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The rainbow vertex connection number of edge corona product graphs

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Abstract. Let G_1,G_2 be a special graphs with vertices of G_1 1,2,..., n and edges of G_1 1,2,… m. The generalized edge corona product of graphs G_1 and $G2$, denoted by $\boldsymbol{G_1} \diamond \boldsymbol{G_2}$ is obtained by taking one copy of graph G_1 and m copy of G_2, thus for each edge e_k= i_j of G, joining edge between the two end-vertices i, j of e_k and each vertex of the k-copy of G_2. A rainbow vertex-coloring graph G where adjacent vertices u-v and its internal vertices have distinct colors. A path is called a rainbow path if no two verticess of the path have the same color. A rainbow vertex-connection number of graph G is minimun number of colors in graph G to connected every two distinct internal vertices u and v such that a graph G naturally rainbow vertex-connected, denoted by rvc (G). In this paper, we determine minimum integer for rainbow vertex coloring of edge corona product on cycle and path such as $\boldsymbol{P_n} \diamond \boldsymbol{P_m}, \boldsymbol{P_n} \diamond \boldsymbol{C_m}, \boldsymbol{C_n} \diamond \boldsymbol{P_m},$ **and** $\boldsymbol{C_n} \diamond \boldsymbol{C_m}$.

1. Introduction

The concept of rainbow connection in graphs as follows at the first time was introduced in 2008 by Chartrand *et a.l* [2]. Let $G = (V,E)$ be a non trivial and connected, the rainbow connection number of G is the minimum number of colors in a rainbow connected edge-coloring of G, denoted by $rc(G)$. The graph G is rainbow-connected if G has a rainbow u-v path for every two vertices u and v of G. The graph G with size m and diameter $diam(G)$ be a connected graph, then

$$m \geq rc(G) \geq diam(G). \tag{1}$$

Krivelevich and Yuster [5] was proposes a new concept of a rainbow connection and called by rainbow vertex-connected. A rainbow vertex-coloring graph G where adjacent vertices u-v and its internal vertices have distinct colors. A path is called a rainbow path if no two verticess of the path have the same color. A path is called a rainbow path if no two verticess of the path have the same color [8]. A vertex-colored graph G is rainbow connected if any two vertices are connected by a rainbow path. A rainbow vertex-connection number of graph G is minimun number of colors in graph G to connected every two distinct internal vertices u and v such that a graph G naturally rainbow vertex-connected, denoted by rvc (G) [1].Several results on rainbow vertex-coloring of some families of graphs such as:

- Cycle C_n of order $n \geq 3$:

$$rvc(C_n) = \begin{cases} 0, & n = 3 \\ 1, & n = 4{,}5 \\ 3, & n = 9 \\ \left\lceil \frac{n}{2} \right\rceil - 1, & n = 6, 7, 8, 10, 11, 12, 13 \text{ } or \text{ } 15 \\ \left\lceil \frac{n}{2} \right\rceil, & n \geq 16 \text{ } or \text{ } 14 \text{ [3]} \end{cases}$$

- Pencil graph Pc_n for $n \geq 2, rvc(Pc_n) = \left\lceil \frac{n}{2} \right\rceil$ if $n \leq 7$ and $rvc(Pc_n) = \left\lceil \frac{n}{2} \right\rceil +$ 1 otherwise [9]
- Path P_n of order $n \geq 3$, $rvc(P_n) = n - 2$ [10]

Furthermore, X. Li and Y. Shi [7] studied the following theorem and gave the lower bound for *rvc*(*G*).

Theorem A. *Let G be a nontrivial connected graph of order n, then*

$$rvc(G) \geq diam(G) - 1. \quad (2)$$

Hou Yaoping and Shiu Wai-Chee [4] observed edge corona product on simple graph. Let graph G_1 is a special graph with vertices 1,2,..., *n* and edges e_1, e_2, …, e_m and G_2 is special graphs too. The generalized edge corona product of graphs G_1 and *G2* denoted by $G_1 \diamond G_2$, is obtained by taking one copy of graphs G_1 and m copy of G_2, thus for each edge $e_k = i_j$ of *G*, joining edges between the two end-vertices *i*, *j* of e_k and each vertex of the *k*-copy of G_2[11].

2. Result

In this section, we determined our result of rainbow vertex coloring of edge corona product on cycle and path as follows $\boldsymbol{P_n \diamond P_m, P_n \diamond C_m, C_n \diamond P_m,}$ **and** $\boldsymbol{C_n \diamond C_m}$, for every n and m are elements of natural number with n ≥ 2 and m ≥ 3.

2.1. The rainbow vertex-coloring of edge corona product on $P_n \diamond P_m$

We start with a rainbow vertex-connection number of edge corona product of a two path on *n* vertices P_n and m vertices P_m. Edge corona product between two of path graph denoted by $P_n \diamond P_m$ with $n \geq 2$ and $m \geq 2$. We present the rainbow vertex-coloring of edge corona product on $P_n \diamond P_m$ graph as follows.

Theorem 2.1. *Let G be a edge corona product of path graph P_n and P_m, the rainbow vertex-connection number of $P_n \diamond P_m$ is rvc($P_n \diamond P_m$)= $n - 2$.*

Proof. Path graph P_n with $n \geq 2$ has a vertex set $V(P_n) = \{u_i\,; 1 \leq i \leq n\}$ and an edge set $E(P_n) = \{u_i u_{i+1}\,; 1 \leq i \leq n - 1\}$. Edge corona product between two of path graph is denoted by $P_n \diamond P_m$ with $n \geq 2$ and $m \geq 2$ has a vertex set $V(P_n \diamond P_m) = \{u_i\,; 1 \leq i \leq n\} \cup \{v_j^k\,; 1 \leq j \leq m\,; 1 \leq k \leq n-1\}$ and a edge set $E(P_n \diamond P_m) = \{u_i u_{i+1}\,; 1 \leq i \leq n - 1\} \cup \{u_i v\,; 1 \leq i \leq n\,;\ 1 \leq j \leq m\,;\ 1 \leq k \leq n-1\} \cup \{v_j^k\, v_{j+1}^k\,;\ 1 \leq j \leq m-1\,;\ 1 \leq k \leq n - 1\}$.

Begin with *rvc(*$P_n \diamond P_m$*)*= $n - 2$. Since $diam(P_n \diamond P_m) = n - 1$, by using Theorem A, we have *rvc(*$P_n \diamond P_m$*)* $\geq n - 2$. Furthermore to prove that *(*$P_n \diamond P_m$*)* $\leq n - 2$, by vertex coloring v according to the following formula as follow:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i\,;\ i = 1, n \\ i - 1, & \text{for } v = u_i\,;\ 2 \leq i \leq n - 1 \\ 1, & \text{for } v = v_j^k\,;\ 1 \leq j \leq m\,;\ 1 \leq k \leq n - 1 \end{cases}$$

It easily to see that the colors of vertices are n-1, that is, $c: V(P_n \diamond P_m) \rightarrow \{1,2,3 \ldots, n-2\}$. Then $rvc(P_n \diamond P_m) \leq n-2$. So, if we combining both of them, we have the vertices minimum colors is $rvc(P_n \diamond P_m) = n-2$.

The following theorem we determine the rainbow vertex connection number of edge corona of a path P_n and a cycle graph C_m. Edge corona product of path and cycle graph denoted by $P_n \diamond C_m$ such that a rainbow connection-number as follows

2.2. The rainbow vertex-coloring of edge corona product on $P_n \diamond C_m$

Theorem 2.2. *Let G be edge corona product of path P_n and cycle C_m graph, the rainbow vertex connection number of $P_n \diamond C_m$ is $rvc(P_n \diamond C_m) = n-2$.*

Proof. Path graph P_n with $n \geq 2$ has a vertex set $V(P_n) = \{u_i\,; 1 \leq i \leq n\}$ and an edge set $E(P_n) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\}$. A cycle graph C_m with $m \geq 3$ has a vertices set $V(C_m) = \{u_i\,; 1 \leq i \leq m\}$ and an edges set $E(C_m) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\} \cup \{u_1 u_m\}$. Then $P_n \diamond C_m$ has a vertex set $V(P_n \diamond C_m) = \{u_i\,; 1 \leq i \leq n\} \cup \{v_j^k\,; 1 \leq j \leq m\,; 1 \leq k \leq n-1\}$ and an edge set $E(P_n \diamond C_m) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\} \cup \{u_i v_j^k\,; 1 \leq i \leq n\,; 1 \leq j \leq m\,; 1 \leq k \leq n-1\} \cup \{v_j^k v_{j+1}^k; 1 \leq j \leq m-1; 1 \leq k \leq n-1\} \cup \{v_1^k\, v_m^k\}$

Begin with $rvc(P_n \diamond C_m) = n-2$. Since $diam(P_n \diamond C_m) = n-1$, by Theorem A, we have $rvc(P_n \diamond C_m) \geq n-2$. Furthermore to prove that $(P_n \diamond C_m) \leq n-2$, by vertex coloring v according to the following formula as follow:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i\,;\ i = 1, n \\ i-1, & \text{for } v = u_i\,;\ 2 \leq i \leq n-1 \\ 1, & \text{for } v = v_j^k\,;\ 1 \leq j \leq m\,;\ 1 \leq k \leq n-1 \end{cases}$$

It easily to see that the color of vertices are n-1, that is, $c: V(P_n \diamond C_m) \rightarrow \{1,2,3 \ldots, n-2\}$. Thus $rvc(P_n \diamond C_m) \leq n-2$. So, if we combining both of them, we have the vertices minimum colors is $rvc(P_n \diamond P_m) = n-2$.

Now we present the rainbow vertex connection number of edge corona product of a cycle graph on *n* vertices C_n and a path on m vertices P_m. Edge corona product of cycle graph and path is denoted by $C_n \diamond P_m$ with $n \geq 3$ and $m \geq 2$, as follows

2.3. The rainbow vertex-coloring of edge corona product on $C_n \diamond P_m$

Theorem 2.3. *Let G be edge corona product of cycle C_n and path P_m , the rainbow vertex connection number of $C_n \diamond P_m$, is*

$$rvc(C_n \diamond P_m) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor, & n \leq 4 \\ \left\lceil \frac{n}{2} \right\rceil, & n \geq 5 \end{cases}$$

Proof. Cycle graph C_m with $m \geq 3$ has a vertex set $V(C_m) = \{x_i\,; 1 \leq i \leq m\}$ and an edge set $E(C_m) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\} \cup \{u_1 u_m\}$. A path graph P_n with $n \geq 2$ has a vertex set $V(P_n) = \{u_i\,; 1 \leq i \leq n\}$ and an edge set $E(P_n) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\}$. Edge corona product of cycle graph and path is denoted by $C_n \diamond P_m$ with $n \geq 3$ and $m \geq 2$ has a vertices set $V(C_n \diamond P_m) = \{u_i\,; 1 \leq i \leq n\} \cup \{v_j^k\,; 1 \leq j \leq m\,; 1 \leq k \leq n\}$ and an edges set $E(C_n \diamond P_m) = \{u_i u_{i+1}\,; 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \cup \{u_i v_j^k\,; 1 \leq i \leq n\,; 1 \leq j \leq m\,; 1 \leq k \leq n-1\} \cup \{v_j^k v_{j+1}^k; 1 \leq j \leq m-1; 1 \leq k \leq n-1\}$

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Begin with *rvc*($C_n \diamond P_m$)= $\left\lfloor \frac{n}{2} \right\rfloor$; $2 \le n \le 4$. Since $diam(P_n \diamond C_m)$= $\left\lfloor \frac{n}{2} \right\rfloor + 1$ by Theorem A, we have *rvc*($C_n \diamond P_m$) $\ge$ $\left\lfloor \frac{n}{2} \right\rfloor$. Furthermore to prove that *rvc*($C_n \diamond P_m$) $\ge$ $\left\lfloor \frac{n}{2} \right\rfloor$, by vertex coloring v according to the following formula as follows:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i \text{ ; } i = 1, n \\ i-1, & \text{for } v = u_i \text{ ; } 2 \le i \le n-1 \\ 1, & \text{for } v = v_j^k \text{ ; } 1 \le j \le m \text{ ; } 1 \le k \le n-1 \end{cases}$$

It is easy to see that the color of vertices are n-1, that is, $c: V(C_n \diamond P_m) \to \left\{1,2,3 \ldots, \left\lfloor \frac{n}{2} \right\rfloor\right\}$. Thus *rvc*($C_n \diamond P_m$) $\le$ $\left\lfloor \frac{n}{2} \right\rfloor$. So if we combining both of them, we have the vertices minimum colors is *rvc*($C_n \diamond P_m$) $=$ $\left\lfloor \frac{n}{2} \right\rfloor$.

On second step we show that *rvc*($C_n \diamond P_m$)= $\left\lceil \frac{n}{2} \right\rceil$; $n \ge 5$. Since $diam(P_n \diamond C_m)$= $\left\lceil \frac{n}{2} \right\rceil + 1$ by Theorem A, we have *rvc*($C_n \diamond P_m$) $\ge$ $\left\lceil \frac{n}{2} \right\rceil$. Furthermore to prove that ($C_n \diamond P_m$) $\le$ $\left\lceil \frac{n}{2} \right\rceil$, by vertex coloring v according to the following formula as follows:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i \text{ ; } i = 1, n \\ i-1, & \text{for } v = u_i \text{ ; } 2 \le i \le n-1 \\ 1, & \text{for } v = v_j^k \text{ ; } 1 \le j \le m \text{ ; } 1 \le k \le n-1 \end{cases}$$

It is easy to see that the color of vertices are n-1, that is, $c: V(C_n \diamond P_m) \to \left\{1,2,3 \ldots, \left\lceil \frac{n}{2} \right\rceil\right\}$. Thus *rvc*($C_n \diamond P_m$) $\le$ $\left\lceil \frac{n}{2} \right\rceil$. So if we combining both of them, we have the vertices minimum colors is *rvc*($C_n \diamond P_m$) $=$ $\left\lceil \frac{n}{2} \right\rceil$.

The following theorem determine the rainbow vertex connection number of edge corona of a denoted by $C_n \diamond C_m$ as follows.

2.4. The rainbow vertex-coloring of edge corona product on $C_n \diamond C_m$

Theorem 2.4. *Let G be edge corona product of cycle graph where n and m are a natural number with $n \ge 3$ and $m \ge 3$, the rainbow vertex coloring number of $C_n \diamond C_m$ is*

$$rvc(C_n \diamond C_m) = \begin{cases} \left\lfloor \dfrac{n}{2} \right\rfloor, & n \le 4 \\ \left\lceil \dfrac{n}{2} \right\rceil, & n \ge 5 \end{cases}$$

Proof. Edge corona product between of two cycle graph and path is denoted by $C_n \diamond C_m$ with $n \ge 3$ and $m \ge 3$ has a vertex set $V(C_n \diamond C_m) = \{u_i \,; 1 \le i \le n\} \cup \{v_j^k \,; 1 \le j \le m \,; 1 \le k \le n\}$ and an edge set $E(C_n \diamond C_m) = \{u_i u_{i+1} \,; 1 \le i \le n-1\} \cup \{u_1 u_n\} \cup \left\{u_i v_j^k \,; 1 \le i \le n \,; 1 \le j \le m \,; 1 \le k \le n-1\right\} \cup \left\{v_j^k v_{j+1}^k ; 1 \le j \le m-1; 1 \le k \le n-1\right\}$

Begin with *rvc*($C_n \diamond P_m$)= $\left\lfloor \frac{n}{2} \right\rfloor$; $2 \le n \le 4$. Since $diam(C_n \diamond C_m)$= $\left\lfloor \frac{n}{2} \right\rfloor + 1$, by Theorem A, we have *rvc*($C_n \diamond C_m$) $\ge$ $\left\lfloor \frac{n}{2} \right\rfloor$. Furthermore to prove that ($C_n \diamond C_m$) $\le$ $\left\lfloor \frac{n}{2} \right\rfloor$, by vertex coloring v according to the following formula as follows:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i \text{ ; } i = 1, n \\ i-1, & \text{for } v = u_i \text{ ; } 2 \le i \le n-1 \\ 1, & \text{for } v = v_j^k \text{ ; } 1 \le j \le m \text{ ; } 1 \le k \le n-1 \end{cases}$$

It is easily to see that the color of vertices are $\left\lceil\frac{n}{2}\right\rceil for\ n \leq 4$, that is, $c: V(C_n \diamond P_m) \rightarrow \left\{1,2,3 \ldots, \left\lceil\frac{n}{2}\right\rceil\right\}$. Thus *rvc*$(C_n \diamond P_m) \leq \left\lceil\frac{n}{2}\right\rceil$. So if we combining both of them, we have the vertices minimum colors is *rvc*$(C_n \diamond P_m) = \left\lceil\frac{n}{2}\right\rceil$.

On second step we show that *rvc*$(C_n \diamond P_m) = \left\lceil\frac{n}{2}\right\rceil; n \geq 5$. Since $diam(P_n \diamond C_m) = \left\lceil\frac{n}{2}\right\rceil + 1$ by Theorem A, we have *rvc*$(C_n \diamond P_m) \geq \left\lceil\frac{n}{2}\right\rceil$. Furthermore to prove that $(C_n \diamond P_m) \leq \left\lceil\frac{n}{2}\right\rceil$, by vertex coloring v according to the following formula as follows:

$$f(v) = \begin{cases} 1, & \text{for } v = u_i\ ;\ i = 1, n \\ i-1, & \text{for } v = u_i\ ;\ 2 \leq i \leq n-1 \\ 1, & \text{for } v = v_j^k\ ;\ 1 \leq j \leq m\ ;\ 1 \leq k \leq n-1 \end{cases}$$

It is easily to see that the color of vertices are $\left\lceil\frac{n}{2}\right\rceil for\ n \geq 5$, that is, $c: V(C_n \diamond P_m) \rightarrow \left\{1,2,3 \ldots, \left\lceil\frac{n}{2}\right\rceil\right\}$. Thus *rvc*$(C_n \diamond P_m) \leq \left\lceil\frac{n}{2}\right\rceil$. So if we combining both of them, we have the vertices minimum colors is *rvc*$(C_n \diamond P_m) = \left\lceil\frac{n}{2}\right\rceil$.

3. Conclusion

In this paper, we have determined the exact values of total rainbow connection number of edge corona product of cycle and path on such as $P_n \diamond P_m, P_n \diamond C_m, C_n \diamond P_m,$ dan $C_n \diamond C_m$ depend on *rvc*(G_1) as follows *rvc*$(P_n \diamond P_m)$ *and* $rvc(P_n \diamond C_m)$ is $n-2$, $C_n \diamond P_m$ dan $C_n \diamond C_m$ is $\left\lceil\frac{n}{2}\right\rceil,\ n \geq 5$. As we did this proof, it was dificult to get a minimum rainbow vertex connection number. Thus, it still gives the following open problem.

Open Problem 1. *Let G be edge corona product of cycle and any graph, determine sharper lower bound of rvc (G)*

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