students can apply such knowledge in real-world problems or everyday life. So that such knowledge can be perceived more of it benefits directly by the students [12]. Math literacy helps a person to get to know the role of mathematics in the world and make considerations and decisions that are needed as citizens [2]. Thus knowledge and understanding about mathematical concepts is important, but what is more important is the ability to enable mathematics literacy was to solve problems that faced in everyday life. [13]. From some opinions above can be conclude that mathematics literacy is essential especially for Junior High School students so that's why PISA use to test students 15 years of age.

Mathematical literacy and students' reasoning and argument capability has been widely investigated in the research. In the research conducted by Nursyahidah concluded that students ability in identifying allegations mathematical, evaluate the mathematical arguments, and build mathematical proof still in a low percentage [14]. The same thing also delivered by Anisah, the majority of Junior High School students still have mathematical reasoning capability that is less because of the difficulty in identifying the problems given on the matter [15]. While mathematics problem that are tested in tests conducted by PISA not only test the students ability of simple mathematics, but the level for 4-6 is the degree to which students tested the ability to think higher level [16]. PISA questions are very demanding reasoning ability and problem solving. A student is said to be able to solve a problem if he is able to apply previously acquired knowledge into a new unknown situation [3]. PISA problem is a problem that is able to hone students' reasoning skills in relating mathematics with everyday life. The main focus of the PISA problem is students ability to use their knowledge and skills in dealing with life's challenges [17]. This research will show how mathematical literacy problem can support the students ability of reasons and argumen. How students resolve mathematical literacy problems given using reasons and argument ability on relation and function topic having previously done the learning with a scientific approach and discovery learning and problem based learning model.

3. Methodology

This research was conducted in one of math class at SMPN 3 Jember. In this School there has been general literacy but mathematics literacy in particular has not been taught. The research was conducted through a learning activity using the scientific approach on function and relation topic. Teaching and Learning was divided into two meetings to support student's reasoning and argument, at the first meeting of teaching definition of relation and function and on the second meeting to teaching function formula. Learning design based on students ability, gender different, weaknesses and advantages of students. To find out how teaching and learning to support reasoning ability and argument of student will be test result of learning. The test was done through the problem on the student worksheet, there are three problems given. The student worksheet that was already done then collected and analyzed by giving a score with a rubric assessment. Each step of work was given a score. Score 4 was given for answers students who can use his reasoning in interpreting mathematical problems and perform the procedure well and can choose strategies to solve them, while the answers for students who are less complete given score 3. Score 2 was given to students who are less precise in using his reasoning in interpreting mathematical problems and implement the procedures well and can choose strategies to solve them. For students who did not use their reasoning in interpreting mathematical problems and perform the procedures well and can choose strategies to solve them given score 1. Then the score is accumulated. Then the analysis continued with the descriptive qualitatively method by presenting example of student's answers of each problems.

4. Results and discussion

The research was conducted through two lessons in math class at Junior High School. 35 Students participated in the lesson. Learning begins by giving a stimulus related to the student's knowledge on the topic taught. Then 35 students in class were divided heterogeneously into groups of 5 students each

group. This heterogeneous division was intended to ensure that the quality between groups is not much different because in that class the students' math skills range from low to high. Furthermore, at the first meeting students were given a worksheet in the design with a scientific approach and model of discovery learning to find a function concept. After discovering function concept the students were instructed to solve mathematical literacy problems that are on the student worksheet. While at the second meeting the student worksheet was designed using a scientific approach and problem based learning model to find the concept of function formula. Then learning was continued with the representatives of the students to write the answer on the board. After students solved the student worksheets that are then collected and analyzed. The results of the analysis are used to find out how to support students' reasoning and argument through mathematical literacy problems.

Table 1 below shows each group score.

Table 1. Group score.

	Problem 1	Problem 2	Problem 3	Score
Group 1	56,25	62,5	62,5	60,42
Group 2	62,5	75	56,25	64,58
Group 3	75	62,5	56,25	64,58
Group 4	87,5	87,5	62,5	79,17
Group 5	68,75	50	62,5	60,42
Group 6	93,75	75	81,25	83,33
Group 7	87,5	75	75	79,17

The score in table 1 above is the result of student work in solving the problems given through the student worksheet. There are three problems used to find out how mathematical literacy problems can support students' reasoning and argument. Based on the above table we can know that in problem 1 there are four groups that get a score of more than equal to 70. In problem 2 there are four groups and in problem 3 only two groups get scores more than equal to 70. Then the score is accumulated and obtained three groups that score more than equal to 70. The given problem is a mathematical literacy problem that is closely related to the students. In problem 1 given the problems associated with farm, students are asked to calculate the number of eggs that can accommodate chicken and duck eggs on the farm mentioned. In the problem 2, the context given relates to school equipment, students are asked to provide purchase options based on the price of the stationery in the store. At problem 3, the context relates to hotel lodging costs, students are required to estimate the cost of lodging for different day duration. To solve problems using process of solving mathematics literacy problems that is formulate, employ and interpret.

The following results will illustrates how students' reasoning abilities on each problems.

4.1. Analysis of students' reasoning for problem 1

Problem 1

Mr. Heri has a chicken and duck farm. The ratio of chickens and ducks is 3: 4. Total of the animals on his farm are 840. Each chickens lay 8 eggs and ducks lay 5 eggs. If all the eggs will are placed in an egg container like the picture on the side. Is placed eggs into their containers a function? Give your reason!

Figure 1 below is one of the students' answers

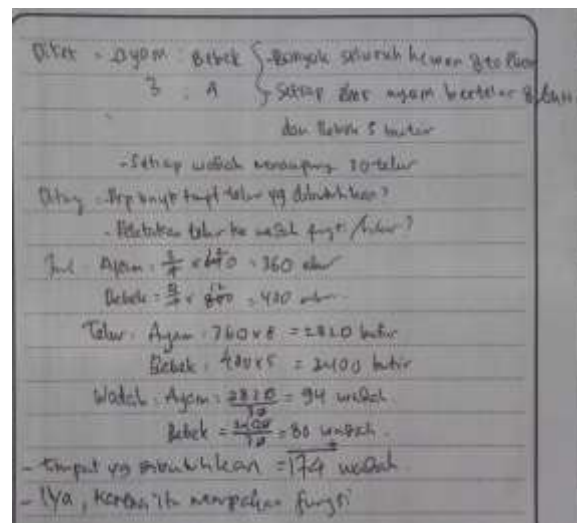


Figure 1. Example of a group answer in problem 1.

Translate:	
Known: Chicken : Duck 3 : 4	<ul style="list-style-type: none"> - Total of animals are 840. - each chicken lay 8 eggs and duck lay 5 eggs
	<ul style="list-style-type: none"> - each egg container holds 30 eggs
Asked : - how many egg containers are needed?	
	<ul style="list-style-type: none"> - laying eggs into the egg container function/not?
Answer: Chicken = $\frac{3}{7} \times 840 = 360$ chickens	
Duck = $\frac{4}{7} \times 840 = 480$ ducks	
Eggs = Chicken = $360 \times 8 = 2820$ eggs	
Duck = $480 \times 5 = 2400$ eggs	
Egg Containers = Chicken = $\frac{2820}{30} = 94$	
Duck = $\frac{2400}{30} = 80$	
- Egg container needed = 174	
- yes, because it is a function	

Figure 1 shows that they wrote down the information obtained from the problem first before solving it. So they are able to understand the problem, but at the completion step they have poor written communication skills because it only mentioned the calculation without explanation of the logical reason. From each step they did not give any information to get to the next step so it will be difficult to understand others who read it. Their reasoning ability was good because it was able to solve the purpose of the problem. They were able to associate well-acquired information so that no steps are passed. They were able to interpret, apply and did problem solving because in the process of solving it they used ratio in determining the number of chickens and ducks. In step 2 there is a miscalculation that causes the answer to be obtained

is not correct. They do not gave a good reason for the second question. Based on this analysis, it can be concluded that they tend to only do the calculations according to the order in the problem. They do not present any reason or explanation on any step in the written formula. The reasoning ability is quite good because it was able to understand the information provided. This is supported by Jurnaidi's opinion who states that students who belong to the category have good mathematical reasoning ability is able to identify the statement and determine the mathematical way of solving the problem [18].

4.2. Analysis of students' reasoning for problem 2

Problem 2

Andi will buy school equipment in a store. The book price is Rp2000,-, Pencil is Rp1500,-, Pen is Rp2000,-, Ruler is Rp1000,- and eraser is Rp500,-. Andi only has Rp10.000,-, mention 3 possibilities of school equipments that can be purchased if Andi must buy at least 2 types of school equipment! Show a relation in the store! Explain!

Figure 2 below is one of the students' answers.

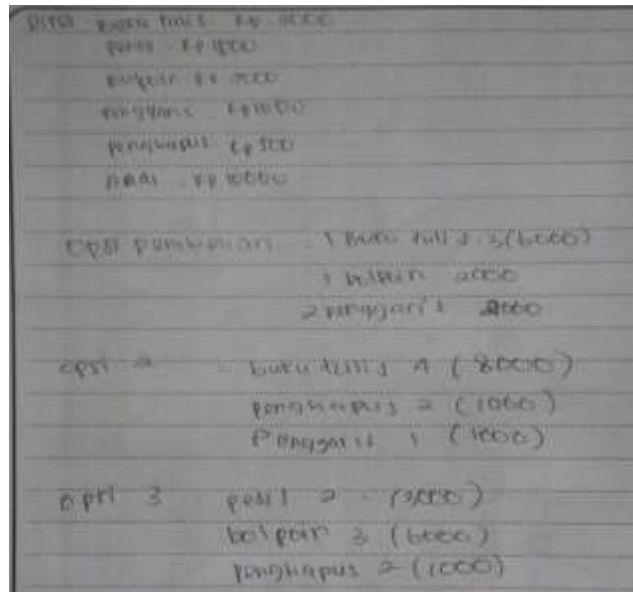


Figure 2. Example of a group answer in problem 2.

Translate:

Known : Books Rp2.000
Pencil Rp1.500
Bulpoin Rp2.000
Ruler Rp1.000
Eraser Rp500
Andi = Rp10.000

Purchase Option = 1 Book 3 (6000)
1 Pen 2000
2 Ruler 2000

Option 2 = Book 4 (8000)
Eraser 2 (1000)
Ruler 1 (1000)

Option 3 = Pencil 2 (3000)
Pen 3 (6000)
Eraser 2 (1000)

Figure 2 shows that they were able to understand the given problem. In the final step they had poor written communication skills because they only write the solution so that it will be a little difficult to understand others who read it. From each purchase option they wrote down their calculation steps. However the information they get from the problems they write at the beginning of the work so that it can help other readers to understand their intentions. Their reasoning abilities were good because they can solve problems. They were able to understand well-received information even though it is less good at communicating it. They did not give a reason for each option they choose. They just wrote down important points for him to make himself easier. Things like this can make students experience errors in reasoning. Based on this analysis, it can be concluded that they tend to write only important points according to the sequence in the problem. They do

not present any reason or explanation on any step written down. The reasoning ability was good enough because it was able to understand the problem purpose and successfully solve it. They can formulating problem into mathematics forms, it can support students' reasoning and argumentation skills. This is accordance with Anisah's opinion that mathematical process to solve mathematic literacy problem is the application of students' reasoning and argumentation skills [15].

4.3. Analysis of students' reasoning for problem 3

Problem 3

The cost of lodging in Hotel Indah for 1 day is Rp80.000 and will get a discount of 5% of lodging fees. Mr. Roni will stay at Hotel Indah on April 3, 2017. How much does Pak Roni spend when staying at the Indah Hotel if Mr. Roni is staying until April 5, April 7, April 8, and April 11? Represent with arrows diagram! Specify the domain, codomain, range and write the function formula from the domain to the codomain!

Figure 3 below is one of the students' answers.

Handwritten student answer for Problem 3:

- 3 April → 3 hari → $3 \times 80.000 = 240.000$
 Biaya yg dibayarkan = $\frac{95}{100} \times 240.000 = \text{Rp. } 228.000$
- 7 April → 5 hari → $5 \times 80.000 = 400.000$
 Biaya yg dibayarkan = $\frac{95}{100} \times 400.000 = \text{Rp. } 380.000$
- 8 April → 6 hari → $6 \times 80.000 = 480.000$
 Biaya yg dibayarkan = $\frac{95}{100} \times 480.000 = \text{Rp. } 456.000$
- 11 April → 9 hari → $9 \times 80.000 = 720.000$
 Biaya yg dibayarkan = $\frac{95}{100} \times 720.000 = \text{Rp. } 684.000$

Figure 3. Example of a group answer in problem 3.

Translate:

5 April → 3 Days → $3 \times 80.000 = 240.000$

$$\text{Cost} = \frac{95}{100} \times 240.000 = \text{Rp}228.000$$

7 April → 5 Days → $5 \times 80.000 = 400.000$

$$\text{Cost} = \frac{95}{100} \times 400.000 = \text{Rp}380.000$$

8 April → 6 Days → $6 \times 80.000 = 480.000$

$$\text{Cost} = \frac{95}{100} \times 480.000 = \text{Rp}456.000$$

11 April → 9 Days → $9 \times 80.000 = 720.000$

$$\text{Cost} = \frac{95}{100} \times 720.000 = \text{Rp}684.000$$

Figure 3 shows that they did not write down the information obtained from the problem first before finishing it. They immediately did the calculations without giving a clear explanation. They were able to understand the problem well so they can solve the problem. They chose a different settlement strategy with another group, they calculated the cost of lodging by multiplying the total stay cost by 95%. 95% obtained from discount given by the hotel by 5% so that Pak Roni only need to pay for 95% only. In the problem of no information regarding payments when checking in and

checking out at certain hours, they use 3 days to describe the length of stay from 3 April to 5 April. However they do not give any reason so it will be difficult to understand others who read it. Their reasoning ability is good because it is able to solve the purpose of the problem. They are able to associate well-received information so that no steps are passed. Based on this analysis, it can be concluded that their reasoning ability is quite good because they are able to understand the information provided but they tend to only do the calculations according to the order in the problem. They do not present any reason or explanation on any step written down. Process of solving this problem is formulate, employ and interpret. They can formulating situations mathematically and employ the procedures to find the answer. They can interpret answers into the real life. This ability is mathematical ability needed for mathematical literacy [4]. It can improve students' reasoning and argumentation skills in accordance with Winarti's opinion that mathematic literacy problem is a problem that is able to hone students' reasoning skills in relating mathematics with everyday life. [17]

5. Conclusion

Based on research results and discussion it can be concluded that the students' reasoning and argumentation skills can be supported through mathematical literacy problem on relations and functions topic. Mathematical literacy problem can hone the students' reasoning and argument ability because the context that encompasses daily problems so that the students better understand the problems, besides the daily problems that the context close to the students can motivate the students to be able to solve every given problem. Process of solving math literacy problems classified as formulate, employ and interpret can be supported students' reasoning and argumentation skills. Based on this research is expected to be an input for teachers to apply mathematical literacy problems in learning to be able to further improve students' reasoning and argument.

The next research the researcher should make the question clearly so that it can be understood by students.

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