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To cite this article: A I Kristiana *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **243** 012077

IOP Conf. Series: Earth and Environmental Science **243** (2019) 012077 doi:10.1088/1755-1315/243/1/012077

Local antimagic r-dynamic coloring of graphs

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Abstract. Let $G = (V, E)$ be a connected graph. A bijection function $f : E(G) \rightarrow \{1, 2, 3, \cdots, E(G)|\}$ is called a local antimagic labeling if for all $uv \in E(G)$s, $w(u) \neq w(v)$, where $w(u) = \Sigma_{e\in E(u)} f(e)$. Such that, local antimagic labeling induces a proper vertex k-coloring of graph G that the neighbors of any vertex u receive at least $\min\{r, d(v)\}$ different colors. The local antimagic r-dynamic chromatic number, denoted by $\chi_r^{la}(G)$ is the minimum k such that graph G has the local antimagic r-dynamic vertex k-coloring. In this paper, we will present the basic results namely the upper bound of the local antimagic r-dynamic chromatic number of some classes graph.

1. Introduction

In this paper, all graph is simple, connected and undirected, $G = (V, E)$, on the vertex set $V(G)$ and the set $E(G)$. For vertex, v of G is denoted by $N(v)$ and the degree of v is denoted by $d(v)$. The maximum and minimum of graph G are denoted by $\Delta(G)$ and $\delta(G)$. Arumugam [4], introduced the concept of local antimagic chromatic number of graphs. Then it is followed by Albirri [2] which did some research of another graphs.

Definition 1.1 *[4] Let $G = (V(G), E(G)$ be a graph of order n and size m having no isolated vertices. A bijection $f : E(G) \rightarrow \{1, 2, 3, ..., m\}$ is called a local antimagic labeling if for all $uv \in E(G)$ we have $w(u) \neq w(v)$, where for $w(u) = \Sigma_{e\in E(u)} f(e)$. A graph G is local antimagic if G has a local antimagic labeling.*

Montgomery [8] introduced the concept of r-dynamic coloring, definition of r-dynamic coloring as follows,

Definition 1.2 *[8] An r-dynamic coloring of a graph G is defined to be a map c from V to the set of colors such that*

- *If $uv \in E(G)$, then $c(u) \neq c(v)$, and*
- *For each vertex $v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$.*

The minimum k such that graph G with an r-dynamic k-coloring is called the r-dynamic chromatic number of graph G, $\chi_r(G)$. This concept was introduced by Montgomery [8]. He found lower bound of the r- dynamic chromatic number, $\chi_r(G) \geq \min\{\Delta(G), r\} + 1$.

In this paper, we introduce the new concept which the combination of local antimagic labeling [4] and r-dynamic chromatic number [3]. The local antimagic labeling induces a proper vertex k-coloring of graph G where the vertex v is assigned the color $w(v)$ such that the neighbors of any vertex v receive at least $\min\{r, d(v)\}$ different colors.

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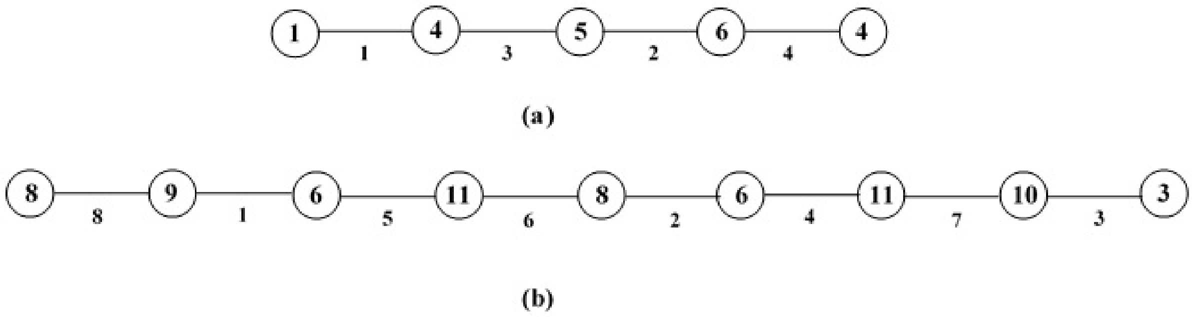

Figure 1. (a) $\chi_r^{la}(P_5) = 4$ (b) $\chi_r^{la}(P_9) = 6$

2. Main Result

In the paper, we have the new concept which combine of local antimagic labeling and r-dynamic chromatic number.

Definition 2.1 *Let $G = (V, E)$ be a graph of size m having no isolated vertices. A bijection $f : E(G) \to \{1, 2, 3, \cdots, m\}$ is called local antimagic r-dynamic coloring, such that:*

- *If $uv \in E(G)$, then $w(u) \neq w(v)$, where $w(u) = \Sigma_{e \in E(u)} f(e)$ and*
- *For each vertex $v \in V(G), |w(N(v))| \geq min\{r, d(v)\}$.*

Definition 2.2 *The local antimagic r-dynamic chromatic number of graph G, denoted by $\chi_r^{la}(G)$ is the minimum k such that graph G has an local antimagic r-dynamic vertex k-coloring induced by local antimagic labelings.*

We find the lower bound of local antimagic $r$dynamic chromatic number of G. We get the local antimagic r-dynamic chromatic number of some classes graphs namely path, cycle, path, star, and complete.

Lemma 2.1 *Let G be a connected graph with order at least 3, then local antimagic r-dynamis chromatic number is $\chi_r^{la}(G) \geq 2$.*

Theorem 2.1 *Let P_n be a path graph with order n, for $n \geq 2$ then local antimagic r-dynamic chromatic number is*

$$\chi_r^{la}(P_n) \leq \begin{cases} 3, & \textit{if } r = 1 \\ 4, & \textit{if } r \geq 2 \textit{ and } n = 5 \\ n, & \textit{if } r \geq 2 \textit{ and } n = 3, 4 \\ \frac{n+4}{3}, & \textit{if } r \geq 2 \textit{ and } n \equiv 5 \, mod \, 6, \; n \neq 5 \\ \frac{n+7}{3}, & \textit{if } r \geq 2 \textit{ and } n \equiv 2 \, mod \, 6 \\ \frac{n+8}{3}, & \textit{if } r \geq 2 \textit{ and } n \equiv 1 \, mod \, 3, \; n \neq 4 \\ \frac{n+9}{3}, & \textit{if } r \geq 2 \textit{ and } n \equiv 0 \, mod \, 3, \; n \neq 3 \end{cases}$$

Proof 2.1 **Case 1**: *For $n \equiv 5$ mod 6, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & \textit{if } i \equiv 2 \, mod 3 \\ \frac{2n-i-1}{3}, & \textit{if } i \equiv 0 \, mod 3 \\ \frac{2n+i-5}{3}, & \textit{if } i \equiv 1 \, mod 3 \\ n-1, & \textit{if } i = 1 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{4n-5}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n+2i-5}{3}, & if\ i \equiv 2\ mod 3 \\ n-1, & if\ i = 1 \\ n-2, & if\ i = n \\ n, & if\ i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}$, $\frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. Furthermore, it shows that every vertex has $|w(u)| \geq min\{r, d(u)\}$. We obtain that $\chi_r^{la}(P_n) \leq \frac{n+4}{3}$.

Case 2: *For $n \equiv 2$ mod 6, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n-i-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+i-5}{3}, & if\ i \equiv 1\ mod 3 \\ n-1, & if\ i = 1 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{4n-5}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n+2i-5}{3}, & if\ i \equiv 2\ mod 3 \\ n-1, & if\ i = 1 \\ n-2, & if\ i = n \\ n, & if\ i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}$, $\frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. Furthermore, it shows that every vertex has $|w(u)| \geq min\{r, d(u)\}$. We obtain that $\chi_r^{la}(P_n) \leq \frac{n+7}{3}$.

Case 3: *For $n \equiv 1$ mod 3, we define a bijection $f : E(P_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n-i+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+i-3}{3}, & if\ i \equiv 1\ mod 3 \\ n-1, & if\ i = 1 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{4n-1}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n+2i-3}{3}, & if\ i \equiv 2\ mod 3 \\ n-1, & if\ i = 1 \\ \frac{n+2}{3}, & if\ i = n \\ n, & if\ i = 2 \end{cases}$$

From the weight of vertex x_i in path P_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}$,

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$\frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(P_n) \leq \frac{n+8}{3}$.

Case 4: *For* $n \equiv 0$ *mod* 3, *we define a bijection* $f : E(P_n) \rightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+i-4}{3}, & if\ i \equiv 1\ mod 3 \\ n-1, & if\ i = 1 \end{cases}$$

From the labels f, *we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{4n-3}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n+2i-4}{3}, & if\ i \equiv 2\ mod 3 \\ n-1, & if\ i = 1 \\ \frac{n}{3}, & if\ i = n \\ n, & if\ i = 2 \end{cases}$$

From the weight of vertex x_i *in path* P_n, *we can see that for every two adjacent vertices have distinct weight namely* $w(v) = n-1, n, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+5}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}$, $\frac{2n+11}{3}, \frac{2n-1}{3}, \frac{4n-5}{3}, \frac{2n+17}{3}, \ldots, \frac{2n-1}{3}, \frac{4n-5}{3}, n-2$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(P_n) \leq \frac{n+9}{3}$.

Case 5: *For* $n = 5$, *we define a bijection* $f : E(P_5) \rightarrow \{1, 2, 3, ..., 4\}$. *We have edge label of path* P_5, $f(e) : 1, 3, 2, 4$ *and vertex weight* $w(v) : 1, 4, 5, 6, 4$. *Based on the vertex weight that for any two adjacent vertices have distict weight and satisfy* $|w(u)| \geq min\{r, d(u)\}$. *Such that, we obtain that* $\chi_2^{la}(P_5) \leq 4$.

Case 6: *For* $n = 3, 4$, *we define a bijection* $f : E(P_n) \rightarrow \{1, ..., n-1\}$. *We have edge label of path* P_3, $f(e) : 1, 2$ *and vertex weight* $w(v) : 1, 3, 2$. *We have edge label of path* P_4, $f(e) : 1, 3, 2$ *and vertex weight* $w(v) : 1, 4, 5, 2$. *Hence, we obtain that* $\chi_r^{la}(P_n) \leq n$.
The proof is complete.

Theorem 2.2 *Let* C_n *be a cycle graph with order* n, *for* $n \geq 3$ *then local antimagic* r*-dynamic chromatic number is*

$$\chi_r^{la}(C_n) \leq \begin{cases} 3, & if\ r = 1 \\ n, & if\ r \geq 2\ and\ n = 3, 4, 5 \\ \lceil \frac{n}{3} \rceil + 2, & if\ r \geq 2\ and\ n \equiv 1, 2, 3\ mod 6 \\ \lceil \frac{n}{3} \rceil + 1, & if\ r \geq 2\ and\ n \equiv 0, 4, 5\ mod 6 \end{cases}$$

Proof 2.2 *For* $r = 1$ *in [4],* $\chi_{la}(C_n) = \chi_1^{la}(C_n) = 3$. *For* $r \geq 2$, *we divide into some cases as follows.*

Case 1: *For* $n \equiv 1$ *mod* 6, *we define a bijection* $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ *as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-2}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, *we obtain the vertex weight* $w(u) = \Sigma_{e \in E(u)} f(e)$ *as follows.*

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i-5}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \\ 2n-1, & if\ i = n \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+3}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+9}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+15}{3}, \ldots, \frac{2n+1}{3}, \frac{4n-1}{3}, 2n-1$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+8}{3}$.

Case 2: *For $n \equiv 2$ mod 6, we define a bijection $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-1}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+8}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+14}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+20}{3}, \ldots, \frac{2n+2}{3}, \frac{4n+1}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+7}{3}$.

Case 3: *For $n \equiv 3$ mod 6, we define a bijection $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+3}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+3}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i+1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v); n+1, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+9}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+15}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+21}{3}, \ldots, \frac{2n+3}{3}, \frac{4n+3}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+6}{3}$.

Case 4: *For $n \equiv 4$ mod 6, we define a bijection $f : E(C_n) \longrightarrow \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-2}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+1}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n-1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i-1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \\ 2n-1, & if\ i = n \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+7}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+13}{3}, \frac{2n+1}{3}, \frac{4n-1}{3}, \frac{2n+19}{3}, \ldots, \frac{2n+1}{3}, \frac{4n-1}{3}, 2n-1$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+5}{3}$.

Case 5: *For $n \equiv 5$ mod 6, we define a bijection $f : E(C_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i-1}{3}, & if\ i \equiv 0\ mod 3 \\ n, & if\ i = n \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+1}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v) = n+1, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+8}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+14}{3}, \frac{2n+2}{3}, \frac{4n+1}{3}, \frac{2n+20}{3}, \ldots, \frac{2n+2}{3}, \frac{4n+1}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+4}{3}$.

Case 6: *For $n \equiv 0$ mod 6, we define a bijection $f : E(C_n) \to \{1, 2, 3, ..., n\}$ as follows*

$$f(x_i x_{i+1}) = \begin{cases} \frac{i+2}{3}, & if\ i \equiv 1\ mod 3 \\ \frac{2n-i+2}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{2n+i}{3}, & if\ i \equiv 0\ mod 3 \end{cases}$$

From the labels f, we obtain the vertex weight $w(u) = \Sigma_{e \in E(u)} f(e)$ as follows.

$$w(x_i) = \begin{cases} \frac{2n+3}{3}, & if\ i \equiv 2\ mod 3 \\ \frac{4n+3}{3}, & if\ i \equiv 0\ mod 3 \\ \frac{2n+2i+1}{3}, & if\ i \equiv 1\ mod 3 \\ n+1, & if\ i = 1 \end{cases}$$

From the weight of vertex x_i in cycle C_n, we can see that for every two adjacent vertices have distinct weight namely $w(v); n+1, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+9}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+15}{3}, \frac{2n+3}{3}, \frac{4n+3}{3}, \frac{2n+21}{3}, \ldots, \frac{2n+3}{3}, \frac{4n+3}{3}$. *Furthermore, it shows that every vertex has* $|w(u)| \geq min\{r, d(u)\}$. *We obtain that* $\chi_r^{la}(C_n) \leq \frac{n+3}{3}$.

Case 7: *For $n = 3, 4, 5$, Define a bijection $f : E(C_n) \to \{1, 2, 3, ..., n\}$. We have edge label of cycle C_n as follows*

- *We have edge label of cycle C_3, $f(e) : 1, 2, 3$ and vertex weight $w(v) : 4, 3, 5$*
- *We have edge label of cycle C_4, $f(e) : 1, 3, 4, 2$ and vertex weight $w(v) : 3, 4, 7, 6$*
- *We have edge label of cycle C_5, $f(e) : 1, 3, 5, 2, 4$ and vertex weight $w(v) : 5, 4, 8, 7, 6$*

Hence, we obtain that $\chi_r^{la}(C_n) \leq n$.
From Case 1-7, we obtain that for $n \equiv 1, 2, 3$ (mod 6), $\chi_r^{la}(C_n) \leq \lceil \frac{n}{3} \rceil + 2$ and for $n \equiv 0, 4, 5$ (mod 6), $\chi_r^{la}(C_n) \leq \lceil \frac{n}{3} \rceil + 1$. The proof is complete.

Theorem 2.3 *Let S_n be a star graph with order $n+1$, for $n \geq 3$ then local antimagic r-dynamic chromatic number is $\chi_r^{la}(S_n) = n+1$.*

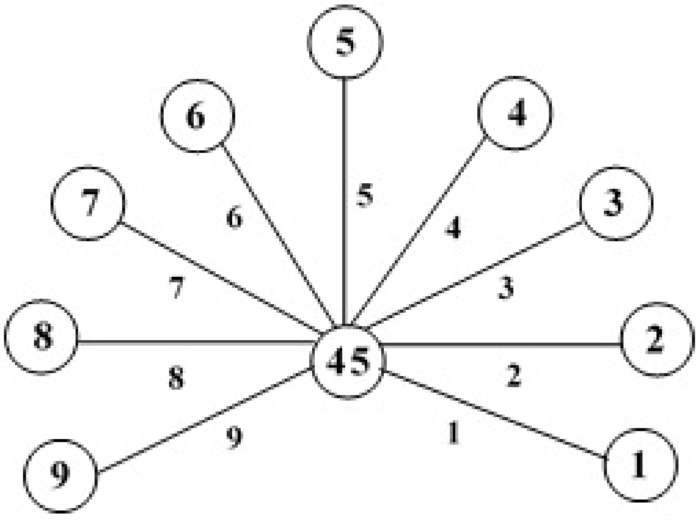

Figure 2. $\chi_r^{la}(S_9) = 10$

Proof 2.3 *Consider the star graph, S_n with central vertex v_0, $d(v_0) = n$ and vertices v_i, $d(v_i) = 1$, $1 \le i \le n$. The order of star graph is $n+1$ and the size of star graph is $|E(S_n)| = n$, namely $e_i = v_0v_i, 1 \le i \le n$. Define a bijection $f : E(S_n) \to \{1, 2, 3, ..., n\}$ as, $f(e_i) = i$, $1 \le i \le n$ such that $w(v_0) = \Sigma_{k=1}k$, $1 \le k \le n$ and $w(v_i) = i$, $1 \le i \le n$. Hence, it shows $|w(N(v_0))| = n \ge min\{r, d(v_0)\}$ and $|w(N(v_i))| = 1 \ge min\{r, d(v_i)\}$. We obtain that $\chi_r^{la}(S_n) = n + 1$.*

Theorem 2.4 *Let K_n be a complete graph with order n, for $n \ge 3$ then local antimagic r-dynamic chromatic number is $\chi_r^{la}(K_n) = n$.*

Proof 2.4 *Consider the complete graph, K_n vertices v_i, $d(v_i) = n - 1$, $1 \le i \le n$. The order of complete graph is n and the size of star graph is $|E(K_n)| = \frac{n(n-1)}{2}$, namely $e_j = v_iv_{i+k}, 1 \le i \le n, 1 \le k \le n - i$. Define a bijection $f : E(K_n) \to \{1, 2, 3, ..., \frac{n(n-1)}{2}\}$ as, $f(e_j) = j$, $1 \le j \le \frac{n(n-1)}{2}$ such that $w(v) \ne w(u)$ every $e = uv, e \in E(K_n)$. Hence, for every $v \in V(G)$, it shows $|w(N(v))| = n - 1 \ge min\{r, d(v)\}$. We obtain that $\chi_r^{la}(K_n) = n$.*

3. Conclusion

We have found the concept local antimagic r-dynamic coloring. We find the basic results namely the upper bound of the local antimagic r-dynamic chromatic number of some classes graph, namely path, cycle, star, and complete graph

Acknowledgement

We gratefully acknowledge the support from University of Airlangga, Surabaya and CGANT University of Jember Indonesian of year 2018.

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