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The analysis of implementation of discovery based learning to improve students higher order thinking skills in solving r-dynamic vertex coloring problem based on their reflective thinking skill

H Umam^{2*}, Dafik^{1,2}, M Irvan³

¹CGANT UNEJ, University of Jember, Jember, Indonesia

²Department of Postgraduate Mathematic Education, University of Jember, Jember, Indonesia

³Department of Elementary School Education, University of Jember, Jember, Indonesia

Email: hanifumam19@gmail.com; d.dafik@unej.ac.id

Abstract. This study aims to analyze the implementation of discovery-based learning in improving students' high-order thinking skills in solving r-dynamic problems based on their reflective thinking ability. The method used in this study is a mixed method, which mix up the qualitative and quantitative methods. The subjects consisted of 18 students in experiment class and 18 students in control class. The post-test result of the experiment class of 11.11% students having low level of high-order thinking ability, and 88.89% having high level of high-order thinking ability with their reflective thinking skill 25% of low level, 68.75% of medium level, 6.25% of high level. In the post-test result of the control class, there are 27.78% students having low level of high-order thinking ability and 72.22% students having high level of high-order thinking ability with their reflective thinking skill 53.85% of low level, 46.15% of medium level. The data analysis presents the score of independent sample t-test of post-test is sig. 0.05 ($p \leq 0.05$), so it is significant. Therefore, there is a different of their high-order thinking skill ability between the experiment and control classes. The regression equation of both variables express 1.497 ($\bar{Y} = -51.067 + 1.497X$). The coefficient of determination is 0.575 of 57.5% effect, contribution of the dependent variable (the result of the test post) by the free variable (Discovery based learning) is 57.5% and 42.5% explained by other variables. It implies that we accept the positive hypothesis as $t_{count} > t_{table}$ ($4.649 > 2.119$). Thus, there is an impact of the implementation of discovery based learning in improving the students' higher order thinking skills in solving the r-dynamic vertex coloring problem based on their reflective thinking skill.

1. Introduction

In teaching and learning process, especially on the subject of Discrete Mathematics the aim is not only to understand the math materials taught by lecturers, but the ability of reasoning, communication, representation, problem solving, and good behaviour after studying mathematics is a general purpose that must be mastered by students [19]. Mathematics education at higher education tends to direct learners to understand mathematics formula by applying it in the subject involving problem solving. This is the special case when the material contains certain formulas, such as arithmetic sequences and



series. It should not be done by a teachers now days, since in this modern era, it demands a global competition, specifically in the development information technology and communication rapidly in all aspects of life. The development of technology has the changed of qualifications and competences of the workforce. The issue of developing 21st century skills should become an attention for education researchers. The North Central Regional Education Laboratory (NCREL) and the Meteri group (2003) identify a framework for 21st century skills that is divided into four categories, to be precise digital age proficiency, inventive thinking, effective communication, and high productivity [16]. Assessment and Teaching for 21st Century Skills (ATCS) summarize four key issues related to the 21st century skills, namely way of thinking, way of working, work tools and life skills. Way of thinking includes creativity, critical thinking, problem solving, decision making and learning [16]. In this study, reflective ability is an ability to relate knowledge acquired to previous knowledge so that the summaries can be obtained to solve new problems. Therefore, the ability to think is very appropriate in solving math problems. The ability to think reflectively required to be meticulous and understanding a material or a problem. Obviously, it is in accordance with the learning of mathematics that must be thorough, skilled and quick in strategizing, especially in solving a problem.

To figure out how much students' reflective thinking ability, an educator must do some activities that can make students demonstrate reflective thinking ability. One of the activities is solving math problems. Problem solving is the most important part in mathematics; this even includes in the mathematics curriculum section. Since in the learning process, it is needed in every problem solving activities. Problem solving can stimulate brain function to develop students' mind power creatively in identifying problems and finding alternatives solution. Krathwohl (2002) in *A revision of Bloom's Taxonomy: an overview - Theory Into Practice* states that indicators for measuring higher-order thinking cover 3 levels that is analysis level, evaluation level and level creation, and 3 levels under those level is a low-ability[11]. Pohl (2000) report that the bloom taxonomy is considered the basis for higher-order thinking[14]. This thinking is based on the fact that some types of learning require more cognitive processes than others, but it has more general benefits in the bloom taxonomy. For example the ability to involve analysis, evaluation and create are considered by higher-order thinking. According to Krathwohl (2002) in *A revision of Bloom's Taxonomy: an overview - Theory Into Practice* the indicators for measuring high-order thinking include analysis, evaluation and creation. However, before accomplishing high level thinking, the three levels of remembering, understanding and applying must be achieved [11]. According to Dewey (2000), the reflective thinking is "active, persistent, and careful consideration of any belief or supposed from the knowledge in the light of the grounds that support it and the conclusion to which it tends". Thus, reflective thinking is active, continuous, persistent, and carefully considering everything that is believed to be true or the format of knowledge for reasons that support it and lead to the conclusion [16]. Sezer (Chee and Pou, 2012) states that reflective thinking is an awareness of what is known and what is needed. In this case, it is necessary to bridge the gap of the learning situation [4]. Meanwhile, according to Gurol (2011) the definition of reflective thinking is a directional process and exactly where individuals analyse, evaluate, motivate, and get deep meaning, using appropriate learning strategies[7]. Kember *et al* (2000) reveal Mezirow's theoretical framework and state reflective thinking can be classified into 4 stages: namely, Habitual Action, Understanding, Reflection, and Critical Reflection [10]. In assessing a reflective thinking by using reflective thinking indicators. YuukMing and Manaf (2014) provide indicators of reflective writing through four levels of descriptive writing, descriptive reflection, dialogic reflection and critical reflection [21]. Additionally, Hutton and Smith (1995) argue that in essence, the first is not reflective at all, but merely reports events or literature [8]

Table 1. Cognitive Level Krathwohl and Reflective Thinking Level Hutton and Smith

Code-Cognitive Level	Code- Reflective Thinking Level
C1 Remember	R1 Descriptive Reflection

C2 Understand	R2 Dialogic Reflection
C3 Apply	R3 Critical Reflection
C4 Analyze	
C5 Evaluate	
C6 Create	

Thinking levels for cognitive abilities are coded by using C. The lowest thinking level is encoded by using C1 and up to the highest level with C6 code: C1, C2, and C3 being low order thinking C4, C5, and C6 being high order thinking. In analysing the interrelationships of high-order thinking skills and reflective thinking, the authors combine the indicators of higher-order thinking skills and reflective thinking in the reflective writing notion used by Hutton and Smith [8]. King (1993) argues that "Higher order thinking skills include critical, logical, reflective thinking, metacognitive, and creative thinking"[17]. This indicates that reflective thinking is contained in higher-order thinking. Therefore, the authors choose one indicator of thinking combining level. On the other hand, there are some subjects whose thinking ability does not meet the indicators of higher-order thinking, but it presents the ability to think reflectively. Thus, the authors also include low-level thinking (C3) in terms of reflective thinking capabilities embodied with reflective writing that is:

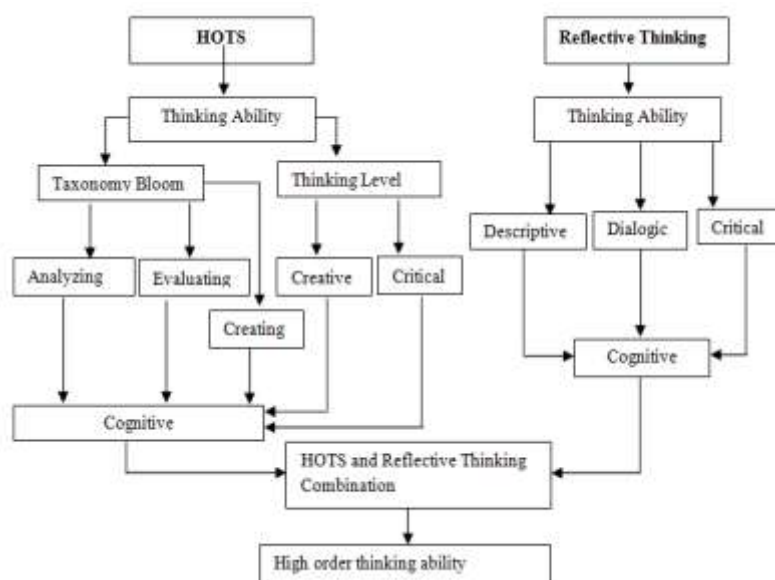


Figure 1. The combination mode of high order thinking and reflective thinking

Table 2. Combination of Thinking Ability and reflective thinking

Code-HOTS and Reflective	Code-LOTS and Reflective
HR1 Analyze Descriptive	L1 Knowledge
HR2 Analyze Dialogic	L2 Comprehension
HR3 Evaluate Descriptive	L3 Application
HR4 Analyze Critical	LR1 Apply Descriptive
HR5 Evaluate Dialogic	LR2 Apply Dialogic
HR6 Create Descriptive	LR3 Apply critical
HR7 Evaluate Critical	
HR8 Create Dialogic	

HR9 Create Critical

*L and LR codes are not included at the level to be considered

Based on the indicators, the lowest thinking level starts from L1 to the highest thinking level of HR 9. The thinking level encoded by L and LR is a thinking level that is not included for higher-order thinking. HR-encoded level is a combination indicator of higher-order thinking and reflective thinking by levels of HR1 to HR3 is a low-thinking level. HR4 to HR6 is the level of thinking being prepared for HR7 to HR9 is a high level of thinking.

Discovery learning model (DL) is a learning design that presents the subject matter through the critical thinking process is an integral part of the learning process. In this pursuit subjects are required to be more active in understanding the lessons and teachers are considered only as a companion of learning. Balim (2009) argues that "using the discovery learning methods in which the students are active and teachers guide them, is believed to increase the student's success and inquiry learning skills more than traditional teaching methods do"[2]. In this study, we analysis of the implementation of discovery based learning to improve students higher order thinking skills in solving r-Dynamic vertex coloring problem based on their reflective thinking skill.

Definition 1.1: (Graph Coloring) Given a graph $G = (V, E)$, a graph coloring of G is a function $f: V \rightarrow C$ from the set V of vertices to a set C of colors such that any two incident vertices are assigned different colors, where $C \subset \mathbb{IN}$ [20]. By r-Dynamic vertex coloring problem, we mean a proper k -coloring of a simple, connected and undirected graph $G = (V, E)$, which has a map $c: V(G) \rightarrow S$, where $|S| = k$, such that any two adjacent vertices receive different colors [1]. An r -dynamic k -coloring is a proper k -coloring c of G such that $|c(N(v))| \geq \min\{r, d(v)\}$ for each vertex v in $V(G)$, where $N(v)$ is the neighborhood of v and $c(S) = \{c(v) : v \in S\}$ for a vertex subset S [9]. The dynamic coloring of a graph G is proper coloring such that every vertex of G with degree has at least two neighbors that are colored differently[12].

Definition 1.2: (k -colorable) A graph G is k -colorable if there is a graph coloring of G with at most k colors [21]. The r -dynamic chromatic number, written as $\chi_r(G)$, is the minimum k such that G has an r -dynamic k -coloring. Note that the 1-dynamic chromatic number of graph is equal to its chromatic number, denoted by $\chi(G)$, and the 2-dynamic chromatic number under the name a dynamic chromatic number, denoted by $\chi_d(G)$ [1].

Definition 1.3: (Chromatic Number) For a given graph, the chromatic number of G , denoted by $\chi(G)$, is the smallest integer k for which G is k -colorable [21]. The r -dynamic chromatic number, introduced by Montgomery and written as $\chi_r(G)$, is the least k such that G has an r -dynamic k -coloring [6]. An r -dynamic coloring is defined as a proper coloring in which any multiple-degree vertex is adjacent to more than one color class. A dynamic coloring is thus a map c from V to the set of colors such that

- (1) If $uv \in E(G)$, then $c(u) \neq c(v)$, and
- (2) For each vertex $v \in V(G)$, $|c(N(v))| \geq \min\{2, d(v)\}$ [13].

Definition 1.4: (Optimal Graph Coloring) For a given graph G , the optimal graph coloring, denoted by $\varrho(G)$, is a graph coloring of G such that $|\varrho(G)| = \chi(G)$ [21].

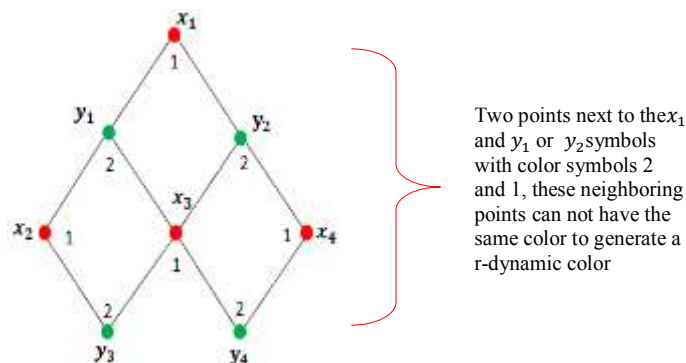


Figure 2. Graph along with r-dynamic coloring

2. Research Methods

This study was conducted by employing mixed method research because the study aimed at testing the effectiveness of discovery model level in learning high order skills as well as understanding the problem they encounter in learning high order is considered to be a mixing of quantitative and qualitative approach. Creswell (1998) argue mixed method as "is an approach to inquiry that combines or philosophical assumption the use of quantitative and qualitative approaches, and the mixing of both approached in a study". This study employed sequential exploratory model [5]. The research model is by qualitative research in the first stage, and then this study continue quantitative method in the second stage. The data will be analyzed as a whole and taken the conclusion from all the data. In qualitative research, the data source was taken from student worksheets and interviews, as well as using conjecture (phase portrait) to assess the process of subject worksheets completion. According to Sutarto et al (2018) conjecture is the statement based on the reasoning process where its truth is not yet certain. Quantitative research was experimental research model by using pre-test and post-test [18]. Barge (2007) argue Pre or post-test procedures are a commonly used method to evaluate learner outcomes of educational programs [3]. Cresswel argue the pretest were administered prior to the beginning of the trainings and the posttest were are administered just prior to the end [5].

Table 3. Experimental model of pre-test and post-test

Class	Pre-test	Treatme nt	Post-test
Experimen t	R_1	X	R_2
Control	R_1	-	R_2

In the quantitative analysis phase, the researchers deployed two tests, namely independent sample t -test result of pre-test and post-test of experiment and control class; and influence test by analysing questionnaires data and post-test study result. In the independent sample t -test phase, it consists of three stages, specifically homogeneity test, test of normality, and independent sample t -test. The test phase on the research was influenced by used validation and rehabilitation test of questionnaire data. After it was valid and reliable, it was conducted the homogeneity test. Then, the data are known to have the same variant done by regression test, regression equation test, correlation coefficient test and coefficient determination test. Triangulation of qualitative and quantitative data was built to shape the data valid.

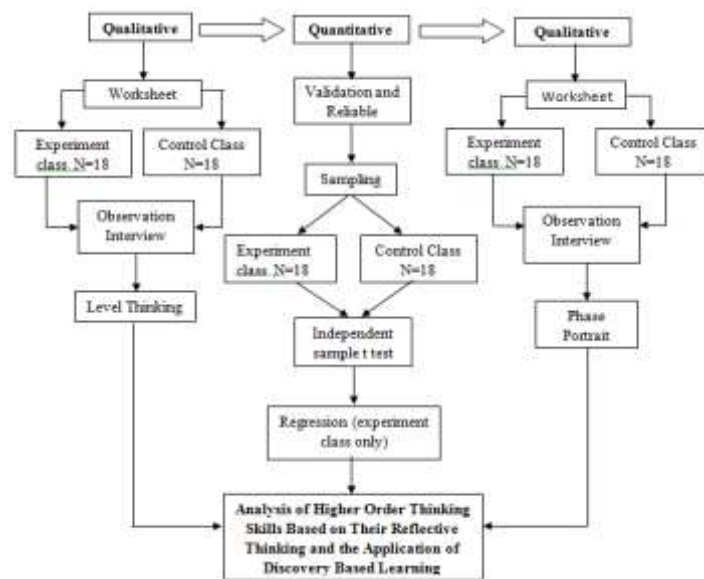


Figure 3. Triangulation Model of Qualitative and Quantitative Data

2.1. Population

Research subjects are 36 university students with who has been tested their thinking ability.

Table 4. Research Subjects

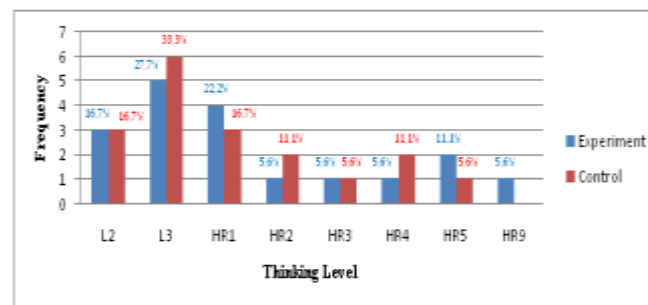
Sample		n = 36
Experiment		18
Control		18
Gender	Male	23
	Female	15

2.2. Instruments

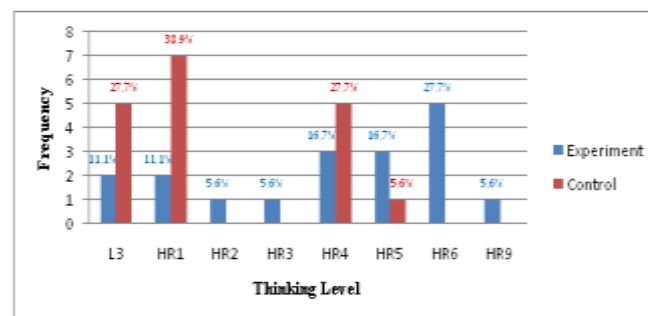
The instrument of analysis include tests, interviews, observation and questionnaires. instruments used for qualitative research are observation, tests, and interviews and the instruments used for quantitative research are questionnaires and tests at the test post stage in the experimental class and use phase portraits at the last stage to know deeply the student's process in completing worksheets

3. Results and Discussion

The first step uses a qualitative method on research subjects of 36 university students who has been tested their thinking ability. 36 subjects who had been tested on pre-test (diagram 1) in the experimental class, there were 16.7% of the subjects at L2 level, 27.7% of subjects at L3 level, 22.2% of subjects at HR1 level, 5.6% of subject at HR2 level, 5.6% of subjects at HR3 level, 5.6% of subject at HR4 level, 11.1% of subject at HR5 level, and 5.6% of subject at HR9 level with 44.4% subject placed at level L, as well as 55.6% at HR levels with 60% of subjects at low levels (HR1 to HR3), 30% of subjects at moderate levels (HR4 to HR6) and 10% of subjects at high levels (HR7 to HR9). The control class, there were 16.7% of subjects at L2 level, and 33.3% of subjects at L3 level, 16.7% of subjects at HR1, 11.1% of subjects at HR2 level, 5.6% of subjects at HR3 level, 11.1% of subjects at HR4 level, and 5.6% of subjects at HR5 level with 50% of subjects at L levels, and 50% at HR levels with 66.7% of subjects at low levels, 33.3% of subjects at moderate levels. There was a subject that is at a high level.

**Diagram 1.** Analyze of Pre-test Results

36 subjects were tested at the post-test (diagram 2) in the experimental class, 11.1% of subjects were at L3 stage, 11.1% of subjects were at HR1 stage, 5.6% of subjects were HR2, 5.6% of subjects were HR7, 16.7% of subjects were HR4, 16.7% of subjects were HR5, 27.7% of subjects were HR6. 5.6% of subjects were HR9 stage subjects with 11.11% of subjects at L, and 88 , 89% in the HR stage with 25% of subjects at low levels, 68.75% of subjects were moderate and 6.25% of subjects at high levels. The control class, there were 27.7% of subjects at L3 stage, 38.9% subjects were in HR1 stage, 27.7% of HR4 stage subjects and 2.5% of HR5 stage subjects with a total of 27.78 subjects who were at level L , and 72.22% with 53.85 subjects at low levels, 46.15% at medium level and none at high levels.

**Diagram 2.** Analyze Result of post-test

Now, we will use a statistic inferential for quantitative analysis of students achievement test. It was conducted to find out whether there are differences or not of students achievement test between a control class and experiment class. Homogeneity test was also done to find out whether the two classes are homogeneous or not. Based on Table 5, the homogeneity test obtained sig value. 0.667. Since the significance greater than 0.05 ($0.667 > 0.05$), it means that pre-test data is a homogeneous variant.

Table 5. Independent Sample Pre-test
Test of Homogeneity of Variance

		Levene	df	df	Sig.
		Statistic	1	2	
Score	Based on Mean	.188	1	34	.667

After homogeneity test analysis was done, it was continued by analysis of normality test. It functioned to gather the data on normal distribution. The data was distributed if the significant score more than 0.05. based on table 6, it presents the significant score for experiment class as $0.122 \geq 0.05$ and he control class as $0.082 \geq 0.05$. Thus, it meant that the data form the two classes were normal distributed.

Table 6. Normality Pre-test

Tests of Normality						
	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Experiment	.181	18	.122	.928	18	.325
Control	.191	18	.082	.918	18	.252

The test of variance analysis showed that the mean score on experiment class 58.055 (SD = 18.402). This score was more than the control class 49.444 (SD = 14.234) and the significance in the table shows the value [$t = 1.570$ and $df = 34$, $p > 0.05$] which mean that the score in the pre-test not different.

Table 7. Independent sample test pre-test

Independent Samples Test						
Class	N	Mean	SD	T	df	sig(2-tailed)
Experiment	18	58.06	18.4	1.57	34	.126
Control	18	49.44	14.2			

To know the difference of learning result of post-test experiment class and control class was done by data analyze using the same procedure. On Table 8, the value of homogeneity test significance $0.164 > 0.05$ which mean that value is homogeneous data variant.

Table 8. Homogeneity of post-test

Test of Homogeneity of Variance					
		Levene Statistic	df1	df2	Sig.
Score	Based on Mean	2.023	1	34	.164

Data analysis on Table 9 presents normality test of learning result for post-test at experimental class of $0.186 > 0.05$ and control class of $0.082 > 0.05$. It means that both data from the experimental class and the control normal.

Table 9. Normality of post-test

Tests of Normality						
	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Experiment	.169	18	.186	.932	18	.214
Control	.186	18	.082	.918	18	.252

		1			1	.25
Control	.191	8	.082	.936	8	2

The variance test data analysis showed average experimental class value of 68,888 (SD = 14,608). This average value is greater than control class of 55,555 (SD = 11,617) and the significance of the table [$t = 3$ and $df = 34$, $p \leq 0.05$]. It means there a difference in the value of the experiment class and control class after the treatment in the experimental class.

Table 10. Independent sample of post-test

Independent Samples Test						
Class	N	Mea n	SD	t	df	sig(2- tailed)
Experi ment	18	68.8	14.6	3	34	.005
Control	8	55.5	11.6			

Regression test is administered to know whether there is influence of applying of Discovery Based learning method to improve of high level thinking ability. Regression test hypothesis is taken after normality test, regression equation test, linear regression test, correlation coefficient test, and coefficient of determination test. The data normality test is done to know whether the data were normal distribution or not. Based on the significance value of kolmogorov-smirnova on Table 11, normality test of discovery based learning significance was $0.091 > 0.05$ and the significance of post-test experimental class study result was $0.063 > 0.05$. The significance value of shapiro-wilk normality test on discovery based learning was $0.18 > 0.05$ and significance value of experimental class of post-test results was $0.119 > 0.05$. Based on the two tests, the significance value was > 0.05 which the data normally distributed.

Table 11. Regression normality

Tests of Normality						
	Kolmogorov-Smirnova			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Discovery Based Learning	.188	18	.091	.928	18	.119
Post-test results	.197	18	.063	.918	18	.12

The regression equation test be used to predict how high the influence of dependent value (learning method) when the independent value (learning outcomes) manipulated.

Data analysis of regression test in table 13 that the searched regression sig value. of the value $0.066 > 0.05$. Thus, the appropriate regression equation for both variables are:

$$\bar{y} = -51.067 + 1.497x$$

\bar{y} = Learning Outcomes

x = Discovery based learning method

According to the previous equation in the regression test, it is obtained by regression coefficient of 1.497 which state that any increase in the use of Discovery Based Learning method of learning affect the learning outcomes of the subjects of 1.497.

Table 12. Regression Test

Coefficient					
Model	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	T	Sig.
(Constant)	51.07	25.9		1.97	.07
DBL	1.5	.32	.75	4.65	.00

Regression linearity test done to determine the regression equation that has been made linear or not. Based on table 12, the value of t_{count} in discovery based learning variable is 4,649, on degrees free (df) = $N - 2 = 18 - 2 = 16$ on 95% significance of table value of 2,119. It can be concluded that $t_{count} > t_{table}$ which means there is influence of Discovery based learning method to high level thinking ability. Based on linearity test using ANOVA, table linearity value ($F = 114.855$) with significance value $0.00 < 0.05$ which means analysis of regression data can be used to know the effect of Discovery based learning method on high-order thinking ability.

Table 13. Regression Linearity Test

ANOVA Table					
Discovery Based Learning * Achievement Post Test	Sum of squares	Df	Mean Square	F	Sig.
	887.91				.000
Between Groups	534.29	8	110.95	23.851	.001
Within Groups	353.62	7	50.52	10.859	
Total	929.78	17	4.651		

The correlation coefficient test aims to find out the strong weakness of the relationship between two variables. Table 14 presents correlation coefficient of 0.758. Based on interpretation table of correlation coefficient value, it shows 0.758 that means there is a strong relationship level, specifically

in the sense of application of Discovery based learning method has a strong relationship with student learning outcomes. Coefficient of determination is done to measure how big contribution between dependent variable and independent variable in the form of per cent. Coefficient of determination (R Square) obtained equal to 0.575 with criterion level of relation. This means that the contribution of the dependent variable (the result of the test post) by the free variable (Discovery based learning) is 57.5% and 42.5% explained by other variables.

Table 14. Correlation Coefficient Test and Determination test

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error
1	.758	.575	.548	9.821

The regression equation test can be used to predict how high the dependent value (learning method) when the independent value (learning outcomes) is manipulated.

Based on Table 12, it demonstrates the regression sig value. of $0.00 > 0.05$. Thus the appropriate regression equations for both variables are:

$$\bar{y} = -51.067 + 1.497x$$

\bar{y} = Learning Result

x = Method of discovery based learning

From the previous equation, regression coefficient is 1.497 which states that any increase in the use of learning methods Discovery Based Learning affects student learning outcomes of 1.497. Hypothesis test is used to know the influence of discovery based learning method to the ability of higher-order thinking. The criteria of the hypothesis follows:

Ho: "There is an influence of Discovery Based Learning method on high-order thinking ability"

Ha: "There is no influence of Discovery Based Learning method on high-order thinking ability"

Ho is accepted when $t_{count} > t_{table}$ which means there is a significant influence between Discovery Based Learning method to high-order thinking. Ho is rejected when $t_{count} < t_{table}$ which means there is no influence of discovery based learning method to the ability of higher-order thinking. In the regression test table on table 12, the value of t_{count} 4.649 and the value of t_{table} on degrees free (df) = $N - 2 = 18 - 2 = 16$ with 95% significance of 2.119 with the provision $t_{count} > t_{table}$ ($4.649 > 2.119$). Then, Ho accepted and Ha rejected with the provision that there is a significant influence between discovery based learning methods on high-order thinking ability.

Furthermore, the analysis of the following worksheet result was conducted to find out the process of completion of the student higher order thinking skills. This result was derived by using an interview and observation on the students worksheet. We choose some subjects, but in this paper we only present one subject.

Subject 1

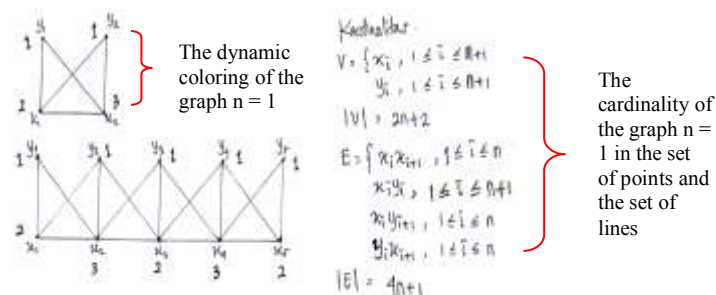


Figure 4. Graph of the subject

The analysis of subjects 1 (figure 4) of the r-dynamic coloring of the graph is appropriate and the color applying on the graph of order $n = 4$. When we verify the result whether it is true or not, we finally conclude that the appropriate r-dynamic coloring meets the requirements for graph coloring of 1-dynamic.

v	$c(v)$	$c(N(v))$	r	$d(v)$	$\min\{r, d(v)\}$	$\{c(N(v))\} \geq \min\{r, d(v)\}$
x_1	2	2	1	3	1	y_6
x_2	3	2	1	5	1	y_6
x_3	2	2	1	5	1	y_6
x_4	3	2	1	5	1	y_6
x_5	2	2	1	3	1	y_6
y_1	1	2	1	2	1	y_6
y_2	1	2	1	3	1	y_6
y_3	1	2	1	3	1	y_6
y_4	1	2	1	3	1	y_6
y_5	1	2	1	2	1	y_6

Table proof
dynamic staining r
on graph of figure 4

Figure 5. The r-dynamic coloring of graph of order $n = 4$

In the vertex x_1 verification, the table marked with number 2, the point has 2 neighboring colors that are symbolized with numbers 1 and 3 and will be proved for 1-dynamic, the point has a maximum of degrees 3 which means that the coloring at that point should not exceed the degrees, since the number of colors that are 2 adjacent colors and the minimum value between the dynamic value and the maximum degree is 1 then the coloring is true in 1-dynamic. For the point x_2 through y_5 all the coloring meets 1-dynamic since all the coloring on the vertices of the graph $\geq \min$ the dynamic value or the maximum degree.

To know deeply the student process of thinking skills, we will use a portrait phase, the portrait phases are taken to draw the process of higher order thinking skills based on their reflective thinking. We have selected six research objects of each experiment class and control class, but in this paper we depict only three subject as an illustration. The following is the higher order thinking skills associated with reflective thinking skills on solving r-dynamic problems.

Table 15. Description of the phase portrait grains

1	= Understanding the definition of vertex coloring	
2a	= Understanding the cardinality of graph	
2b	= Understanding the strategy to assign the r-dynamic coloring	
3	= Understanding the characteristic of special graph	a
4a	= Understanding the graph expansion technique	
4b	= Defining the notation of vertices and edges	
4c	= Developing a vertex and edge sets and determining the order and size	
5	= Assigning the r-dynamic coloring	
6a	= Confirm the fullfulness of the property of r-dynamic coloring	
6b	= Formulate the algorithm	
7a	= Developing a truth table of the r-dynamic coloring	
7b	= developing a bijective function	
7c	= Justifying the fullfulness of the property of r-dynamic coloring	
8a	= Formulate the lemma or theorem	
8b	= Prove the obtained lemma or theorem	
8c	= Take into a general cases	

8d = Conjecturing a general result of a general graph

Based on the above indicators, we did interview on the selected research subjects. The interviews included some key questions that are directly related to the workflow of completion line. This following is one of the interviewed students script.

Researcher : In understanding the definition of dynamic coloring difficult? which parts are considered very difficult?

Subject : The properties of the r -dynamic is the critical part since having a k -coloring is not enough to finish the coloring we need to satisfy a further condition.

Researcher : How do you start assign a color? Do you have a specific strategy to get it? Please tell me.

Subject : Yes we have and we need to have it, otherwise it will be harder to obtain it. By applying the greedy algorithm we can get a k -coloring, further we rearrange such that it will be easily to expand, then finally we check the fullfilness of the r -dynamic properties.

Researcher : Once you get the r -dynamic coloring, can you always expand it. How to handle the obtained colors which is not able to be expanded?

Subject : When we have a color which has meet the r -dynamic coloring properties, we try to assign color for the bigger order of a graph, further by recognize the pattern then we extend the colors, we also expand the cardinality of the graph. When the graph is considered a special graph then expansion of cardinality does not give a complexity, however if the graph is an operation of special graph then it will have more complexity. It also affects to finding the r -dynamic assigning.

Researcher : Once you can extend the colors and the cardinality, do you have a difficulty to start deriving a bijective function? What strategy did you use?

Subject : This is also crucial for me sir. I have not get used to find a bijective function sir, but we continuously derive the function. I frequently get a mistake in defining the domain x of vertices, and also get confused selecting the sequence of cics olors which form an arithmetic sequence.

When the interview is continued, then we can get Figure 6 showing the process of higher order thinking skills of subject 1 in solving the r -dynamic problem. In step 1 to stage 4b subject 1 thinks straight according to what is explained, but in step 4b subject 1 returns to 3, in step 4c jump to step 7a, after the interviewer's instructions he explores and chooses stage 5, then jumps to step 6b and continues 7a, 7b. In step 7b the flow of his thought goes back to 6a and then continues to 7c, 8a. In step 8a, skip to 8c and then go back to 8b and choose 8d.

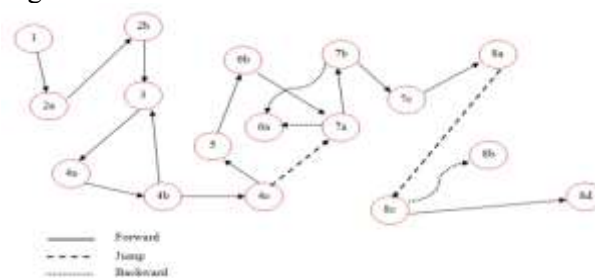


Figure 6. The phase portrait of subject 1

Subject 2

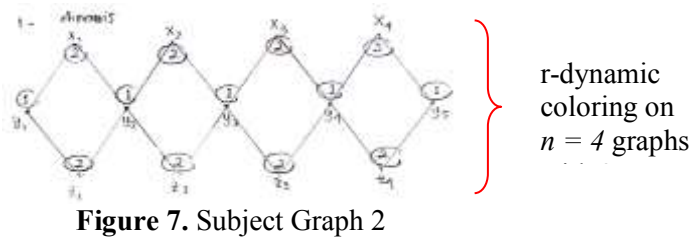


Figure 7. Subject Graph 2

The above graph is the r -dynamic coloring done by subject 2. The graph of order $n = 4$ by assigning 3 different point colors of which there is no adjacent vertices have been assigned a same colors, as well as it satisfies the properties of r -dynamic. It is true and subject 2 attain level HR3, namely Evaluating Descriptive. By applying the same technique to the subject 2, we get the following phase portrait depicted in Figure 8 showing the process of higher order thinking skills. Subject 2 undergoes repetition at stages 4c, 6a, 7a, 7b and 8c, as well as jumps at stage 1, 4c

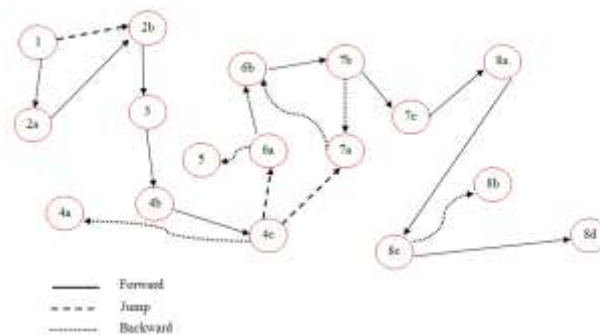


Figure 8. Phase portrait of subject 2

Subject 3

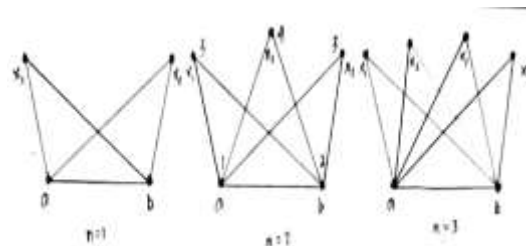


Figure 9. Phase Potrait subject 3

The above graph is the r -dynamic coloring done by subject 3. The graph of order $n = 1, 2, 3$ by assigned 4 different colors of which no adjacent vertices has been assigned a same colors, as well as it satisfies the properties of r -dynamic. It is true and subject 3 attain level HR2, namely Analysis Dialogic. By applying the same technique to the subject 2, we get the following phase portrait depicted in Figure 10 showing the process of higher order thinking skills. Subject 3 also undergoes repetition at stages 5, 6b, 8a and 8b and jumps at stage 4c.

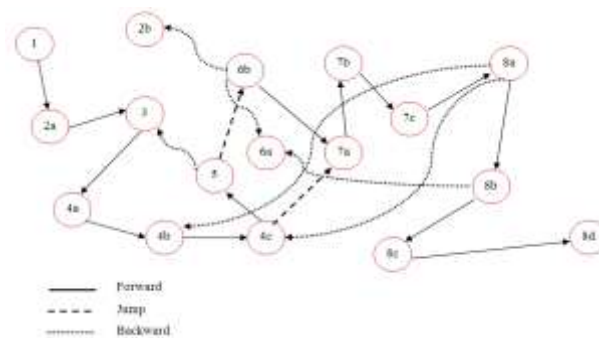


Figure 10. Phase Potrait subject 3

Combining the three above portrait phase, now we have Figure 11 depicting a combination of the three subjects of their higher order thinking skills process in global form. We can see that the in-degree and out-degree sequence of this portrait phase is not regular, it shows that the students thinking from one step to ther step is vary depend on how deep their understanding on the *r*-dynamic problems. The interesting one is the four in-degree occurs in stage 6a and four out-degree occurs in stage 8a. It implies that the thinking skills process of students does not consistently step up from lower stage to higher stage, but conversely it goes forward and backward.

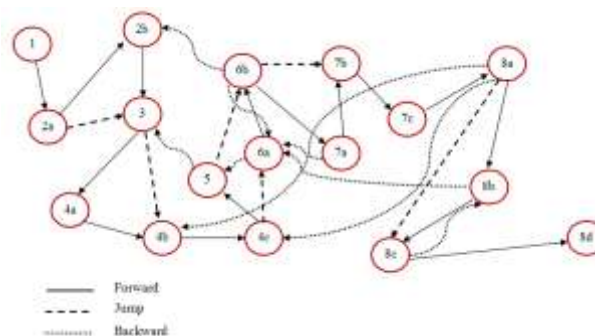


Figure 11. Phase Potrait combination

Discussion

This study was conducted to analyze the higher-order thinking skill based on their reflective thinking under the implementation of discovery based learning. The research findings of pre treatment stage, it shows that, in the experiment class there are 8 students at LOTS stage and 10 students at HOTS stage. For the control class, there were 9 students at LOTs stage and 9 students of HOTS stage. This result is inline with Ming and Manaf research (2014), they stated that the scientific approach will boost the students thinking skills.

After the implementation of discovery based learning on experiment class, the learning outcomes at the post-test stage of the experimental class gave 2 students at the LOTs stage and 16 students at the HOTS stage. For the control class there were 5 students at the LOTs stage and 13 students at the HOTS. The result of independent sample test analysis showed that the students' learning outcomes at the pre-test stage were not significantly different, but in the post stages the students' learning outcomes showed significant value ($p \leq 0.05$) which means that the post-test stage showed a different learning outcome averages. It was shown by *t*-test and regression analysis. Regression analysis reveals that discovery based learning has 57,5% influence with significance of $t_{count} > t_{table}$ ($4.649 > 2.119$). This study is in line with Hobri et al (2018) as well as with Balim (2009). They claim that by teacher

centered learning students will be passively receptive, they can not be expected to enter into higher order thinking skills level.

4. Conclusions

We have analyzed the students higher order thinking skills based on their reflective thinking skills under the implementation of discovery based learning. The results shows, in the experiment class, that the level of higher order thinking skills who has a good reflective thinking skill in solving r -dynamic coloring problem are 11.11% of students having low level of HOTS, and 88.89% having high level of HOTS. The level of higher order thinking skills who has a fair reflective thinking skill are 25% of students having low level of HOTS, 68.75% of students having fair level of HOTS, 6.25% of students having high level of HOTS. In the control class, the level of higher order thinking skills who has a moderate reflective thinking skill are 27.78% of students having low level of HOTS, and 72.22% having high level of HOTS.

The data analysis also shows that there are a significant differences of HOTS in solving r -dynamic coloring problem regarded to the score of students achievement test. The results of students post in term of independent sample t -test significance value is ≤ 0.05 . It implies that there is an effect of the application of discovery based learning to improving the students higher order thinking skills in solving r -dynamic coloring problem with regression test of the learning model for the experimental class $> (4,649 > 2,119)$. It concludes that we accept H_0 and reject H_a .

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