

PAPER • OPEN ACCESS

GMM estimation of simultaneous spatial panel data dynamic models with high order models approach

To cite this article: D E Kusriani *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **243** 012048

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

GMM estimation of simultaneous spatial panel data dynamic models with high order models approach

D E Kusrini^{1*}, Setiawan², B N Ruchjana¹, H Kuswanto¹

¹Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

²Unniversitas Padjadjaran, Bandung, Indonesia

*Email: dwi_endah@statistika.its.ac.id

Abstract. This paper shows the process of changing Simultaneous Spatial panel data dynamic Equations (SSPDD) into a high order model, which is then, parameterized in the high order model using the General method of Moment (GMM). The estimation steps start by forming the reduced form equation, and then form the instrumental Variable (IV) vector which is used to form the moment matrix $g_{nt}(\theta)$. This moment matrix is used to calculate the estimated value of the model parameters. In this paper, the results of parameter estimation have not been tested with data or simulations so that it is recommended to do further testing using simulation data.

1. Introduction

Econometric spatial modeling at this time has arrived at the simultaneous modeling stage, with panel data as the data used, however panel data dynamic spatial modeling is nevertheless rarely carried out.

Recent research on the model was carried out through [6] who used the Quasi Maximum Likelihood (QMLE) method to estimate the parameters with n and T scenarios that tend to be infinite. In the simultaneous equation system, one of the things that is usually done is to form a "reduced form" equation to get the parameters of the structural equation. The results of this method can produce interesting results if there are only 2 equations, but if the equation is more than two ($m > 2$) it will be difficult to do the "reduced form" formation. Therefore [6] form a multivariate model for the simultaneous model to be "quasi reduced form". The identification of the parameters of the quasi reduced form for the structural model parameters equals to the traditional linear simultaneous equation model. GMM estimation used by [5] for the spatial autoregressive model in the interrelated networks system. The model used by [5] is an equation with several models whose parameter estimation is performed simultaneously. In the equation, each spatial lag has a different coefficient that is difficult to estimate by using Maximum Likelihood method (MLE) because when the distribution form of disturbance is normal, the Jacobian transformation will be complicated. In addition, the form of disturbances is assumed to correlate between equations so that the estimates made by [3] cannot be applied directly to this model. The equation model used by [5] is a static model that is a model with no time lag on its endogenous variables.

Based on the above description, this paper describes the steps taken by [5] on the spatial simultaneous dynamic equations model because according to [5] when the number of rows and columns of spatial weight matrix (W) are equal (symmetrical), the system equation of interrelated networks can be simultaneous equations model. The spatial simultaneous dynamic equation model will be brought into the High Order model form nearing the form of the Spatial Dynamic Data Panel Equation model that is developed by [4]. Furthermore, the GMM estimates developed follow the way of [4].

2. Simultaneous Spatial Panels Data Dynamic (SSPDD) Models

Spatial data modeling in its development no longer involves a single equation but has developed towards simultaneous modeling, one of them was done by [2] and [1]. [2] used the simultaneous spatial data panel model with SEM model approach for regional growth model, and the estimation method used is Generalized Spatial Three-Stage Least Squares (GS3SLS). While [1] use simultaneous spatial autoregressive model with random effect and parameter estimation method used is Error Component 3 Stage Least Squares (EC3SLS) method. Simultaneous spatial panel data dynamic models have been developed by [6], namely Multivariate and Simultaneous Equation Dynamic Panel Spatial Autoregressive Models, with QMLE parameter estimation approach.

The parameter estimation method is also developed from MLE, QMLE, to IV (Instrumental Variable) GMM method. Each estimation method has its advantages and disadvantages. The OLS method obviously cannot be used in the dynamic model because the variable y_{t-1} is treated as a fixed variable while y_t is a random variable. Using the MLE method to estimate parameters of the spatial panel data dynamic model has difficulties (not feasible) in the calculation when N and T are large [4]. This difficulty is caused by a spatial weight matrix that cannot be normalized when the time effect (T) is removed which will make it difficult to estimate the parameters, when there are many matrixes of spatial weights in the model.

QMLE and MLE methods have the same weaknesses when large T but smaller than N with fixed effects are simultaneously estimated with other parameters in the model, because the asymptotic distribution and consistency of the estimator will be biased and will not centre on zero. The convergence rate of QMLE will probably not be worth as much as \sqrt{nT} [6]. The GMM equation is expected to solve all the above difficulties as a consistent GMM estimate of \sqrt{nT} , asymptotically normal and relatively efficient. The Simultaneous Spatial Panel Data Dynamic Model (SSPDD) used is the SAR model adapted from [4] and [1] models with the number of equations $m = 2$ as follows:

$$y_{n1,t} = \delta_{11}W_n y_{n1,t} + \eta_{11}W_n y_{n1,t-1} + \tau_{11}y_{n1,t-1} + k_{11}y_{n2,t} + \beta_{11}X_{n1,t} + c_{n1} + v_{11,t} \quad (1)$$

$$y_{n2,t} = \delta_{12}W_n y_{n2,t} + \eta_{12}W_n y_{n2,t-1} + \tau_{12}y_{n2,t-1} + k_{12}y_{n1,t} + \beta_{11}X_{n1,t} + c_{n2} + v_{12,t} \quad (2)$$

$|\tau| + |\delta| + |\eta| < 1$

with $y_{nm,t}$ is an $N \times 1$ vector of the response variable, where $y_{n1,t} = (y_{11t}, \dots, y_{n1t})$, $y_{n2,t} = (y_{12t}, \dots, y_{n2t})$, and $y_{nm,t-1}$ is an $N \times 1$ vector of variable predetermined lag 1, and $y_{n1,t-1} = (y_{11t-1}, \dots, y_{n1t-1})$, $y_{n2,t-1} = (y_{12t-1}, \dots, y_{n2t-1})$. $v_{nt} = (v_{1t}, \dots, v_{nt})$ is an $N \times 1$ column vector of error with $v_{nt} \sim N(0, I\sigma_v^2)$. W_n = an $N \times N$ spatial weights matrix X_{nt} = an $N \times K$ explanatory variable matrix, $c_{nm} = N \times 1$ column vector of individual effects, $m = 1, 2$.

3. Methods

The methodology of the research in this paper follows the following steps:

- 1) Change the dynamic panel data spatial simultaneous equation (SSPDD) into the form of High Order Model in the following manner:
 - a. Form a matrix Z_{n1t} and Z_{n2t}
 - b. Form the equation (1) and (2) into a matrix $Y_{N,t}$
 - c. Forming a matrix in the previous step becomes a matrix of the High Order Spatial Autoregressive model
- 2) Transform the High Order Spatial Autoregressive model to eliminate the fixed effect with an orthonormal matrix $F_{T,T-1}$, $F_{n,n-1}$ to eliminate individual effects variables τ_{n0} and time effect α_{t0} in ways:
 - a. Form matrix $J_T = (I_T - \frac{1}{T} \mathbf{1}_T \mathbf{1}'_T)$
 - b. Look for the eigenvector matrix $F_{T,T-1}$ form J_T
 - c. Form an orthonormal matrix $[F_{T,T-1}, \frac{1}{\sqrt{T}} I_T]$
 - d. Forming an endogenous variable transformation matrix

$$Y_{nT}^* = [y_{n1}^*, y_{n2}^*, \dots, y_{nT-1}^*] = [y_{n1}, y_{n2}, \dots, y_{nT}] F_{TT-1}$$

and endogen lag matrix

$$Y_{nT-1}^{(*,-1)} = [y_{n0}^{(*,-1)}, y_{n1}^{(*,-1)}, \dots, y_{nT-2}^{(*,-1)}] = [y_{n0}, y_{n1}, \dots, y_{nT-1}] F_{T,T-1}$$

- e. Form a transformation matrix for predetermined variables $Z_{Nj}^*, V_{nT}^*, c_n^*$ by multiplying each with $F_{T,T-1}$
- 3) Form the reduced form equation from the variables that have been transformed in the previous step by:
 - a. Calculate matrix $S_{nT}(\delta)$ with $\delta = (\delta_{11}, \delta_{12})'$; $\eta = (\eta_{11}, \eta_{12})'$
 - b. Calculate matrix A_n
 - c. Making reduced form matrix from the equation High Order Spatial Autoregressive.
- 4) Estimate reduced form model parameters that have been obtained in the previous step with the following steps:
 - a. Form the following Instrumental Variable Q_{nT} .
 - b. Calculate matrix $J_{nT-1} = I_{T-1} \otimes J_n$
 - c. Form a linear moment component

$$Q_{nT-1} J_{nT-1} V_{nT-1}^*(\theta)$$
 - d. Form a quadratic moment component

$$V_{nT-1}^*(\theta) J_{nT-1} P_{nL,T-1} J_{nT-1} V_{nT-1}^*(\theta)$$
 for $l = 1, 2, \dots, m$.
 - e. Based on the components of the linear moment and the quadratic moment then form the moment condition matrix $g_{nT}(\theta)$
 - f. Calculate matrix $\Delta_{nm,T}$
 - g. Calculate matrix ω_{nmT}
 - h. Determine the moment matrix Σ_{nT} based on the matrix $\Delta_{nm,T}$ and ω_{nmT} .
 - i. Form matrix $g_{nT}(\theta) \Sigma_{nT}^{-1} g_{nT}(\theta)$
 - j. Get an $\hat{\theta} = (\delta, \eta, \tau, \beta', k)'$ value estimator by decreasing $\min_{\theta \in \Theta} g_{nT}(\theta) \Sigma_{nT}^{-1} g_{nT}(\theta)$.

4. Result and Discussion

According to [5], The Simultaneous Equation Model is one form of an Interrelated Network model if the spatial weight matrix W of the first equation and the second equation have the same dimensions. So, in this paper The SSPDD equation is then brought into matrix form as the step by [5] as follows: where and. By changing notation, the equation 3 above will be: Furthermore, equation (4) is formed into the High Order Spatial Autoregressive model as established [4] as follows:

$$\begin{pmatrix} y_{n1t} \\ y_{n2t} \end{pmatrix} = \delta_{11} \begin{pmatrix} W_n & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{n1t} \\ y_{n2t} \end{pmatrix} + \eta_{11} \begin{pmatrix} W_n & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{n1t-1} \\ y_{n2t-1} \end{pmatrix} + \delta_{12} \begin{pmatrix} 0 & 0 \\ 0 & W_n \end{pmatrix} \begin{pmatrix} y_{n1t} \\ y_{n2t} \end{pmatrix} \\ + \eta_{12} \begin{pmatrix} 0 & 0 \\ 0 & W_n \end{pmatrix} \begin{pmatrix} y_{n1t-1} \\ y_{n2t-1} \end{pmatrix} + \begin{pmatrix} Z_{n1t} & 0 \\ 0 & Z_{n1t} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \eta_{12} \begin{pmatrix} 0 & 0 \\ 0 & W_n \end{pmatrix} \begin{pmatrix} y_{n1t-1} \\ y_{n2t-1} \end{pmatrix} \\ + \begin{pmatrix} Z_{n1t} & 0 \\ 0 & Z_{n1t} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \tau_{11} & 0 \\ 0 & \tau_{12} \end{pmatrix} \begin{pmatrix} y_{n1t-1} \\ y_{n2t-1} \end{pmatrix} + \begin{pmatrix} c_{n1} \\ c_{n2} \end{pmatrix} + \begin{pmatrix} v_{11t} \\ v_{12t} \end{pmatrix} \quad (3)$$

where $Z_{n1t} = (X_{n1t}, y_{n1t})$ and $Z_{n2t} = (X_{n2t}, y_{n2t})$. By changing notation, the equation 3 above will be:

$$Y_{Nt} = \delta_{11} W_{N1} Y_{Nt} + \eta_{11} W_{N1} Y_{Nt-1} + \delta_{12} W_{N2} Y_{Nt} + \eta_{12} W_{N2} Y_{Nt-1} + Z_{Nt} \beta + \tau_0 Y_{Nt1} + C_N + V_{Nt} \quad (4)$$

With $Y_{Nt} = \begin{pmatrix} y_{n1t} \\ y_{n2t} \end{pmatrix}$, $\delta_j W_{Nj} = \begin{pmatrix} \delta_{11} & 0 \\ 0 & \delta_{12} W_{N2} \end{pmatrix}$,

$$\eta_j W_{Nj} = \begin{pmatrix} \delta_{11} & 0 \\ 0 & \delta_{12} W_{N2} \end{pmatrix} \\ Y_{Nt-1} = \begin{pmatrix} y_{n1t} \\ y_{n2t} \end{pmatrix}, Z_{Nt} = \begin{pmatrix} Z_{n1t} & 0 \\ 0 & Z_{n2t} \end{pmatrix}, V_{Nt} = \begin{pmatrix} v_{n1t} \\ v_{n2t} \end{pmatrix}, \beta_0 = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}, \tau_0 = \begin{pmatrix} \tau_{11} & 0 \\ 0 & \tau_{12} \end{pmatrix}$$

Furthermore, equation (4) is formed into the High Order Spatial Autoregressive model as established [4] as follows:

$$Y_{Nt} = \sum_{j=1}^p \delta_{j0} W_{Nj} Y_{Nt} + \sum_{j=1}^p \eta_{j0} W_{Nj} Y_{Nt-1} + Z_{Nt} \beta_0 + \tau_0 Y_{Nt-1} + c_{N0} + v_{Nt}, t = 1, 2, \dots, T \quad (5)$$

$Y_{Nt} = (y_{11t}, y_{21t}, \dots, y_{n1t}, y_{12t}, y_{22t}, \dots, y_{n2t})'$ or can be formed into $Y_{Nt} = (Y_{11t}, Y_{21t}, \dots, Y_{Nt})'$ is an $N \times 1$ vector endogenous variable and Z_{Nt} is a $N \times K$ exogenous variable matrix.

$V_{Nt} = (v_{11t}, v_{21t}, \dots, v_{n1t}, v_{12t}, v_{22t}, \dots, v_{n2t})'$ is an $N \times 1$ disturbance term vector where $v_{it} \sim N(0, \sigma_0^2)$. While $W_{nj} Y_{nt}$ and $W_{nj} Y_{nt-1}$ are dependent spatial lag variable, where W_{nj} is an $N \times N$ spatial weights matrix non stochastic and form based on yit between spatial units with $j=1, 2, \dots, p$. The matrix W_{nj} can be a rownormalized matrix or not. $\delta_{j0} = (\delta_{10}, \delta_{20}, \dots, \delta_{p0})'$ and $\eta_{j0} = (\eta_{10}, \eta_{20}, \dots, \eta_{p0})'$ is a spatial autoregressive parameter and $c_{N0} = (c_{10}, c_{20}, \dots, c_{n0})'$ is a column vector of individual effects.

Model (5) will be a high order SAR model if $p \geq 2$ and then to avoid unnecessary variables, the individual effect variables c_{N0} must be eliminated by transforming them with the matrix F_{TT-1} obtained from the matrix $[F_{TT-1}, \frac{1}{\sqrt{T}} l_T]$ which is the orthonormal matrix of the eigenvector $J_T = (I_T - \frac{1}{T} l_T l_T')$. F_{TT-1} is a $T \times (T-1)$ eigenvectors matrix that is connected with eigenvalues of one and l_T is an vector one with T dimension.

The dependent variable matrix $[Y_{n1}, Y_{n2}, \dots, Y_{nt}]$ can be transformed to be an $N \times (T-1)$ matrix so,

$$[Y_{n1}^*, Y_{n2}^*, \dots, Y_{nt-1}^*] = [Y_{n1}, Y_{n2}, \dots, Y_{nt}] F_{TT-1}$$

and

$$[Y_{n1}^{(*,-1)}, Y_{n2}^{(*,-1)}, \dots, Y_{nt-2}^{(*,-1)}] = [Y_{n0}, Y_{n1}, \dots, Y_{nt-1}] F_{TT-1}$$

$$Y_{nt-1}^* \neq Y_{nt-1}^{(*,-1)}$$

Multiplication by F_{TT-1} above applies also to variables V_{nt}^* and Z_{nt}^* . Since $l_T' F_{TT-1} = 0$ then $[c_{n0}, \dots, c_{n0}] F_{TT-1} = 0$ so that individual effects can be eliminated by orthonormal matrix transformations. After transformation process then the equation or model (5) will be,

$$Y_{Nt}^* = \sum_{j=1}^p \delta_{j0} W_{Nj} Y_{Nt}^* + \sum_{j=1}^p \eta_{j0} W_{Nj} Y_{Nt-1}^* + Z_{Nt}^* \beta_0 + \tau_0 Y_{Nt-1}^* + V_{Nt}^*; t = 1, 2, \dots, T-1 \quad (6)$$

With

$$V_{nt}^* = \left(\frac{T-t}{T-t+1} \right)^{\frac{1}{2}} \left[V_{nt} - \frac{1}{T-t} \sum_{h=t+1}^T V_{nh} \right]$$

And

$$Y_{nt-1}^{(*,-1)} = \left(\frac{T-t}{T-t+1} \right)^{1/2} \left[Y_{nt-1} - \frac{1}{T-1} \sum_{h=t}^{T-1} Y_{nh} \right]$$

Elimination of time effect is not done on this model because the model used is a fixed effect model. Estimation parameter for model (6) cannot be done by MLE method according to [4] with the following consideration:

a) If W_{nj} can not be roughly normalized the SAR structure can not be defined.

b) The existence of a variable with time lag as an explanatory variable then the variable will be correlated with the disturbance term after being transformed by F_{TT-1}

For this reason, the GMM method is used to estimate model (6) because the method does not require a SAR form for $J_n Y_{nt}^*$ and is free of asymptotic bias. To estimate the model parameter (6), it is necessary to reduce the form from the following equation (5).

$$Y_{nt}^* = A_n Y_{nt}^{(*,-1)} + S_n^{-1} (Z_{nt}^* \beta_0 + V_{nt}^*) \quad (7)$$

With

$$S_n(\delta) = I_n - \sum_{j=1}^p \delta_j W_{nj}, \quad S_n \equiv S_n(\delta_0)$$

And

$$A_n = S_n^{-1} \left(\tau_0 I_n + \sum_{j=1}^p \eta_j W_{nj} \right)$$

For each spatial lag $W_{nj}y_{nt}^*$ for $j = 1, 2, \dots, p$ by defining $G_{nj} \equiv W_{nj}S_n^{-1}$ then it is obtained,

$$W_{nj}y_{nt}^* = G_{nj}(R_{nt}^* \lambda_0) + G_{nj}V_{nt}^* \quad (8)$$

With $\lambda_0 = (\tau_0, \eta'_0, \beta'_0)$ and $R_{nt}^* = [Y_{nt-1}^{(*,-1)}, W_n Y_{nt-1}^{(*,-1)}, Z_{nt}^*]$ is a predetermined variable in equation (8) with:

$$W_n Y_{nt-1}^{(*,-1)} = (W_{n1} Y_{nt-1}^{(*,-1)}, \dots, W_{np} Y_{nt-1}^{(*,-1)})$$

For the linear moment, then the preparation of data and form the moment condition. A matrix of Instrumental Variables (IV) is formed by $J_n Q_{nt}$ which $J_n = I_n - \frac{1}{n} l_n l_n'$ cause $J_n l_n = 0$ and Q_{nt} having a fixed dimension of column q greater than or equal to $k_x + 2p + 1$ the example Q_{nt} is:

$$[Y_{nt-1}, W_n Y_{nt-1}, W_n^2 Y_{nt-1}, Z_{nt}^*, W_n Z_{nt}^*, W_n^2 Z_{nt}^*] \quad (9)$$

To estimate the equation (6) it should be noted that between $Y_{nt-1}^{(*,-1)}$ and V_{nt}^* are correlated, therefore it is necessary for IV $Y_{nt-1}^{(*,-1)}, W_{nk} Y_{nt-1}^{(*,-1)}$ and $W_{nj} Y_{nt}^*$ on each t. So that required IV for eksplanatory variable as follows:

$$J_n [Y_{nt}^*, Y_{nt-1}^{(*,-1)}, W_n Y_{nt-1}^{(*,-1)}] \quad (10)$$

Let $V_{nT-1}^*(\theta) = (V_{n1}^{*'}(\theta), \dots, V_{nT-1}^{*'}(\theta))'$ which $V_{nt}^*(\theta) = S_{nt}(\lambda) Y_{nt}^* - Z_{nt}^* \delta$ with $\theta = (\delta_0, \lambda_0)'$ and $\lambda_0 = (\tau_0, \eta'_0, \beta'_0)$. Estimate IV that connected with moment linear is

$$Q'_{nT-1} J_{nT-1} V_{nT-1}^*(\theta)$$

The vector $P_{nt} V_{nt}^*$ can not be correlated with $J_n V_{nt}^*$ in equation (6), for a NXN non stochastic matrix P_{nl} according to the content of $tr(P_{nl} J_n) = 0$, here the value,

$$P_{n1} = W_n - tr(W_n J_n) / (W_n J_n) / (n - 1)$$

and

$$P_{n2} = W_n^2 - tr(W_n^2 J_n) / (W_n J_n) / (n - 1)$$

While the $P_{nt} V_{nt}^*$ may be correlated with $G_{nj} V_{nt}^*$ in equation (7).

Defined $P_{nl, T-1} = I_{T-1} \otimes P_{nl}$, then the quadratic moment is $V_{nT-1}^*(\theta) J_{nT-1} P_{nl, T-1} J_{nT-1} V_{nT-1}^*(\theta)$ for $l = 1, 2, \dots, m$ and the moment of its condition with the approximate number of finite moments is,

$$g_{nt}(\theta) = \begin{pmatrix} V_{nT-1}^*(\theta) J_{nT-1} P_{n1, T-1} J_{nT-1} V_{nT-1}^*(\theta) \\ \vdots \\ V_{nT-1}^*(\theta) J_{nT-1} P_{nm, T-1} J_{nT-1} V_{nT-1}^*(\theta) \\ Q'_{nT-1} J_{nT-1} V_{nT-1}^*(\theta) \end{pmatrix} \quad (11)$$

For the GMM estimate a covariance matrix is required from the function of moment $E(g'_{nT}(\theta_0) g_{nT}(\theta_0))$ which can be approximated by,

$$\Sigma_{nT} = \sigma_0^4 \begin{pmatrix} \frac{1}{n(T-1)} \Delta_{nmT} & 0_{mxq} \\ 0_{qxm} & \frac{1}{\sigma_0^2} \frac{1}{n(T-1)} Q_{nT-1} J_{nT-1} Q_{nT-1} \\ + \frac{1}{n(T-1)} \begin{pmatrix} (\mu_4 - 3\sigma_0^4) \omega_{nmT} \omega_{nmT} & 0_{mxq} \\ 0_{qxm} & 0_{qxq} \end{pmatrix} \end{pmatrix} \quad (12)$$

With

$$\begin{aligned} \omega_{nmT} &= [vec_D(J_{nT-1} P_{n1, T-1} J_{nT-1}), \dots, vec_D(J_{nT-1} P_{nm, T-1} J_{nT-1})], \\ \Delta_{nmT} &= [vec_D(J_{nT-1} P_{n1, T-1} J_{nT-1}), \dots, vec_D(J_{nT-1} P_{nm, T-1} J_{nT-1})] \\ &\quad \times [vec_D(J_{nT-1} P_{n1, T-1}^s J_{nT-1}), \dots, vec_D(J_{nT-1} P_{nm, T-1}^s J_{nT-1})] \end{aligned}$$

The $vec_D(\cdot)$ operator is a column vector formed from the diagonal element of the given square matrix, and a $A_n^s = A_n + A_n'$ matrix for all square matrices.

Optimal estimation of GMM is derived from,

$$\hat{\theta}_{onT} = argmin_{\theta \in \Theta} g_{nT}(\theta) \sum_{nT}^{-1} g_{nT}(\theta) \quad (13)$$

Has asymptotic distribution,

$$\sqrt{n}(\hat{\theta}_{onT} - \theta_0)^d \rightarrow N\left(0, \text{plim}_{n \rightarrow \infty} \frac{1}{T-1} (D_{nT} \Sigma_{nT}^{-1} D_{nT})^{-1}\right) \quad (14)$$

In [4] it is also mentioned that the resulting GMM estimator is consistent by \sqrt{nT} , asymptotically normal, and efficient. Instrumental Variables (IV) used for obtaining the best linear and quadratic moment conditions, in which the number of IV will increase as the time period increases. Some studies also prove that the resulting GMM estimates are consistent, when T is relatively small compared to N.

5. Conclusion

This paper introduces the Simultaneous Spatial Panel Data Dynamic (SSPDD), where the model can be used to analyse the empirical problems widely involving the interaction between the individual and the time in a different area. The SSPDD model is subsequently brought into the High Order model where it cannot be estimated using the MLE method due to the complexity of the computation process, so GMM estimation was developed to obtain the model parameter estimation.

The model and parameter estimation method produced in this paper still require further testing, especially the asymptotic distribution of the estimated parameters that have been generated. So, data simulation has not been done in this paper because it is still in the process of working for the next research.

Acknowledgement

Our thanks go to the Ministry of Research and Technology of Higher Education of the Republic of Indonesia who funding this paper as part of a dissertation from the lead author.

References

- [1] Baltagi, Badi H and Ying D 2012 EC3SLS Estimator For A Simultaneous System of Spatial Autoregressive Equations With Random Effects *Forthcoming in Econometric Reviews*
- [2] Gebremariam G H, Gebremedhin T G, Schaeffer P V, Phipps T T, Jackson R W 2011 *Regional Science* **Vol 51** No. 1 102-120
- [3] Lee L F and Liu X 2010 *Econometric Theory* **Vol 26** 187-230
- [4] Lee L F dan Yu J 2014 *Econometrics* **180** 174-197
- [5] Wei W, Lung-Fei L and Yan B 2018 *Regional Science and Urban Economics* **Vol 69** 10-36
- [6] Yang K and Lee L F 2015 Multivariate and Simultaneous Equation Dynamic Panel Spatial Autoregressive Models: Stability and Spatial Cointegration *Job Market Paper* (The Ohio State University, Ohio: Department of Economics)
- [7] Elhorst J P 2014 *Spatial Econometrics: From Crosssectional to Panels Data* (Springer Heidelberg: New York)
- [8] Elhorst J P 2003 *International Regional Science Review* **Vol. 26** 244-268