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Study of central and non-central collisions in billiard games

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Abstract. The purpose of the billiard games is to enter the object ball into the billiard table hole. If the hole position is aligned straight with the cue ball and object ball then the player can hit it centrally, but when the cue ball and object ball positions are not straight towards the hole then the player must hit the object ball in a non-central manner so that the ball object can bounce off by forming a certain angle and into the hole. The difference in object ball direction after the collision is an application of the collision concept that is a central collision and non-central collision. There has been a study of central and non-central collision reviews on billiard games to find out the validity of the law of conservation of momentum, coefficients of restitution (e) and disintegration energy (Q) . The data in this study was obtained by using an application tracker video analysis. The result of this study shows that on central collision in the billiard game the law of conservation of momentum is not applicable, $e \leq 0,510$ and $Q = -0,00786$ Joule. In the non-central collision, the law of conservation of momentum is not applicable, $e \leq 0,537$ and $Q = -0,00605$ Joule. The central and non-central collision in billiard games does not apply the law of conservation of momentum because it is influenced by many factors including the limited accuracy of the tracker application and the effects of external forces.

1. Introduction
Physics is a branch of natural science that underlies the development of science and technology. Physics is very important in human life because of many events in life involving both conscious and unconscious physics. One of the physics concepts that we often encounter in everyday life is the concept of the collision. A standard discussion topic in freshman mechanics courses is elastic collisions [9]. The collision is the meeting of two or more objects at the same time. One example of events around human life related to collision is billiard games [8].

Billiard games are played with some equipment including billiard sticks, billiard table and billiard balls consisting of object ball and cue ball. Billiard games is basically an application of various concepts of physics one of which is the collision. When the white ball is cue ball beaten by a stick and then rolled to hit the ball numbered is object ball, it is necessary to know things that can be controlled for object ball can fit into the billiard table hole. One of the controls that can be done so that the object ball can fit into the billiard table hole is by controlling the direction and speed of the object ball after the collision [3]. Billiard games can occur central and non-central collision. On the process of the collision will apply some of the concepts of physics such as the law of conservation of momentum, energy, elastic and inelastic collisions, etc [1].

1.1. Billiard games

Billiard games is a sport that falls into the category of sports concentration. This sports branch is played on the table and with special auxiliary equipment and its own rules. This game is divided into various types such as Carom, English Billiard and Pool. Mass of billiard sticks is 28.35 grams and has a length of 147-152 cm. The billiard table has a rectangular shape with a length of 270 cm and a width of 135 cm. Billiard balls have a massive ball shape with a diameter of 5.7 cm. Mass of billiard is not the same that is mass of cue ball is 0.17 kg and mass of object ball is 0.16 kg [12].

1.2. Central collision

The central collision occurs when the position of the centre of the mass of both balls that collide in line or are in the field of one dimension. The central collision in billiard games occurs when the cue ball collides with an object ball whose collision point is aligned to the centre of the mass of both balls [2]. The central collision process on billiard games can be analyzed using energy conservation laws and conservation laws of momentum. Central collision in the straight path on the x-axis can be seen in figure1.
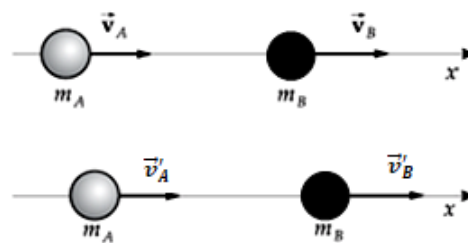

Figure 1. Central collision on the x-axis, with $\vec{v}$ (velocity of billiard balls before collision) and $\overrightarrow{v^l}$ (velocity of billiard balls after collision)

1.3. Non-central collision

The non-central collision occurs when the centre position of the two colliding balls is not in line. The non-central collision on billiard games occurs when the cue ball strikes the object ball but the two ballpoints of both balls are not aligned. The non-central collision will produce an angle between 0˚ to 180˚ [7],
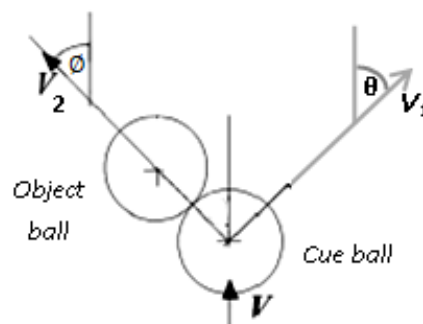

Figure 2. The non-central collision on billiard games with θ (angle of cue ball cuff ball towards cue ball direction before collision) and Ø (object ball scattering angle to cue ball direction before collision)

In event of a non-central collision between the cue ball and object, ball applies the law of conservation of momentum and the law of conservation of kinetic energy. The non-central collision on billiard games takes place in a two-dimensional plane, so the law of conservation of momentum on non-central collisions will be projected on x-axis and y-axis.

1.4. The coefficient of restitution

In contextual events, elastic collisions are very difficult to find because of many factors that effect. Therefore, at the central collision and non-central collision is also defined quantity without unit called coefficient of restitution. The coefficient of restitution e is defined as the ratio between the numerical value of the velocities after and before a collision [10]. In a non-central collision to determine the coefficient of restitution, it must change the collision occurring in two dimensions into one-dimensional collisions, this is necessary because the coefficient of restitution applies only to one-dimensional collisions. The coefficient of restitution equation is as follows,

$$e \le \frac{v_2' - v_1'}{v_2 - v_1} \tag{1}$$

a value of $e = 1$ represents a perfectly elastic collision, and a value of $e = 0$ indicates a totally inelastic interaction [5].

1.5. Disintegration energy

The collision of any two balls or any other massive objects is always accompanied by a loss of energy [4]. In billiard games, when a cue ball collides with the object ball, there will be some kinetic energy of the system that is converted into other energies, such as heat energy and sound energy. If the final kinetic energy of the system is less than the initial kinetic energy then there has been the absorption of energy by the system and if the final kinetic energy of the system is greater than the initial kinetic energy of the system, then the energy released by the system occurs. The difference between the initial kinetic energy and the final kinetic energy is called disintegration energy (Q) [11]. From the explanation above equation disintegration energy at central collisions were as follows,

$$Q = K' - K \tag{2}$$

And the equation of the disintegration energy at non-central collision is as follows,

$$Q = K_1' \left(1 + \frac{m_1}{m_2}\right) - K_1 \left(1 - \frac{m_1}{m_2}\right) - 2 \left(\frac{m_1^2 K_1 K_1}{m_2^2}\right)^{\frac{1}{2}} \cos \theta \tag{3}$$

equation (3) indicates that the calculated disintegration energy is in a state of a two-dimensional field

2. Main Body

2.1. Methodology

The study of central and non-central collision in billiard games is by analyzing the data obtained from the tracker video analysis [6]. The data obtained directly are mass and the diameter of the billiard balls, while the data obtained from the video analysis using the application tracker is the instantaneous velocity of the billiard balls before the collision, the instantaneous velocity of the billiard balls after the collision and the angle of the billiard balls after the collision. The collision video used in this study was a central collision and non-central collision on $\frac{5}{8}$ part
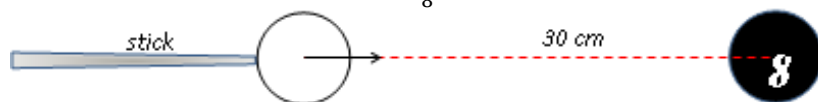

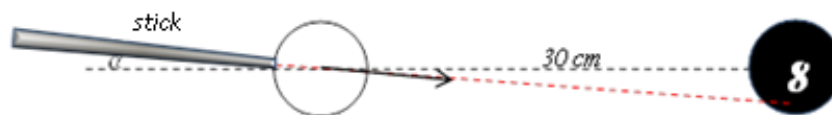**Figure 3**. The schematic video of central collision on billiard games

Figure 4. The schematic video of the non-central collision on the $\frac{5}{8}$ section with $\alpha = 4,1°$ (α is the angle between the direction billiard sticks against the second centre of the mass centre of the ball).

the data obtained are used to assess the central and non-central collisions of billiard games. To find out the validity of the law of conservation of momentum using the following equation is,

$$P_{before} = P_{after} \qquad\qquad (4)$$

and to analyze the coefficients of restitution using equation (1), and to analyze disintegration energy using equation (2) and (3).

2.2. Results and Discussion

The resulting research data are the mass and the diameter of the cue ball and object ball, the momentary speed of cue ball and object ball before and after the collision and the angle of the cue ball and object ball scatter after the collision.

Table 1. Data of mass and diameter of billiard balls

	Mass (kg)	Diameter (m)
Cue ball	0,170	$5,7 \times 10^{-2}$
Object ball	Type of billiard balls	$5,7 \times 10^{-2}$

Table 2. Data of velocity of billiard balls on a central collision

Type of billiard balls	Velocity before collision (m/s)	Velocity after collision (m/s)
Cue ball	0,506	0,135
Object ball	0	0,393

Table 3: Data of velocity of billiard balls on a non-central collision

Type of billiard balls	Velocity before collision (m/s)	Velocity after collision (m/s)
Cue ball	0,498	0,225
Object ball	0	0,363

Table 4. Data of scattering angle cue ball and object ball on a central and non-central collision

Type of billiard collision	Scattering angle of the cue ball after the collision(θ)	Scattering angle of the object ball after collision ($\emptyset$)
Central	$0°$	$0°$
Non – central	$37,2°$	$22°$

Based on the research data that has been obtained it will be studied several quantities included in the concept of collisions such as the law of conservation of momentum, coefficient of restitution and disintegration energy at central collisions and not-central collisions as follows.

2.2.1. The law of conservation of momentum

Theoretically, the collision occurring on a billiard game must apply the law of conservation of momentum. At the central collision is valid the law of conservation of momentum in one-dimensional whereas in the non-central collision apply the law of conservation of momentum in two-dimensional. In billiard games, a central collision can occur when the cue ball and object ball when colliding the centre of the mass of both balls in a line,
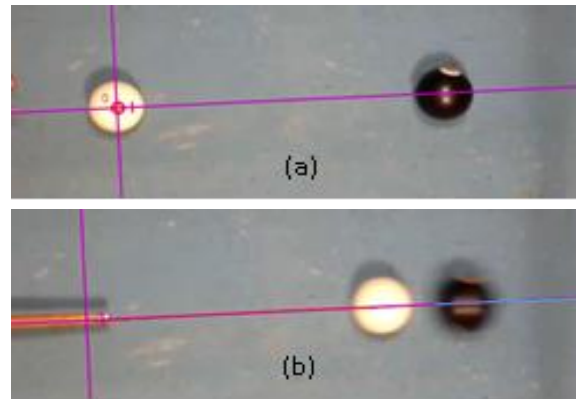
Figure 5. Central collision, (a) before the collision; (b) after the collision

Based on the data in the table. 2, it is known that the momentary velocity cue ball before the collision is $v_{cb} = 0.506 \, m/s$ and before the collision is object ball in rest condition so that $v_{ob} = 0 \, m/s$. After the collision object ball bouncing with velocity $v'_{ob} = 0.393 \, m/s$ and the cue ball is still moving at a velocity is $v'_{cb} = 0.135 \, m/s$ in the direction of movement of the cue ball before the collision. The total momentum before the collision on the central collision is,

$$\sum \vec{P}_{before} = m_{cb} v_{cb} + m_{ob} v_{ob} \tag{5}$$
$$= (0.17 \times 0.506) + 0$$
$$= 0.08602 \, kg \, m/s$$

and the total momentum after the collision is,

$$\sum \vec{P}_{after} = m_{ob} v'_{ob} + m_{cb} v'_{cb} \tag{6}$$
$$= (0.17 \times 0.135) + (0.16 \times 0.393)$$
$$= 0.08583 \, kg \, m/s$$

Based on the results of the calculation of equations (5) and (6), can be noted that the magnitude of total momentum before and after the collision, in this case, has a different value, so it can be stated that in this case does not apply the law of conservation of momentum. This can occur due to many factors that affect the influence of external forces on the system and calibration on the application tracker is still less through. The non-central collision on billiard games, cue ball and object ball after the collision will be scattered with a certain angle to the direction of cue ball before the collision,
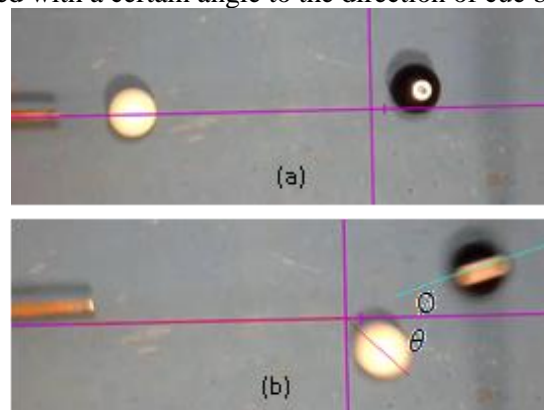

Figure 6. The non-central collision of 5/8 part (a) before the collision; (b) after the collision

The theoretical law of conservation of momentum also applies to the non-central collision. The non-central collision on billiard games takes place in a two-dimensional plane, so the total momentum on the centralized collision will be projected on the horizontal axis (x) and the vertical axis (y). Based on data in Table .3, that there is a momentary velocity cue ball and object ball before the collision is

$v_{cb} = 0.498 \, m/s$ and $v_{0b} = 0 \, m/s$, and the momentary velocity after the collision is $v'_{cb} = 0.225 \, m/s$ and $v'_{ob} = 0.363 \, m/s$. The angle of the cue ball and object ball after the collision is $\theta = 37.2°$ and $\phi = 22°$, it can be determined the total momentum before the collision on the $x -$ axis,

$$\sum \vec{P}_{x_{before}} = m_{cb}v_{cb} + m_{ob}v_{ob} \tag{7}$$
$$= (0.17 \times 0.498) + 0$$
$$= 0.08466 \, kg \, m/s$$

and the total momentum after the collision on the $x -$ axis,

$$\sum \vec{P}_{x_{after}} = m_{cb}v'_{cb} \cos\theta + m_{ob}v'_{ob} \cos\phi \tag{8}$$
$$= \left(0.17 \times 0.225 \times \cos 37.2°\right) + \left(0.16 \times 0.363 \times \cos 22°\right)$$
$$= 0.08431 \, kg \, m/s$$

while the total momentum before the collision on the $y -$ axis,

$$\sum \vec{P}_{y_{before}} = m_{cb}v'_{cb} \sin\theta \tag{9}$$
$$= 0.17 \times 0.225 \times \sin 37.2°$$
$$= 0.02312 \, kg \, m/s$$

and total momentum after collision on $y -$ axis,

$$\sum \vec{P}_{y_{after}} = m_{ob}v'_{ob} \sin\phi \tag{10}$$
$$= 0.16 \times 0.363 \times \sin 22°$$
$$= 0.02175 \, kg \, m/s$$

Based on the results of the calculation of equations (7), (8), (9) and (10), can be noted that the total momentum before the collision and after the collision on the x-axis and the $y -$ axis have unequal values ie total momentum before the collision is greater than after the collision. The big difference in total momentum can be caused by several factors such as the error of the tracker application, the researcher's fault and the effect of the external force.

2.2.2. The coefficient of restitution
In contextual events, elastic collisions are very difficult to find because of many affecting factors. The coefficient of restitution is a quantity that characterizes the degree of importance of a colliding object. At the central collision, the coefficient of restitution can be searched using the following equation,

$$e \leq -\frac{v'_{0b} - v'_{cb}}{v_{ob} - v_{cb}} \tag{11}$$
$$e \leq -\frac{0.393 - 0.135}{0 - 0.506}$$
$$e \leq 0.510$$

The magnitude of the coefficient of restitution on the central collision is about 0.510 which is classified as a partially elastic collision.

In the non-central collision to determine the equation of the coefficient of restitution, it must change the collision occurring in two dimensions (x-axis and y-axis) into one-dimensional collisions (iaxis) since the coefficient of restitution applies to one-dimensional collisions. The non-central collision projection diagram on one dimension (i-axis) can be seen in the following figure.
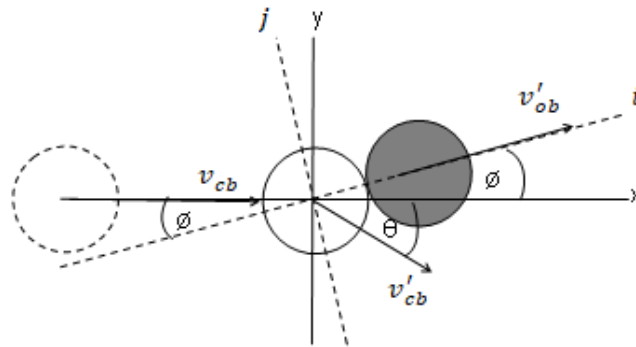
Figure 7. Diagram of the non-central collision of 5/8 part which projected on the one-dimensional (the white ball is cue ball and black ball is object ball)

Once project, then on i – axis their component of the of the vector velocity is $\vec{v}_{cb} \cos\phi$, $\vec{v}'_{ob}$ and $\vec{v}'_{cb} \cos(\phi + \theta)$ whereas on the j–axis there is a vector velocity component $\vec{v}_{cb} \sin\phi$ and $\vec{v}'_{cb} \sin(\phi + \theta)$. In j-axis large velocity components must have the same values and have the opposite direction so that the resultant velocity on the jaxis becomes zero. This causes the system to not move on the j – axis. The non-central collision on 5/8 part, a component of vector velocity on positive j-axis is,

$$\vec{v}_{cb} \sin\phi = 0.498 \times \sin 22^{\circ} \tag{12}$$
$$= 0.18655 \ m/s$$

And the component of vector velocity on negative j – axis is,

$$\vec{v}'_{cb} \sin(\phi + \theta) = 0.225 \times \sin 59.2^{\circ} \tag{13}$$
$$= 0.19326 \ m/s$$

Based on the results of the calculation of equations (12) and (13), can be seen that the major components of velocity on the j negative axis outweigh the j positive axis then the event of a collision the system is experiencing a shift towards j negative axis.

On the i-axis can be seen that the centre of the mass cue ball and the object ball are already in one dimension (i-axis), so the equation of the coefficient of restitution on the non-central collision in general is,

$$e \leq -\frac{v'_{ob} - v'_{cb}}{v_{ob} - v_{cb}}$$

$$e \leq -\frac{v'_{ob} - v'_{cb} \cos(\phi + \theta)}{v_{cb} \cos\phi} \tag{14}$$

$$e \leq \frac{0.363 - (0.225 \times \cos 59.2^{\circ})}{0.498 \times \cos 22^{\circ}}$$

$$e \leq 0.537$$

so the value of the coefficient of restitution in the case of the noncentral collision 5/8 part is $e \leq 0.537$ which belongs to the partial elastic collision.

2.2.3. Disintegration energy

Disintegration energy is the large difference in kinetic energy after the collision and before the collision. Mathematically disintegrated energy has a negative sign and a positive sign. The negative sign explains that the system releases energy while the positive sign describes the system getting energy. In the billiard games, disintegrating energy can be either heat energy or sound energy that arises when the cue ball and object ball collide. In the central collision of billiard balls can be calculated large disintegration energy that is, by using the equation (2) can be determined, is the disintegration energy is,

$$Q = K' - K \tag{15}$$

$$Q = \left(\tfrac{1}{2} m_{cb} v'^2_{cb} + \tfrac{1}{2} m_{ob} v'^2_{ob} \right) - \left(\tfrac{1}{2} m_{cb} v^2_{cb} + \tfrac{1}{2} m_{ob} v^2_{ob} \right)$$

$$Q = \left(\tfrac{1}{2} \times 0.17 \times 0.135^2 + \tfrac{1}{2} \times 0.16 \times 0.393^2 \right) - \left(\tfrac{1}{2} \times 0.17 \times 0.506^2 - 0 \right)$$

$$Q = (0.01390 - 0.02176)$$

$$Q = -0.00786 \, Joule$$

Whereas the disintegration energy at non-central collision to its value can be calculated using equation (3). Energy equation of desintegration on equation (3) is a new equation which can be obtained as follows. At the moment the cue ball collides with a cue ball, then the cue ball will bounce with an angle θ towards the direction of the cue ball before the collision are moving with the speed v'_{cb}. While the object ball will bounce with an angle ϕ towards the direction of the cue ball before the collision are moving with the speed v'_{ob} as shown in Figure 7. Known equations the speed of the cue ball after the collision on the non-central collision is,

$$v'_{cb} = \left(\frac{m^2_{cb} v^2_{cb} - 2 m_{cb} m_{ob} v_{cb} v'_{ob} \cos\phi + m^2_{ob} v'^2_{ob}}{m^2_{cb}} \right)^{1/2} \tag{16}$$

Equation speed of object ball after the collision on the non-central collision is,

$$v'_{ob} = \left(\frac{m^2_{cb} v^2_{cb} - 2 m^2_{cb} v_{cb} v'_{cb} \cos\theta + m^2_{cb} v'^2_{cb}}{m^2_{ob}} \right)^{1/2} \tag{17}$$

The kinetic energy before the collision of billiard balls on a non-central collision is,

$$K = K_{cb} + K_{ob}$$

$$K = \frac{1}{2} m_{cb} v^2_{cb} \tag{18}$$

The value of $K_{ob} = 0$, because before the collision object ball in a motionless conditions. While the kinetic energy after the collision of billiard balls on a non-central collision is,

$$K' = K'_{cb} + K'_{ob}$$

$$= \frac{1}{2} m_{cb} v'^2_{cb} + \frac{1}{2} m_{ob} v'^2_{ob} \tag{19}$$

Substitution equations (15) and equation (16) into equation (18),

$$K' = \frac{1}{2} m_{cb} \left(\frac{m^2_{cb} v^2_{cb} - 2 m_{cb} m_{ob} v_{cb} v'_{ob} \cos\phi + m^2_{ob} v'^2_{ob}}{m^2_{cb}} \right)$$

$$+ \frac{1}{2} m_{ob} \left(\frac{m^2_{cb} v^2_{cb} - 2 m^2_{cb} v_{cb} v'_{cb} \cos\theta + m^2_{cb} v'^2_{cb}}{m^2_{ob}} \right)$$

$$= \frac{m_{cb} v^2_{cb}}{2} \left(1 + \frac{m_{cb}}{m_{ob}} \right) + \frac{1}{2} \left(\frac{m^2_{cb} v'^2_{cb}}{m_{ob}} + \frac{m^2_{ob} v'^2_{ob}}{m_{cb}} \right)$$

$$- v_{cb} \left(m_{ob} v'_{ob} \cos\phi + \frac{m^2_{cb} v'_{cb} \cos\theta}{m_{ob}} \right) \tag{20}$$

Then the substitution the equation (19) and equation (17) into equation (2) so that the resulting equation (3). The disintegration energy in the non-central collision 5/8 part, can be calculated using equation (3) is,

$$Q = \left[\frac{1}{2m_{cb}m_{ob}} \left(m_{cb}^3 v_{cb}^2 + m_{cb}^3 v_{cb}'^2 + m_{ob}^3 v_{ob}'^2 \right) \right] - \left[\frac{v_{cb}}{m_{ob}} \left(m_{ob}^2 v'_{ob} \cos\phi + m_{cb}^2 v'_{cb} \cos\theta \right) \right]$$

$$Q = \frac{1}{2x0.17x0.16} x \left(0.17^3 x0.498^2 + 0.17^3 x0.225^2 + 0.16^3 x0.363^2 \right) -$$

$$\frac{0.498}{0.16} \left(0.16^2 x0.363 x \cos 22° + 0.17^2 x0.225 x \cos 37.2° \right)$$

$$Q = (0.03689 - 0.04294)$$

$$Q = -0.00605 \text{ Joule} \tag{21}$$

Based on the results of the calculation of equations (2) and (3), can be seen that the magnitude of disintegration energy at the central collision is greater than the disintegration energy at the non-central collision. This happens because the large restitution coefficient on the central collision is smaller than the non-central collision, so it can be made that the greater the value of the restitution coefficient of a system the smaller the disintegration energy generated by the system.

3. Conclusion
Based on the results and discussion of central and non-central collisions reviews on billiard games, it can be concluded as follows :
   a. The result of the central and non-central collision review of the billiard games shows that the law of conservation of momentum does not apply to central collisions and non-central collisions on billiard games, this is because it is influenced by many factors including the limited accuracy of the tracker application and the effects of external forces.
   b. The value of the coefficient of restitution of the central collision is $e \leq 0.510$ and the non-central collision is $e \leq 0.537$, in dicating that the central and non-central on billiard games which is classified as a partially elastic collision.
   c. The disintegration energy at the central collision is greater than the disintegration energy at the non-central collision which means that at the central collision more energy is released by the system than by the non-central collision system.

On further research about the central and non-central collision in billiard games, we can expand the study of central and non-central collisions by using a variation on the more balls hit, a billiard ball and the number of impacts, as well as examine the relationship between the coefficient of restitution by the disintegration energy.

4. Acknowledgement

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