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Linked stress release model and its application on earthquake data in java and sumatra

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Abstract. Elastic rebound theory explains stress release model in which stress will increase deterministically and decrease stochastically because of earthquake. Based on earthquakes occurrences, an area interacts with the other areas. Stress movement and interaction among areas have not been considered in stress release model. In this article, stress release model is developed into linked stress release model by considering interaction among areas. This model can be stated through conditional intensity function depending on accumulated stress release when earthquake occurs in a sub-region within a time period. The conditional intensity function is presented by exponential hazard function where each region has different hazard function stating that each region has specific characteristic. This research data consists of earthquake data occurring in Java Island and Sumatra Island during the period of January 2010 to December 2016. By using maximum likelihood estimate method, estimation of conditional intensity function of linked stress release model for earthquakes in each region is obtained. Probability of earthquake occurrence in southern Java is the same as probability in northern Sumatra. Most of proportion of accumulated stress in northern and southern Java is derived from their own regions while most of proportion of accumulated stress in northern and southern Sumatra is derived from other region.

1. Introduction

Earthquake is a natural phenomenon that is random, in terms of both space and time. Recently occurrences of earthquakes are being analyzed from seismic and stochastic aspects. Stochastic process is one of fields of data analysis in statistics that can be used to predict or explain phenomena in daily life. One of stochastic models which can explain about earthquakes is point process. It is a stochastic model, which describes natural phenomena, which are random in space and time. Generally, points in point process represent time and location, and sometimes other information. In seismology, earthquake occurrences in certain location and time can be considered as points, while the size related to earthquake occurrence is magnitude or depth [3] [4].

Elastic rebound theory proposed by Reid [1] is a classic model of earthquake mechanism. This model shows that elastic stress in seismic area accumulates due to tectonic plate movement and is released because of excessive energy. This theory demonstrates that large earthquake is possibly followed by passive period, while in fact, large earthquake can be followed by active period and sometimes by aftershocks with relatively the same magnitude.

Stress release model is a development of rebound elastic theory in which the size of stress increases deterministically and declines stochastically due to earthquake. Zheng and Vere-Jones [8]



applied stress release model on data of earthquakes in China, Persia, and Japan while Imoto [6] applied the model on Nankai Japan. One of the most interesting phenomena observed in their research was that large earthquake occurrence commonly followed by the other large occurrence in another area. On the analysis of data on earthquakes, an area interacts with the other area. Zheng and Vere-Jones [8] recorded the presence of classification of treatments for stress movement and interaction. Stress movement and interaction among areas are not considered in stress release model. Therefore, stress release model needs to be developed into linked stress release model, particularly for interaction among areas.

2. Linked Stress Release Model

Elastic rebound theory explains that elastic stress in active seismic area accumulates due to tectonic movement and releases because of excessive energy, like time and slip-predictable models. Many characteristics of earthquakes are not recorded in those models and those have to be taken into account through stochastic model. Vere-Jones [2] suggested stress release model as the development of stochastic model for earthquakes proposed by Knopoff.

In stress release model, the stress level increases deterministically with loading rate ρ and initial stress $X(0)$. Stress is released when it exceeds energy limit of plate in the form of earthquake. Loading rate in stress release model describes stress accumulation caused by large-scale tectonic plate movement. It is assumed that initial stress $X(0)$ is positive and loading rate is a constant and positive. Stress will increase linearly with loading rate in time period t with initial stress $X(0)$, and therefore the following equation is obtained,

$$X(t) = X(0) + \rho t \quad (1)$$

Let S_i is a stress released at event i with $i = 1, 2, \dots, n$, and n is the number of events within a certain time interval $(0, t)$. The value of stress S_i released during an earthquake is estimated from magnitude. There is a correlation between magnitude and stress, that is, magnitude M is proportional to logarithm of seismic energy released during earthquake occurrence E , and the relationship can be stated by

$$M = \frac{2}{3} \log E + c \quad (2)$$

where c is a constant. With assumption that the decline of energy S_i during earthquake is proportional to square root of released energy, we obtain

$$S_i \sim E^{\frac{1}{2}} \quad (3)$$

Based on equations (2) and (3), we have

$$S_i = 10^{0.75} \quad (4)$$

If $M = m_i - m_0$, where m_i is magnitude when earthquake i occurs and M_0 is magnitude lower bound, equation (4) becomes

$$S_i = 10^{0.75(m_i - m_0)}$$

Let t_i and S_i are time and stress released which relate to earthquake i respectively, accumulation of released stress in each earthquake during period $(0, t)$ can be stated by

$$S(t) = \sum_{i; t_i} S_i$$

An important variable in stress release model is the level of stress in a certain area, which controls probability of earthquake occurrence. Stress level $X(t)$, increases deterministically and declines stochastically due to earthquake. From equation (1), we obtain

$$X(t) = X(0) + \rho t - S(t) \quad (5)$$

Great earthquake is commonly followed by great earthquake taking place in surrounding area and this occurrence resists aftershocks. Interaction among areas can influence the time and magnitude of earthquake occurrence. Zheng and Vere-Jones [8] recorded the presence of stress movement and interaction among areas, which have not been taken into consideration in stress release model. Thus, the model can be developed with the factor of interaction among areas (Bebbington and Harte [5]).

Let $X_i(t)$, is stress in certain region within time t . We assume that accumulated released stress during earthquake occurrence in j area within period $(0, t)$ is stated as $S(t; j)$ and stress initiated in region which later moves to region has a fixed proportion that is positive or negative notated by θ_{ij} . From equation (5), the following equation is obtained:

$$X_i(t) = X_i(0) + \rho_i(t) \sum_j \theta_{ij} S(t, j) \quad (6)$$

Equation (6) is called linked stress release model. If aftershock is ignored, $\theta_{ii} = 1$ for all i , it becomes equation (5).

3. Conditional Intensity Function

Intensity of an earthquake occurrence is determined through hazard function $\psi(c)$, that is probability of an occurrence within certain time interval $(t, t + \Delta)$ is close to $\psi[X(t, t)]\Delta$ for a quite small Δ . Hazard function is an exponential function, $\psi(x) = \exp(\alpha + \beta x)$, with constant α is initial stress and constant is combination between energy and heterogeneity of crust in the region.

Each region has exponential risk function with different parameters stating that tectonic in each area has various characteristic. Stress energy causing earthquake and tectonic loading rate are various depending on each seismic region. Hence, conditional intensity for region i can be formed from data of previous earthquakes, that is

$$\lambda_i(t) = \psi(X_i(t))$$

from equation (6) we obtain

$$\lambda_i(t) = \exp(\alpha_i + \beta_i X_i(0)) + \beta_i \rho_i(t) - \frac{\sum_j \theta_{ij} S(t, j)}{\rho_i}$$

If $\alpha_i \beta_i X_i(0) = \alpha_i \cdot \beta_i \rho_i = b_i$, and $\frac{\theta_{ij}}{\rho_i} = c_{ij}$

$$\lambda_i(t) = \exp(\alpha_i + \beta_i(t - c_{ij} S(t, j))) \quad (7)$$

Parameters of conditional intensity function (7) are estimated using maximum likelihood estimate method by calculating loglikelihood function:

$$\log L = \sum_{i=1}^N \log \lambda_i(t) - \sum_{i=1}^N \int_{T_1}^{T_2} \lambda_i(t) dt$$

With time interval (T_1, T_2) consisting of $t_i (i = 1, 2, \dots, N)$, $T_1 < t_1 < t_2 < \dots < t_N < T_2$.

4. Application of Linked Stress Release Model

Indonesia occupies one of the most active tectonic zones in the world related to the convergence of three great tectonic plates, forming complex plate convergence routes (Bird [7]). Those three great tectonic plates are Indo-Australia plate, Eurasia plate, and Pacific plate. On the meeting of the plates, crashing energy accumulates until the earth layer can no longer defend the accumulation, and it is later released into earthquake. Subduction between two plates causes subduction between Indo-Australia and Eurasia plates and the formation of ring of fire of Bukit Barisan in Sumatra Island, along Java Island, and Parit Jawa (Sunda). The condition makes both Java Island and Sumatra Island more potentials to earthquake occurrence than any other areas. This section describes application of linked stress release model on data of earthquakes occurring in Java and Sumatra.

We use secondary data obtained from the United States Geological Survey. The data contain t_i , m_i , and areas i, j with t_i stating the time when earthquake i occurs and m_i stating the magnitude of earthquake i . The time range of earthquake occurrence is January 2010 to December 2016 with the minimum magnitude 4.5.

Data Analysis on Java Earthquake

In analyzing data, we divide Java Island into two regions: north and south. North region is located at 6 – 8 South Latitude and 105 – 114 West Longitude, while south region is located at 8 – 10 South Latitude and 105 – 114 West Longitude covering south sea of Java Island. In Java Island, there are

215 earthquake occurrences. Table 1 presents all parameter estimates of conditional intensity function of linked stress release model for earthquakes occurred in Java.

Tabel 1. Parameter estimate of conditional intensity function for Java

earthquake		
Region	Parameter	Estimate
North	a_1	-1.56989102
	b_2	0.01174827
	c_{11}	2.06251188
	c_{12}	-0.55649740
South	a_2	-1.49448515
	b_2	0.00989052
	c_{21}	0.36208426
	c_{22}	0.86738167

Based on conditional intensity function of linked stress release model on equation (7) and parameter estimation in Table 1, conditional intensity functions of linked stress release model on data of earthquakes occurring in the northern and the southern respectively are

$$\lambda_1(t) = \exp(-1.5676 + 0.0117(t - (2.0596 - 0.5538)S(t))) \quad (8)$$

$$\lambda_2(t) = \exp(-1.4945 + 0.0099(t - (0.3678 + 0.8610)S(t))) \quad (9)$$

Magnitude data of earthquakes in Java presented in Figure 1 and conditional intensity function of stress release model of data on earthquakes in Java displayed in Figure 2. There are four earthquakes with the largest magnitude in Java Island; located in them South Sea in 25 January 2014 with the magnitude of 6.3, 4 June 2012 with the magnitude of 6.2, 28 November 2015 with the magnitude of 5.9, and in Sunda Strait 14 April 2012 with the magnitude of 6.2. Based on Figure 2, it can be seen that the intensity in south region was relatively higher than the north region. However, in the end of 2016 the intensity in south region quite declined compared to the north region.

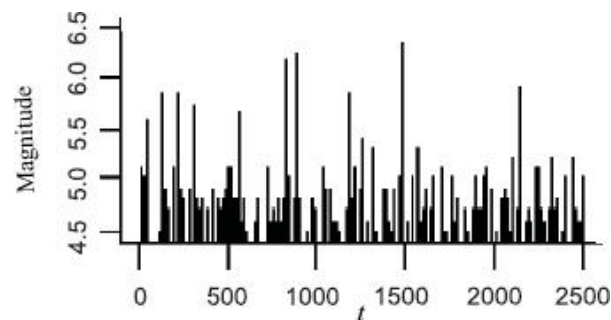


Figure 1. Magnitude data of Java earthquake

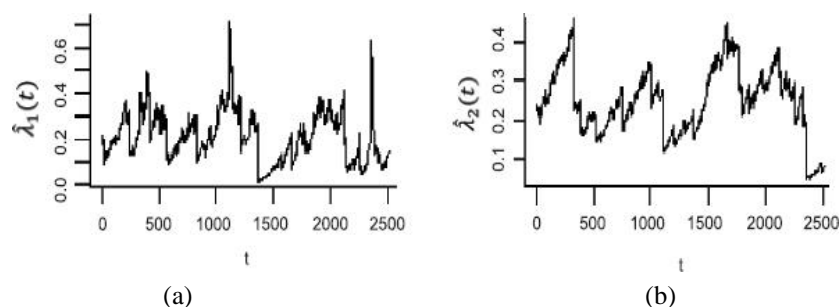


Figure 2. Conditional intensity function for northern Java (a) and southern Java (b)

Data Analysis on Sumatra Earthquake

We divide Sumatra Island data into two regions: north and south. The north region is located at 6 – 0 North Latitude and 95 – 109 West Longitude covering Banda Aceh, Nias, Simeulue, North Sumatra, Sinabang, Padangsidempuan, and Meulaboh. Meanwhile, the south region is located at 0 – 6 South Latitude and 95 – 109 West Longitude covering Mentawai Islands, Bengkulu, Kuripan, Sungaipenuh, Sigli, Muara Siberut, Pasarbaru, Pagaram, Kaliandak and Sikabaulan. There are 938 occurrences of earthquake in Sumatra Island. All parameter estimates of the conditional intensity function for Sumatra Island are shown in Table 2.

Table 2. Parameter estimate of conditional intensity function for Sumatra Earthquake

Region	Parameter	Estimate
North	a_1	-1.498636119
	b_2	0.00952085
	c_{11}	0.367809141
	c_{12}	0.861021432
South	a_1	-1.567641189
	b_2	0.011768394
	c_{21}	2.059594285
	c_{22}	-0.553780067

Based on equation (7) and Table 2, we obtain conditional intensity functions for north and south regions respectively:

$$\lambda_3(t) = \exp(-1.4986 + 0.0095(t - (0.3678 + 0.8610)S(t))) \quad (10)$$

$$\lambda_1(t) = \exp(-1.5676 + 0.0118(t - (2.0595 - 0.5538)S(t))) \quad (11)$$

Figure 3 shows magnitude of earthquake data in Sumatra and Figure 4 shows the conditional intensity for earthquake data in Sumatra. There are five earthquakes with the largest magnitude; in North Sumatra in 6 April 2010 with the magnitude of 7.4, 9 May 2010 with the magnitude of 7.0, Mentawai Islands in 25 October 2010 with the magnitude of 7.4, 5 March 2010 with the magnitude of 6.8, and Sumatra region in 5 May 2010 with the magnitude of 6.6. Based on Figure 4, it can be seen that the intensity in north region was relatively higher than the one in south region. Like in Java Island, in the end of 2016 the intensity in north region quite declined if compared to the one in south region.

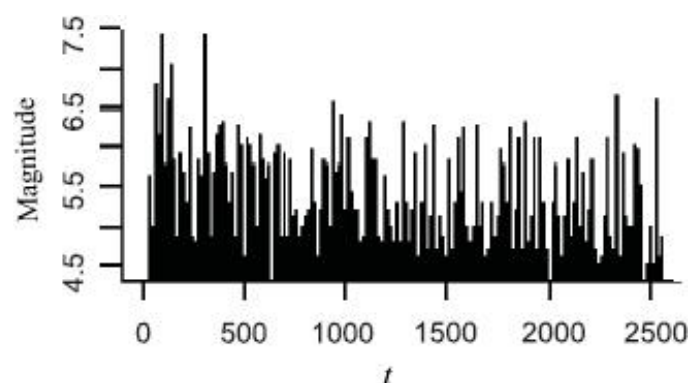


Figure 3. Magnitude data of Sumatra earthquake

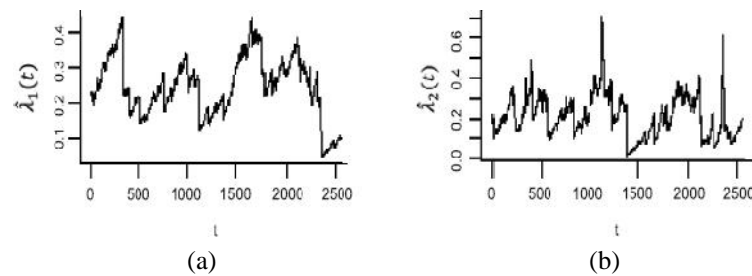


Figure 4. Conditional intensity function for northern Sumatra (a) and southern Sumatra (b)

Interaction among Areas

Based on parameter estimation in Table 1, Table 2, and conditional intensity function on earthquake data in Java and Sumatra, we can see that the intensity of earthquake in southern Java is similar to the intensity in northern Sumatra (equation (9) and equation (10)) and the intensity in northern Java is similar to the one in southern Sumatra (equation (8) and equation (11)). Table 1 shows that C_{11} is greater than C_{12} and C_{22} is greater than C_{21} . It means that most of proportion of stress in northern (and also southern) Java is derived from stress in its own region. Table 2 shows that C_{12} is greater than C_{11} , it means that most of proportion of stress in northern Sumatra is derived from the stress in the southern Sumatra. We can also see that C_{21} is greater than C_{22} , it means that most of proportion of stress in southern Sumatra is derived from the one in the north.

5. Conclusion

Linked stress release model is a development of stress release model by considering the interaction among areas. Conditional intensity function of the model can be presented by hazard function of stress movement and interaction among areas. In application to earthquake data in Java Island and Sumatra Island we see similarity of intensity between southern Java and northern Sumatra. Earthquake intensity of these regions are greater than northern Java and southern Sumatra. Most proportion of stress in northern (and also southern) Java is derived from stress in its own region. In Sumatra, most proportion of stress in northern (and also southern) Java is derived from stress in other region.

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References

- [1] Lu C, Harte D, and Bebbington M 1999 *A Planets Space* 51 907-916.
- [2] Vere-Jones D 1995 *J. Statist* 11 503-538.
- [3] Schoenberg F P 2003 *Journal of the American Statistical Association* 98 no. 464.
- [4] Pratiwi H, Slamet I, Saputro D R S, and Respatiwan 2017 *Journal of Physics: Conf. Series* 855 012033.
- [5] Bebbington M and Harte D 2001 *J. Appl. Probab.* 38A 176-187.
- [6] Imoto M 2001 *Tectonophysics* 338 (3) 287-295.
- [7] Bird P 2003 *Geochem Geophys Geosyst* 4 (3) 1-52.
- [8] Zheng X and Vere-Jones D 1994 *Tectonophysics* 2 101-121.