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# Susceptible infected AIDS treatment (SIAT) model

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**Abstract**. HIV is an infectious disease caused by Human Immunodeficiency Virus. This virus attacks the immune system. A person who has been infected will gradually become AIDS. Mathematical modeling to elaborate the spread of HIV/AIDS disease has been described through susceptible infected AIDS (SIA) model which classifies the population into three compartments, ie compartment of individuals susceptible to disease (S), compartment of HIV infected individuals (I), and compartment of individuals AIDS (A). Until now HIV/AIDS disease cannot be cured, but patients can undergo antiretroviral (ARV) therapy to reduce the growth of virus in the body. People who undergo antiretroviral therapy are classified into treatment individuals (T). The spread of HIV/AIDS with treatment can be represented in SIAT model. The purposes of this research are to formulate the SIAT model, apply the model in Indonesia, determine the pattern of disease spread, and know the role of 90-90-90 programme in controlling the spread of HIV/AIDS disease. The SIAT model is a first order nonlinear differential equation system. Based on the implementation, it's estimated that by 2030 there will be 1706630 people with HIV and 1432720 people who undergo antiretroviral therapy. Based on the simulation, the 90-90-90 programme have an important role in controlling the spread of HIV/AIDS disease.

## 1. Introduction

Human Immunodeficiency Virus or HIV is a virus which destroy the immune system. This virus attacks T cell or CD4 cell in the immune system. If the immune system is weak then the body capability to against the infection will be decreased. That effected various disease easy to enter into the body.

According to Data and Information Center, Ministry of Health of The Republic of Indonesia [1], Acquired Immunodeficiency Syn-drome or AIDS is a collection of diseases due to the decline of the immune system caused by HIV. A person who has been infected HIV is declared as AIDS if the number of T cell < 200/mm2. Until now HIV/AIDS disease can't be cured. Every year the number of people living with HIV/AIDS always increase. The spread of HIV/AIDS disease adverse impact to health, social, public eco-nomic. Disease spread behavior can be described through mathe-matical model.

In 1986, Anderson [2] introduce SIA model which the population is classified into three compartment, they are a compartment of individuals susceptible to disease (S), a compartment of HIV in-fected individuals (I), and a compartment of individuals AIDS (A). This model assume transmission through sexual intercourse with individual HIV. Haryanto et al. [3] develop the model by adding the transmission from individual AIDS.

Directorate General of Disease Control and Environmental Health, Ministry of Health of The Republic of Indonesia [4] noted that in 2016 there were 41250 people living with HIV and 806 died of AIDS.

The government issued a 90-90-90 program to control the spread of HIV/AIDS disease. Antiretroviral therapy (ARV) is one of the treatment of HIV/AIDS disease. Although HIV/AIDS disease can't be cured, with this therapy HIV/AIDS patient has a better chance to survive. Different from SIA model by Haryanto et al. [3] which did not provide any treatment efforts in the model of the spread of HIV/AIDS disease, this research developed SIA model with attention to the influence of ARV therapy. People who undergo ARV therapy is classified into compartment of individu-als who treatment (T). Therefore we developed the spread of HIV/AIDS with antiretroviral therapy by the SIAT model and we applied the model in Indonesia. By this application then the role of 90-90-90 program in controlling the spread of HIV/AIDS disease is known.

## 2. The SIA Model

In the model of HIV/AIDS disease spread, Anderson [2] divided the population into three compartments, ie a compartment of individuals susceptible to disease (S), a compartment of people living with HIV (I), and a compartment of people living with AIDS (A). The SIA model by Roy only concern the transmission through sexual intercourse with HIV patient. Basically, the transmission of HIV is not only trough sexual contact, but also through blood, syringes, and breast milk. Individual infected plays an important role in the spread of the disease, so contact with patients of HIV is one that needs attention.

Therefore, in 2012 El-Hia et al. [5] developed the SIA model by not only paying attention to sexual transmission, but also considering other transmission factors. Furthermore, by 2015 Haryanto et al. [3] developed the SIA model by El-Hia et al. [5] by adding the transmission of individual AIDS (A) to susceptible individuals (S). The interaction between individuals A and S will decrease the number of individuals in compartment S. If ω is the transmission rate of the individual A to the individual S, then the number of individuals S is reduced by ωSA/N. Individual I can also transmit HIV virus to individual S. If β is the transmission rate of the individual I to the individual S, then the number of individuals S is reduced by βSI/N. Someone who has been infected will go into the compartment of I, so the number of I increases as much as ωSA/N and βSI/N individuals.

The complete SIA model by Haryanto et al. [3] is

$$\frac{dS}{dt} = \lambda - \beta\frac{SI}{N} - \omega\frac{SA}{N} - \mu S$$
$$\frac{dI}{dt} = -\beta\frac{SI}{N} + \omega\frac{SA}{N} - \gamma I - \mu I \qquad (1)$$
$$\frac{dA}{dt} = \gamma I - \mu_1 A - \mu A$$

where S(0), A(0) $\geq$ 0, I(0) > 0 and β, ω, μ, $\mu_1$, γ > 0. The five parameters consecutively state the transmission rate from individual I to individual S, the transmission rate from individual A to individual S, natural mortality rate, death rate due to AIDS, and the rate of outbreak from infected individual becomes a person with AIDS.

## 3. The SIAT Model

HIV/AIDS became one of the major illnesses in the world's health problems. Patients with HIV usually can survive until 12 years after infection. This virus makes their body weaker because of the immune system's declining function from day to day. This makes the patient can not run their daily activities optimally. Antitetrovi-ral therapy (ARV) means treating HIV infection with several drugs. HIV virus belongs to the retrovirus, so the drug is called an antiretroviral (ARV) drug. ARV do not kill the virus, but this ther-apy can decelerate the growth of the virus (Spiritia [6]). People who undergo antiretroviral therapy have a better chance of surviv-ing longer than those who do not undergo therapy. People on ARV therapy are classified as individuals undergoing treatment denoted by T. If α states the rate of therapy, then the number of newly treated individuals is α (I+S).

An individual susceptible (S) can be infected because contact with an individual HIV (I) or individual who already infected AIDS (A). If contact rate are denoted by ω, then the compartment of S is reduced by ωSI/N + ωSA/N. In addition to contact, the disease of

HIV can also be transmitted from the mother with HIV/AIDS to the baby due to transmission during pregnancy, or labor, so there is a possibility that the baby is also infected with the virus. It is assumed that the population is not constant, so the birth rate is not the same as the death rate. If the birth rate is θ, then the num-ber of births is θN. In this study, birth was classified into two, ie birth without HIV and birth with HIV. If p is the proportion of individuals who born with HIV, the compartment of I increases by pθN, and the compartment of S increases by (1-p)θN.

Virus that have entered the body will attack CD4 cells and make the human immune system become weak. Then within a certain period of time the infected individual HIV will catch AIDS. According to Haryanto et al. [3], the rate of outbreak from infected individual to the compartment of AIDS which will affect the decrease in the number of individuals in the compartment of I. This will indirectly increase the number of individuals in the compart-ment of S by as much as γI.

If the natural death rate is denoted by μ, then the individual group S, I, A, T will decrease by μS, μI, μA, μT. Until now there is no cure for AIDS disease, so gradually the patient of AIDS will die caused by the disease. The death rate due to AIDS ($\mu_1$) can affect the decline in the number of individuals in the AIDS compartment. The complete SIAT model can be written as

$$\begin{aligned} \frac{dS}{dt} &= (1-p)\theta N - \omega\frac{SI}{N} - \omega\frac{SA}{N} - \mu S \\ \frac{dI}{dt} &= p\theta N + \omega\frac{SI}{N} - \omega\frac{SA}{N} - (\gamma+\mu+\alpha)I \\ \frac{dA}{dt} &= \gamma I - (\mu+\mu_1+\alpha)A \\ \frac{dT}{dt} &= \alpha(I+A) - \mu T, \end{aligned} \qquad (2)$$

with S (0), A (0), T (0) ≥ 0, I (0) > 0 and θ, ω, μ, $\mu_1$, γ, α > 0. The six parameters consecutively are birthrate, contact rate, natural mortality rate, death rate due to AIDS, rate of outbreak, and rate of therapy. The model (3.1) is a one-order nonlinear differential equation system with four equations.

## 4. Application

The SIAT (3.1) model is applied in Indonesia. The data used in determining the parameters are annual data from Directorate General of Disease Control and Environmental Health, Ministry of Health of The Republic of Indonesia [7] and World Bank [8] in 2007-2016. The number of people living with HIV (I) and AIDS (A) on the data was assumed to have not taken antiretroviral ther-apy. The result of parameters estimation are the birth rate value (θ) is 0.02037, the proportion of births with HIV (p) is 0.0001, contact rate (ω) is 0.340171, natural mortality rate (μ) is 0.007116, death rate due to AIDS ($\mu_1$) is 0.04219 , the rate of outbreak (γ) is 0.177612 and the therapeutic rate (α) is 0.117671. Based on the values of these parameters, the model of the spread of disease HIV/ AIDS in Indonesia is

$$\begin{aligned} \frac{dS}{dt} &= 0.020368N - 0.340171\frac{SI}{N} - 0.340171\frac{SA}{N} - 0.007116S \\ \frac{dI}{dt} &= 0.000002N + 0.340171\frac{SI}{N} + 0.340171\frac{SA}{N} - 0.302399I \\ \frac{dA}{dt} &= 0.177612I - 0.166977A \\ \frac{dT}{dt} &= 0.117671(I+A) - 0.007116T. \end{aligned} \qquad (3)$$

The system (4.1) is a one-order nonlinear differential equation system. The initial value used refers to the number of individuals S, I, A, and T in 2007 (t = 0). Based on the data obtained initial value ie

(S(0), I(0), A(0), T(0)) = (232959637, 11440, 11411, 6653). (4)

The solution of system (4.1) with initial value (4.2), for the first 23 years (2007-2030), is determined by the fourth-order Runge-Kutta algorithm. The solution shows the pattern of spread of HIV/AIDS disease in Indonesia (seen in Figure 1).

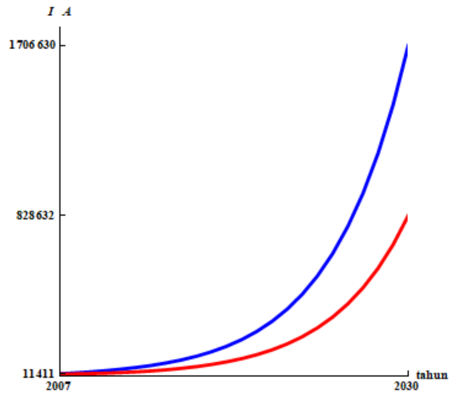


**Figure 1.** The number of individual I (blue), A (red) in 2007-2030

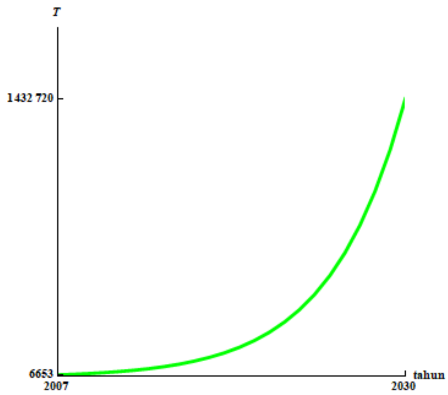


**Figure 2.** The number of individual T (green) in 2007-2030

Figure 1 show that as time passes, the number of individuals I, A, and T continues to increase. The number of individuals I who initially (2007) only 11440 patients, estimated in 2030 there are 1706630 people living with HIV. This shows that the number of individuals I has increased to 150 times the initial condition. Furthermore, the individual A in 2007 had only 11411 patients then is expected to increase to 73-fold by 2030, to 828632 people living with AIDS.

In contrast to I and A, the individual in T compartment has the most significant increase, see Fig. 2. This is shown from 6653 patients who undergo therapy in 2007, see equation (5.2), and then in 2030 estimated there are 1432720 individuals, resulting in an individual increase up to 216 times the initial condition.

## 5. Simulation

In 2013, the United Nations (UN) created a program to control the spread of HIV/AIDS disease known as the 90-90-90 program. The purpose of this program is that by 2020, 90% of people can be diagnosed, 90% of those diagnosed can be on antiretroviral therapy, and 90% of those undergoing therapy can suppress the viral load. This is the UN's attempt to achieve the target of three zero by 2030, one of them is no more contagion of HIV. Therefore, the simulation of parameters in the model (4.1) is used to determine the role of 90-90-90 program in controlling the spread of HIV/AIDS disease.

Before the simulation, it is predicted that in 2030 there 700786 new cases of HIV. Through 90-90-90, 90% of people will be diagnosed or know the status of their HIV, thereby making the person more careful in making contact to prevent the transmission of the virus. Then, 90% who have been diagnosed will undergo antiretroviral therapy, so the requirement of this therapy will increase from year to year. The most influential possible parameter values are α and ω. The result of simulation for αvalue (the therapeutic rate) and ω (contact rate) is shown in Table 1.

**Table 1.** The simulation of parameter value of α and ω toward the number of new cases of HIV in 2030

| $\omega$ | New Cases | $\omega$ | New Cases |
|---|---|---|---|
| 0.3 | 260190 | 0.15 | 6349 |
| 0.25 | 74327 | 0.1 | 2210 |
| 0.2 | 21218 | 0.05 | 1021 |

| $\alpha$ | New Cases |
|---|---|
| 0.2 | 124673 |
| 0.3 | 17506 |
| 0.4 | 3982 |
| 0.5 | 1898 |
| 0.6 | 1398 |
| 0.7 | 1194 |
| 0.8 | 1079 |
| 0.9 | 1004 |
| 0.2 | 124673 |

Based on Table 1, the raising of the therapeutic rate or the decreasing of contact rate will decrease the spread of HIV/AIDS disease, so can reduce new cases of HIV. If many people living with HIV/AIDS in 2007 who undergo ARV therapy and can increase the therapeutic rate become 0.9 then it is estimated that by 2030 only there are 1004 new cases of HIV. Furthermore if many people living with HIV/AIDS have known their HIV infection and keep the contact with individual susceptible (S) and contact rate can decrease until 0.05 then it is estimated that by 2030 only there are 1021 new cases of HIV. The simulation of alpha value and omega value show that 90-90-90 is very important in controlling the spread HIV/AIDS disease.

## 6. Conclusion

Based on the discussion, it is obtained three conclusions.

[1] The SIAT model is

$$\frac{dS}{dt} = (1-p)\theta N - \omega\frac{SI}{N} - \omega\frac{SA}{N} - \mu S$$

$$\frac{dI}{dt} = p\theta N + \omega\frac{SI}{N} - \omega\frac{SA}{N} - (\gamma + \mu + \alpha)I$$

$$\frac{dA}{dt} = \gamma I - \left(\mu + \mu_1 + \alpha\right)A$$

$$\frac{dT}{dt} = \alpha(I + A) - \mu T,$$

with S (0), A (0), T (0) ≥ 0, I (0) > 0 and θ, ω, μ, $\mu_1$, γ, α > 0. The six parameters consecutively are birthrate, contact rate, natural mortality rate, death rate due to AIDS, rate of outbreak, and rate of therapy.

[2] The SIAT model is applied in Indonesia and the pattern of prevalence of HIV/AIDS is increasing from year to year, and it is estimated that by 2030 there will be 700786 new cases and 1706630 people living with HIV.

[3] The 90-90-90 program is one of the government's efforts to control the spread of HIV/AIDS disease. The simulation re-sults by lowering the value of ω to 0.05 and increasing the value of α to 0.9, indicating that the 90-90-90 program have an important role in controlling the spread of HIV/AIDS disease.

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