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# The ability of high school students' critical thinking in solving trigonometric problems

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**Abstract.** Critical Thinking has been one of the tools used in our daily life's to deal with the challenges for survival. Developing critical thinking abilities is something which is very important in the world of education today. Problem solving can support increased critical thinking abilities This study aims at describing the students' critical thinking skills in solving Trigonometry problems. Critical thinking ability in this research employs an indicator which consists of *Focus*, *Reason*, *Inference*, *Situation*, *Clarity*, and *Overview*. The type of this research is descriptive research with qualitative approach. The research subjects consist of 3 students, i.e. high-ability student (ST), medium (SS) and low (SR). The results of this study indicate that there are 5 indicators that can be achieved by the high-ability student (ST) namely *Focus*, *Reason*, *Situation*, *Clarity*, and *Overview*. Meanwhile the medium students are capable of achieving 2 indicators only, namely *Focus* and *Reason* and low ability student achieves 1 indicator of critical thinking ability, namely *Reason*.

## 1. Introduction

One of the important capabilities a person must possess is critical thinking[1]. Critical thinking skills are preferred over algorithmic cognitive abilities. It is also expressed by Amit and Azikovitsh (2015) that the current reform of mathematics education throughout the world includes the transition from algorithmic cognitive abilities towards higher cognitive abilities, i.e. critical thinking skills. The ability to think critically has been recognized as an important ability for the success of learning, working and living in the 21st century [2]–[4]. High critical thinking and good communication skills will easily make someone to changes and be appreciated in both academic and workplace contexts [5].

An understanding of critical thinking has been largely defined by some experts [6]–[8]. According to Facione (2015) the most basic concept of critical thinking ability is the ability of interpretation, analysis, evaluation, conclusions, explanation and self-regulation [6]. Ennis (1996) defines critical thinking as "reflective thinking that makes sense and focuses on what decision-making should trust or what to do " [7]. The indicators of critical thinking according to Ennis (1996) include Focus, Reason, Inference, Situation, Clarity, and Overview. Meanwhile, according to As'ari (2016) a critical thinker is always challenging, analyzing, and assessing the truth of information thus it can be maintained and accounted [9]. Therefore, critical thinking ability is needed by students to use new information or manipulate existing knowledge and information to obtain a reasonable response in the new situation [10], [11].

The Ministerial Regulation of National Education No. 22 of 2006 on Content Standards reads that "mathematics subjects should be given to all learners from elementary school to equip them with



logical, analytical, systematic, critical, creative and cooperative thinking" [12]. This is in line with the competence of mathematics content contained in the Ministerial Regulations of Education and Culture of 2016 article 21 on the level of basic education (Class VII-IX) stating that students are expected to demonstrate certain attitudes including logical, critical, analytical, meticulous and thorough, responsible, responsive, and not easy giving up in solving a problem [13].

Fisher (2001) argues that critical thinking skills need to be taught to students because so far the teacher only teaches about the content of the subject matter and pays less attention on the students' critical thinking skills [14]. This is because critical thinking is widely viewed as a basic competency, as well as reading and writing, which should be taught. In addition, according to Johnson (2009) critical thinking is an essential ability that the students should have to enable them to solve the problems they face [15]. Based on the aforementioned opinion, the ability to think critically should be one of the activities developed and taught in every subject. This responsibility for developing and teaching goes to the teacher because she or he has the freedom to design learning before the learning process takes place. To design an appropriate learning in developing and teaching critical thinking, a teacher can observe the students' thinking ability. It is intended to emphasize more attention to students' critical thinking skills in every math class.

To know students' critical thinking ability, the researcher observes the student's activity in solving problem. This is in accordance with the opinion stated by Sabandar (2009) and Johnson (2009) that there is a relationship between critical thinking and how to solve the problem [15], [16]. The indicators of critical thinking skills used in this study are indicators proposed by Ennis, including Focus, Reason, Inference, Situation, Clarity, and Overview. The study is, therefore, intended to look students' (having high, moderate and low ability) critical thinking skills.

## 2. Literature Review

### 2.1. Critical Thinking

According to Ennis (1996) critical thinking is a logical and reflective thinking that focuses on decision-making patterns of what to believe and what to do [7]. In addition to being logical and reflective, two things require an attention from the definition of critical thinking proposed by Ennis (2011), are to believe or to do [17]. According to Facione (2011), critical thinking is seen as an objective and self-directed decision that results in interpretation, analysis, evaluation, and conclusions, as well as an explanation of the evolutionary, conceptual, methodological, criteria, or contextual considerations that underlie such as judgments [6]. Krulik and Posamentier (1995) argue that critical thinking is analytical thinking and reflection involving activity testing, questioning, linking and evaluating all aspects of a situation or problem [18]. Meanwhile, according to Paul & Elder (2002), critical thinking is the interpretation and evaluation in thinking and used to build the assumptions and application of concepts [19]. Fisher (2011) states that critical thinking is skill and active interpretation and evaluation of observations and communications, information and argumentation", meaning that critical thinking is a skill of interpretation and actively evaluates the results of observation, communication, information and argumentation [14]. Walpern (1998) argues that critical thinking is the thinking needed to solve problems, formulate conclusions, calculate possibilities, and make decisions. Based on the definitions of the critical thinking, it can be concluded that critical thinking is logical and reflective thinking in explaining concepts, solving problems, and making credible and believable decisions.

### 2.2. Indicator Critical Thinking Skills

According to Ennis (1996) people who have critical thinking skills also ideally have some basic indicators or elements that are abbreviated to FRISCO (*Focus, Reason, Inference, Situation, Clarity, and Overview*) [7]. Focus is to identify all information well contained in a problem. Reason is providing logical reasons in a problem. The conclusion (inference) that if the reason is right, is it enough to arrive at the conclusion given? Situation is matching the actual situation. Clarity is there

must be clarity about the terms used in the argument so that no errors in making conclusions. Overview is a re-checking all steps taken to get a conclusion. The indicator of critical thinking ability in this research can be seen in table below.

**Table 1.** Indicator of Critical Thinking Ability

Indicators of Critical Thinking Ability	Explanation
<i>Focus</i>	Determining the information needed to make decisions in solving a problem.
<i>Reason</i>	Providing reasons based on relevant facts/evidence on each step in making decisions and conclusions.
<i>Inference</i>	From the statements made can be drawn the right conclusions.
<i>Situation</i>	Using all information based on the situation described in the problem (physical environment and social environment) and describing the problem situation contained in the problem
<i>Clarity</i>	Reassert the statement of a problem so it can be drawn the right conclusions
<i>Overview</i>	Review and thoroughly examine the decisionstaken.

### 3. Methodology

This is a descriptive research with qualitative approach. The research subjects are high school students in Pasuruan region and 3 students were selected. Research subjects were taken to be the selected respondents based on the category of capability covering high, moderate and low. The instrument used is the critical thinking ability of Trigonometry material made in the form of a description problem that refers to the indicator of critical thinking ability, i.e. *Focus*, *Reason*, *Inference*, *Situation*, *Clarity*, and *Overview*.

Furthermore, the process of interviewing the subject was done to find a deeper explanation of the result of the test with questions that focus on each indicator of critical thinking ability. The result of the interview is then transcribed, coded, analyzed, and used to support the process of decrypting students' critical thinking ability by the researcher. The subject selection path is presented in figure 3.1.

### 4. Result and Discussion

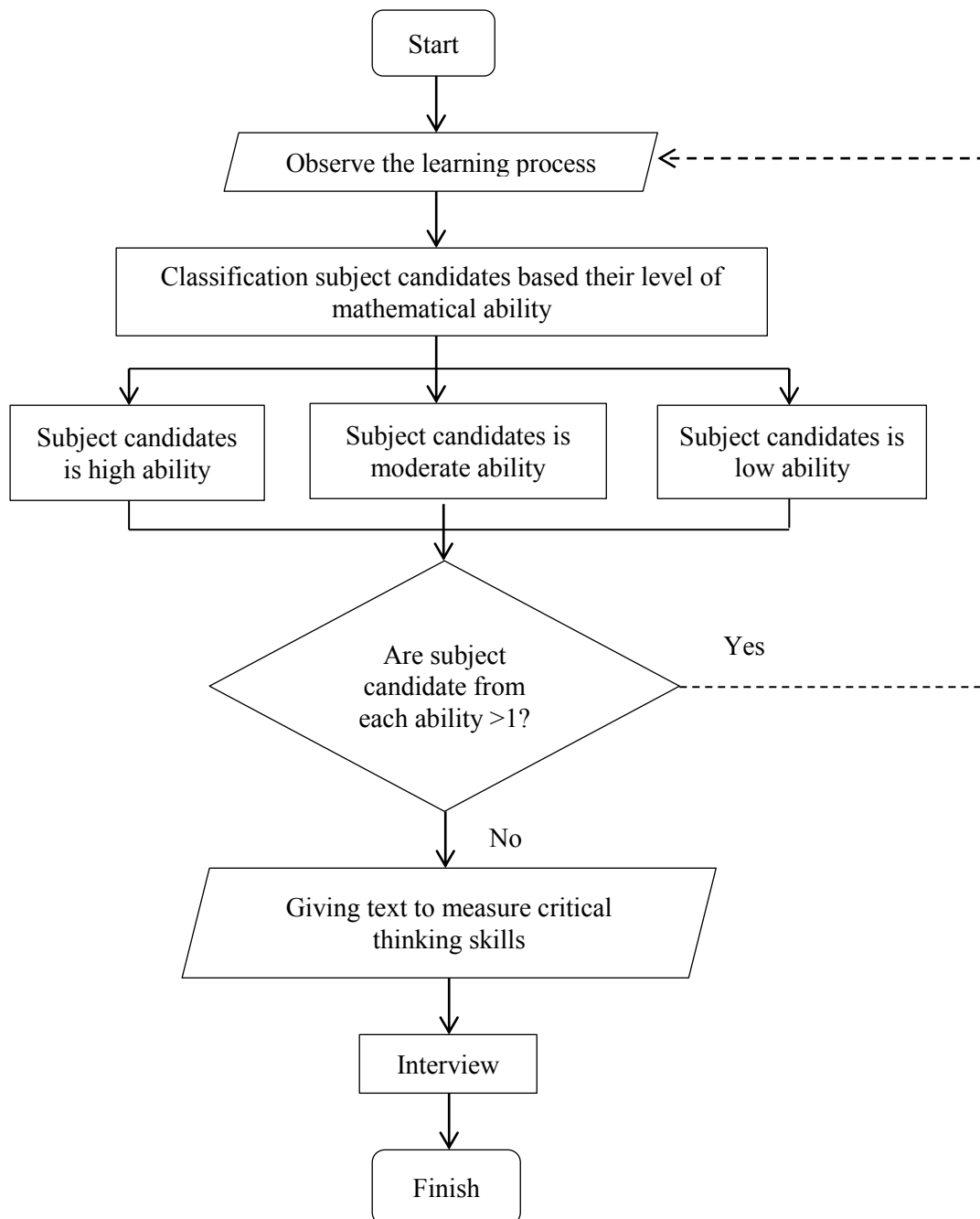
The following to explain students' critical thinking skills of high ability subject (ST), moderate (SS) and low (SR) based on the indicators of critical thinking ability.

#### 4.1. Focus

The focus of this study, is the students' determination of the information needed to make decisions in solving a problem. ST and SS started the writing down all the required information correctly, while SR gave no answer.

The answers from ST and SS are almost the same so that the researcher will write the result of the interview conducted to one of the subjects. The following is the transcription from the interview with SS.

- P* : What do you do when you are given such a problem? will you do it immediately or will you observe it first?  
*SS* : I will observe it first because this type of problem cannot be done immediately.  
*P* : What kind of information do you use in answering the problem in the question?  
*SS* : I will look for an angle from a known angle in the problem.



**Figure 1.** Subject selection flow

The data above shows that ST and SS have been able to determine the information in the problem then applied information to find the angle in question. The subjects are able to identify the facts given in the matter clearly and logically as to identify problems and understand questions in the matter. It can be seen from whose ability to write down the elements that are known from the question and asked in the question. However, the subject only wrote the elements used to solve the problem. The subject could retell the information contained in the matter of trigonometric material using his own words but there are some sentences that are still adopted from the problem. Yet, the sentences used to retell tend to be brief and clear, but the subject is very cautious that make him take longer time.

In addition, they also observe the questions given by researchers before answering the questions. This is in accordance with As'ari's statement (2016) that people with the ability to think critically when given a problem, will not directly give an answer to it but observe the problem further and find more information in the matter.

#### 4.2. Reason

The reasons for this study is that students are able to provide reasons based on relevant facts/evidence at each step in making decisions and conclusions. Based on the result of the test, ST, SS, and SR are able correctly answer test. Yet particularly this indicator had not been clearly seen in SR, compared to ST and SS's work. The following is the results of SS work.

Benar karena  $\tan \alpha = x$  Dan  $= x + \text{tinggi orang}$   
 $1.8$   
 $3 = x$   
 $4 \quad 8$   
 $6 = x$   
 $= 6 + 1.8$   
 $= 7.8 \text{ m}$

**Figure 2.** Results of SS's work

SS does not describe the situation indicator. In the calculations, he wrote the unit in the final result instead of writing down the process. The above picture shows that SS knows the concept and did not really apply it to the given problem. This is in line with what was revealed by Carson (2007) that although students know a concept, they will not necessarily apply how to use it [20].

#### 4.3. Inference

Inference meant in this research is student's ability to make statement that can be used to draw the right conclusion. Based on the results of subject's work, all ST, SS, and SR did not give appropriate inference. They simply concluded the question that had been asked without making any possible statement to be concluded.

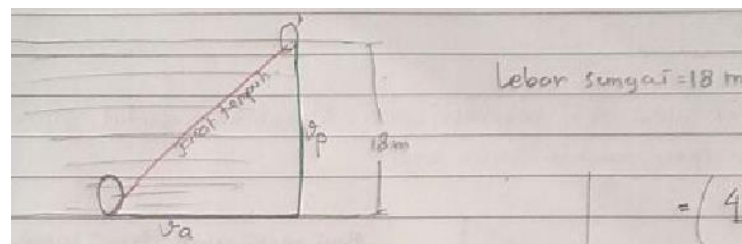
Student's Work	The Translation of the student's work
<p>Ada berapa cara untuk menentukan segitiga baru yang sebangun dengan <math>\triangle ABC</math>?</p> <p>Menurut saya ada dua cara untuk menentukan segitiga baru yang sebangun dengan <math>\triangle ABC</math>, yaitu:</p> <ol style="list-style-type: none"> <li>1) Mempunyai perbandingan panjang sisi yang sama</li> <li>2) mempunyai sudut yang sama besar</li> </ol> <p>∴ Menurut saya, jika akan menentukan bangun datar yang sebangun harus ada 2 hal tersebut, jika salah satu dari hal tersebut tidak ada maka bangun tersebut dikatakan tidak sebangun.</p>	<p>How many ways can you mention to list the new triangles which are congruent with triangle ABC?</p> <p>It seems to me there are two ways to specify that a new triangle is congruent with the triangle ABC, namely:</p> <ol style="list-style-type: none"> <li>1) It has the same side length comparison</li> <li>2) It has the same great angle</li> </ol> <p>In my opinion, if you specify a triangle unvarying there should be 2 things. If one does not exist then it is not congruent</p>

**Figure 3.** Subjek's work

Based on the data above, the subjects of the inference are in accordance with the question and the reasons used to make the conclusions are also appropriate to support the conclusions made and is still relevant tends to be briefly clear. The indicators of reason and inference are interrelated. The accuracy of the outcome of inference is greatly influenced by the outcome of the reasoning process.

#### 4.4. Situation

The situations intended in this study are to use all information based on the circumstances illustrated in the problem (physical environment and social environment) and to illustrate the problem situations contained in the problem. Based on the results of the subject's work, ST has a high analysis in describing the situation in the question compared with the results written by SS and SR, as shown in the picture below.



**Figure 4.** ST's Work

The picture depicts that ST has been able to describe the situation correctly in the given matter. In addition, ST is also able to solve problems with appropriate conditions given to the problem. Here are the results of the researcher's interview with ST.

*P: Please explain what you mean in your answer?*

*SS : I calculated the distance or the side of triangle by using Pythagoras Theorem then calculated it using comparison.*

*P : Are there other factors that affect your answer after knowing the problem on the question?*

*SS : There may be, there must be another point of view that affects the problem.*

*P : What considerations do you give when answering the problems that exist in the question?*

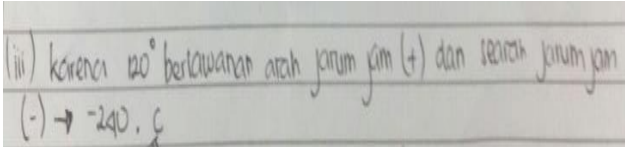
*SS : Distance to speed, because it can affect the results and speed of river flow.*

Based on the conversation above, the subject is able to use all the important information well with a relatively short time required and can consider other factors that exist in the problem. In addition, the subject was able to analyze the existing problem. Therefore based on the Ennis' (1996) indicator of critical thinking ability this subject has met the indicators situation, which is able to describe the problems that exist in the question well [7].

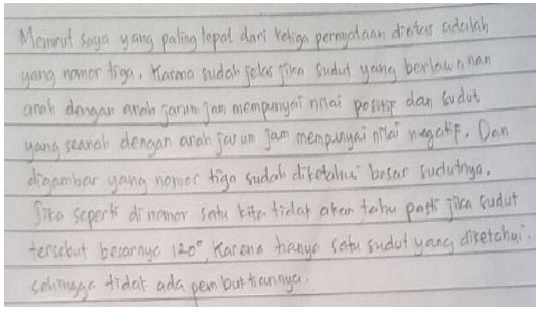
#### 4.5. Clarity

Clarity intended in this study is to re-affirm the statement of a problem so the right conclusion can be taken. From the results of subject's work, all subjects of this study have responded appropriately. Still, for an explanation of why to choose the answer, only ST gave the most appropriate answer, while SS and SR only answered correctly but did not give clear explanation of why the statement was chosen.

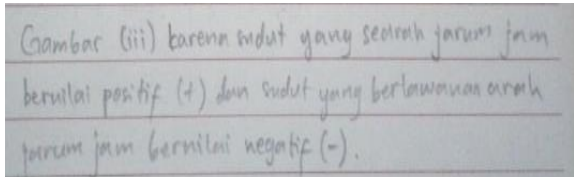
Here are results of SS, SR, and ST's work

Student's Work	The Translation of the student's work
	(iii) Because $120^\circ$ of the counter-clockwise (+) and clockwise (-) $\rightarrow -240$

**Figure 5.** SS's work

	In my opinion the most appropriate statement among the three sentences is the third statement because it's obvious if the angle opposite to the direction of the clock it has positive value and angle clockwise it has negative value and the picture number three already known large angle. In statement one we will not know for sure that the magnitude of the angle $120^\circ$ because only one angle is known
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**Figure 6.** ST's work

	Pictures (iii) since the angle clockwise is positive (+) and angle counterclockwise is negative (-)
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**Figure 7.** SR's work

Apparently, the three subjects answered correctly as observed in the picture. However, only ST gives the correct reason. ST is able to give further explanation (justify or clarify) about his final conclusion, able to explain the term contained in the question so it can be concluded that ST's ability in clarity indicator is the best among all the subjects.

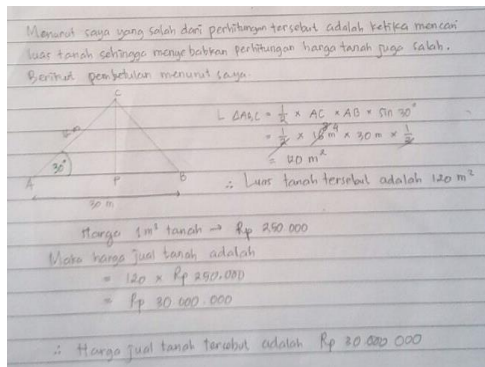
#### 4.6. Overview

Overview, in this study, is to review and thoroughly examine the decisions taken. From the work of the subject, only ST can meet the overview indicator. He is able to analyze deeper in the problems and able to check the errors contained in the answers given in the question, as seen in the following picture. While SS and SR failed to provide deeper analysis so that the answers generated were still wrong.

Student's Work	The Translation of the student's work
	It seems to me that the calculation is wrong while looking for land area causing land price calculation is also wrong. The following is the justification.
	$\text{area of a triangle} = \frac{1}{2} \times AC \times AB \times \sin 30^\circ = \frac{1}{2} \times$



$$16 \text{ m} \times 30 \text{ m} \times \frac{1}{2} = 120$$



So the area of a triangle is  $120 \text{ m}^2$

**Figure 8.** The result ST test

To know whether the subject is doing overview or not, the researcher interviewed the subject. The following is the result of the interview.

*P: What do you do when presented such questions and answers?*

*ST : I will observe and recalculate the answers in the question*

*P : Do you think the answers that have been presented are correct? declare your reason.*

*ST : Not true, because it is supposed to find the area of the triangle with the rules of sin instead of cos.*

Based on the results of the interview above, it appears that the subject re-checked the questions and answers given by researchers. Then, he analyzed and recalculated what was asked in the question. The subject is able to check the truth of the solution obtained by linking to the context of the given problem situation and ensure that the answer obtained is correct.

The aforementioned data are simplified into a table to know which indicators have been achieved by each subject.

**Table 2.** Achievement of Critical Thinking Criteria Indicators

Indicators of Critical Thinking Ability	ST	SS	SR
<i>Focus</i>	Fulfilling, the subject uses the information needed in the problem to solve the problem completely	Fulfilling, the subject uses the information needed in the problem to solve the problem	Not fulfilled
<i>Reason</i>	Fulfilling, the subject is able to provide the correct reason for the conclusion	Fulfilling, the subject is able to provide a reason for the existing conclusion	Fulfilling, but does not give clear explanation on the reason
<i>Inference</i>	Not Fulfilling, because the conclusions were taken directly without giving reasons in advance in	Not fulfilled	Not fulfilled

making conclusions			
<i>Situation</i>	Fulfilling, It can already describe the situation correctly to the problems that have been given and able to analyze more deeply on the problems encountered in the matter	Not fulfilled	Not fulfilled
<i>Clarity</i>	Fulfilling, the subject is able to give a proper explanation of the conclusions taken	Not fulfilled	Not fulfilled
<i>Overview</i>	Fulfilling, the subject is able to check or review the problems that have been given in the matter	Not fulfilled	Not fulfilled

From the results above it can be concluded that student with high academic ability also has a high critical thinking skills and vice versa, students with low academic ability is also low in critical thinking skills. This is in line with Sadler & Fowler's (2006) opinion which states that students with high academic ability can outperform other students in justifying arguments. It can be seen from ST who is high academically, and he is able to reach 5 indicators of critical thinking ability [21]. Meanwhile the subjects who are low in their academic are only able to achieve 1 indicator of critical thinking skills.

O'Rourke (2005) argues that students who are able to examine and evaluate an argument/claim, will identify and evaluate an argument, and can provide supporting evidence, performs an important part of critical thinking [22]. In otherhand, it also revealed by Indrawatiningsih (2018) that argument identification is the main indicator of the ability of argumentation. In addition, the ability to argue is the core of critical thinking [23]–[25]. In this case on ST, he can examine, assess and make the right conclusions of the problems. In addition, he can also give precise reasons for the conclusions that have been taken so it can be concluded that he is good at critical thinking and has the ability to think critically.

Therefore, critical thinking skills must be applied and developed in the core curriculum, teaching and learning process to produce students who have the quality of future leaders' thinking. It is important to develop students' critical thinking skills in all subjects, especially in mathematics.

## 5. Conclusion

Based on the results and discussion above, some conclusions can be drawn, as follows: (1) Students who have high academic ability (ST) have reached 5 indicators of critical thinking ability namely *Focus*, *Reason*, *Situation*, *Clarity*, and *Overview*. This can be seen from how problems that have been given by researchers. He is able to determine the information needed to make decisions in solving a problem, provides a reason based on relevant facts/evidence at each step in making decisions and conclusions, can illustrate the problem situations contained in the problem and is able

to review the overall decision taken . (2) Students who have medium academic ability (SS) have only reached 2 indicators; *Focus and Reason*. (3) Students with low academic ability (SR) could achieve 1 indicator that is *Reason*.

The results of this study provide an overview to teachers and re-searchers about the students' ability to think critically of senior secondary schools' in solving the Trigonometry problem. It is expected that by knowing the critical thinking skills based on each indicator, teachers and researchers can design and develop learning that can facilitate students to practice critical thinking so that students will be accustomed to thinking critically.

For further research, it is suggested that the researcher can target students' critical thinking ability, to let the teachers know the students' level of being critical; therefore, when designing the learning they can improve students' critical thinking ability. The intended levels of critical thinking skills are low, medium, and high level.

### Acknowledgement

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