PAPER • **OPEN ACCESS**

On the local fractional metric dimension of corona product graphs

To cite this article: S Aisyah *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **243** 012043

View the article online for updates and enhancements.



IOP ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

This content was downloaded from IP address 211.159.171.58 on 21/09/2019 at 07:40

On the local fractional metric dimension of corona product graphs

S Aisyah[1], M I Utoyo[2*], L Susilowati[2]

[1]Departement of Mathematics, Kaltara University, Indonesia
[2]Departemen of Matematics, Airlangga University, Indonesia

*E-mail: m.i.utoyo@fst.unair.ac.id; aisyah_rasyid84@yahoo.com

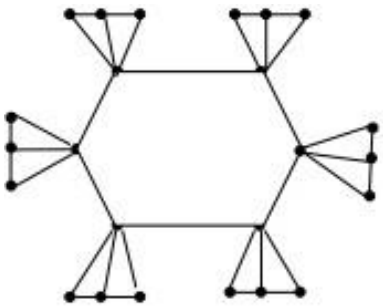
Abstract. A vertex in a connected graph is said to resolve a pair of vertices $\{u, v\}$ in if the distance from to is not equal to the distance from v to x. A set of vertices of is a resolving set for G if every pair of vertices is resolved by some vertices of S. The smallest cardinality of a resolving set for G is called the metric dimension of G, denoted by $\dim(G)$. For the pair of two adjacent vertices $\{u, v\}$ is called the local resolving neighbourhood and denoted by $R_1\{u, v\}$. A real valued function g_1: $V(G) \rightarrow [0,1]$ is a *local resolving* function of G if for every two adjacent vertices $u, v \in V(G)$. The *local fractional metric dimension* of G is defined as $\dim_{ft}(G) = \min\{|g_1|: g_1 \text{ is local resolving function of } G\}$ where $|g_1| = \sum_{v \in V} g_1(v)$. Let and be two graphs of order n_1 and n_2, respectively. The corona product $G \odot H$ is defined as the graph obtained from G and H by taking one copy of G and n_1 copies of H and joining by an edge each vertex from the ith-copy of H with the ith-vertex of G. In this paper we study the problem of finding exact values for the fractional local metric dimension of corona product of graphs.

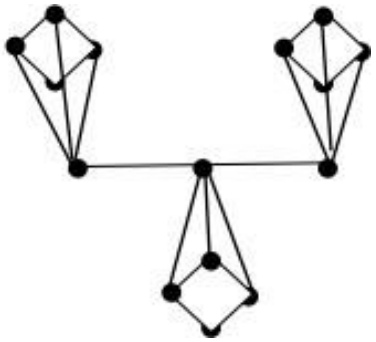
1. Introduction

Let $G = (V(G), E(G))$ be a finite, simple, undirected, connected graph. For a graph G, is the vertex set of G and $E(G)$ is the edge set of G, respectively. For any two vertices x and y of G, $d_G(x, y)$ denotes the distance between x and y, denotes the set of vertices z such that $d_G(x, z) \neq d_G(y, z)$. A resolving set of G is a subset W of $V(G)$ such that $W \cap R_G\{x, y\} \neq \emptyset$ for any two distinct vertices x and y of G [6]. The metric dimension of G, denoted by $\dim(G)$, is the minimum cardinality of the resolving sets of G. Metric dimension was first introduced in the 1970s, independently by Harary and Melter [3] and by Slater [4].

Let g: $V(G) \rightarrow [0,1]$ be a real valued function. For $W \subseteq V(G)$, define $g(W) = \sum_{v \in W} g(v)$. We call g is resolving function of G if $g(R_G\{x, y\}) > 1$ for any two distinct vertices x and y of G. The fractional metric dimension, denoted by $\dim(G)$, is given by $\dim_f(G) = \min\{|g|: g \text{ is resolving function of } G\}$, where $|g| = g(V(G))$. Arumugam and Mathew [1] formally introduced the fractional metric dimension of graphs and obtained some basic results. For an ordered set $X = \{x_1, x_2, \dots, x_k\} \subseteq V(G)$ of vertices, we use the ordered k-tuple $r(u|X) = (d(u, x_1), d(u, x_2), \dots, d(u, x_k))$ as the representation of u with respect to X. If every two adjacent vertices of G has distinct representation with respect to X then X is called as a *local resolving set* for G. A minimum local resolving set is called local basis for G. A local metric dimension for G, denote by $\dim(G)$, is the number of vertices in a local basis for G by Okamoto et al. [5]. From the idea of Arumugam and Mathew [1], Okamoto et al. [5] we define the *local resolving neighbourhood* as follow. For the pair $\{u, v\}$ of two adjacent vertices of G, we define the local resolving neighbourhood

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd





2

2. Main Result

In this section, we investigate the value of $\dim_{fl}(G \odot H)$ when G and H is a complite graph. We first recall some of the bounds on the local fractional metric dimension of graphs [13].

Theorem 1. For any graph G of order n,

a. For the path graph (P_n), $\dim_{fl}(P_n) = 1$
b. For K_n, $\dim_{fl}(K_n) = \frac{n}{2}$
c. For the cycle graph (C_n) on $n \geq 3$ vertices, we have

$$\dim_{fl}(C_n) = \begin{cases} \frac{n}{n-1}, & if\ n\ is\ odd \\ 1, & if\ n\ is\ even \end{cases}$$

d. For the star graph (S_n), $\dim_{fl}(S_n) = 1$
e. For the bipartite graph $(K_{n,m})$, $\dim_{fl}(K_{n,m}) = 1$

To begin with, we consider some straightforward cases. If H is an empty graph, then $K_1 \odot H$ is a star graph and $\dim_{fl}(K_1 \odot H) = 1$. Moreover, if H is a complete graph of order n, then $K_1 \odot H$ is a complete graph of order $n+1$ and $\dim_{fl}(K_1 \odot H) = \frac{n+1}{2}$.

Theorem 2. Let G be a connected non-trivial graph. For any empty graph H,

$$\dim_{fl}(G \odot H) = \dim_{fl}(G)$$

Proof: Taken any of the resolving function $f: V(G \odot H) \rightarrow [0,1]$. Taken any two adjacent vertices $x, y \in V(G \odot H)$ then there are two possibilities there are (1). $x, y \in V(G)$ and (2). $x \in V(G)$ and $y \in H(G)$. If

a. $\{x, y\} \in V(G)$ then there exist $U_{0i} = \{v_{0i}: v_{0i} \in V(G)\}$ with $i = 1,2, \ldots, n$. Based on the definition of the resolving set, is obtained $U_{0i} \subseteq R\{x, y\}$.
b. $x \in U_{0i}$ and $y \in U_{ji}$ with $j = 1,2, \ldots, m$. Based on the definition of the resolving set, is obtained $R\{x, y\} = V(G \odot H)$.

As a result, we can conclude that $\dim_{fl}(G \odot H) = \dim_{fl}(G)$.

Theorem 3. Let G be a connected graph and H be a graph. Then $\dim_{fl}(G \odot H) = |V(G)| \dim_{fl}(H)$

Theorem 4. Let H be a non-empty graph. The following assertions hold.

i. If the vertex of K_1 does not belong to any local metric basis for $K_1 + H$, then for any connected graph G of order n,

$$\dim_{fl}(G \odot H) = n.\ \dim_{fl}(K_1 + H)$$

ii. If the vertex of K_1 belongs to a local metric basis for $K_1 + H$, then for any connected Graph G of order $n \geq 2$,

$$\dim_{fl}(G \odot H) = \text{n. } (\dim_{fl}(K_1 + H) - 1)$$

3. Conclusion

In this paper, we get the value of $\dim_{fl}(G \odot H)$ where G and H are a special graph, i,e complete graphs, complete bipartite graphs, cycle graphs, star graphs, and path graphs.

Open Problem 3.1. Determine value $\dim_{fl}(G \odot H)$ where G and H are general graph.

Open Problem 3.2. To further be able to do research local fractional metrics dimension of comb product graph.

Acknowledgement
We gratefully acknowledge the support from DIKTI Indonesia trough the "Penelitian Disertasi Doktor" RISTEKDIKTI 2018 research project.

References

[1] Arumugam S, and Mathew V 2012 *Discrete Mathematics* **312** 1584–1590

[2] Arumugam S, Mathew V, and Shen J 2013 *Discrete Mathematics Algorithms Appl,* **5 (2)** 1350037 8

[3] Harary F, Melter R A 1977 *Ars Combin.* **2** 191–195 4 318

[4] Slater P J 1975 *Congr. Number.* **14** 549–559

[5] Okamoto F, Phinezy B and Zhang P 2010 *Mathematica Bohemica* **135** no. 3 239-255

[6] Feng M, Wang B Lv K 2014 *Discrete Appl. Math* **170** 55–63

[7] Frucht R, Harary F 1970 *Aequationes Mathematicae* **4 (3)** 322–325

[8] Feng M, Wang B Lv K On the fractional metric dimension of corona product graphs and lexicographic product graphs arXiv:1206.1906v1.

[9] Kang C X The fractional strong metric dimension of graphs *Lecture Notes in Comput. Sci.* 8287 84–95

[10] Kang C X, Yero I G, and Yi E The fractional strong metric dimension in three graph products arXiv:1608.0549v1.

[11] Yi E 2015 *Acta Mathematica Sinica* **Vol .31** No. 3 367-382

[12] Aisyah S, Utoyo M I, and Susilowati L Local Fractional Metric Dimension of Graphs *Discrete Mathematics* (On Submitted).

[13] Iswadi H, Baskoro E T, Simanjuntak R 2010 *Far East Journal of Mathematical Sciences* **52** 155 – 170