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To cite this article: Kusno *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **243** 012017

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On the modeling of the object surface reliefs of marble handicrafts using quartic curves and circles

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Abstract. This paper deals with the construction of the various reliefs of marble handicrafts. The purpose of this discussion is to obtain some curve formulas and procedures for designing the reliefs of the marble surface. The method is as follows, we present the quartic curve of the algebraic form into the Hermit model and we simulate some procedures to construct the reliefs that are generated by the space curves and the circles. Then, making some prototypes of the marble object reliefs are introduced. The results found one formula of geometric quartic curve with its boundary conditions of Hermit type, some procedures and formulas to evaluate and model the reliefs of marble surfaces, and the prototypes of the marble handicrafts.

1. Introduction

This paper deals with the construction of the various reliefs of marble handicrafts. The purpose of this discussion is to obtain some curve formulas and procedures for designing the reliefs of the marble surface. The method is as follows, we present the quartic curve of the algebraic form into the Hermit model and we simulate some procedures to construct the reliefs that are generated by the space curves and the circles. Then, making some prototypes of the marble object reliefs are introduced. The results found one formula of geometric quartic curve with its boundary conditions of Hermit type, some procedures and formulas to evaluate and model the reliefs of marble surfaces, and the prototypes of the marble handicrafts.

Some techniques relating to the reliefs surface modeling have been introduced by some authors. We can design a china vase surface relief by using approximation of variable radius-offset curves [7]. The marble surfaces and the objects image can be defined respectively by some surfaces of revolution and projections of spheres and torus [2,4,8]. Furthermore, it can be formed by free form deformation of object surfaces shape [3] or by connecting between some Bezier patches [6,9]. Using the method of smoothing curve can model and generate the relief surface in direction of revolution axes [1]. Different from the presented methods, we are interested in using the parametric quartic curve and the circle for modeling the marble object surfaces.

This paper is organized in the following steps. In the first step we will evaluate a Hermit quartic curve formula to model the marble surface reliefs. In the second step we will present the methods for constructing some symmetrical marble object reliefs. In the third step, the simulation of some marble objects relief and creating a new prototypes are shown. Finally, the results will be summarized in the conclusion section.



2. Research method

This research uses the following methods: (a) evaluating the equation systems which are derived by the boundary condition of quartic curve Hermit model; (b) analyzing some procedures to simulate the obtained quartic curve profiles and making a program to construct the reliefs that are generated by the space curves; (c) designing the marble object reliefs and the prototypes.

3. Formulation of the quartic curve to design the surface reliefs of the marble objects

This section discusses the formulation of geometric quartic curve that is defined by three data vector points and two selected direction vectors. We propose the problem as follows. Suppose the quartic parametric curve in the algebra form

$$P(u) = a_4 u^4 + a_3 u^3 + a_2 u^2 + a_1 u + a_0 \quad (1)$$

with $0 \leq u \leq 1$. The first derivation $P'(u)$ of the variable u is

$$P'(u) = 4 a_4 u^3 + 3 a_3 u^2 + 2 a_2 u + a_1 \quad (2)$$

From equation (2), we give the boundary conditions for some points and vectors in space (the same as the condition case of cubic Hermite curve) $A = P(0) = P_0 = a_0$; $B = P(1) = P_1 = a_4 + a_3 + a_2 + a_1 + a_0$ with the determined tangent vectors $P'(0) = P_0' = a_1$ for the direction vector of the curve at control point A and $P'(1) = P_1' = 4 a_4 + 3 a_3 + 2 a_2 + a_1$ for the direction vector of the curve at control point B . Along the curve between the end control points A and B , we determine an additional condition namely a free control point X for modifying the curve form $P(u)$ in the following form $X = P(x) = P_x = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$ with $0 < x < 1$. Based on these data, the problem is how to get a new curve which has the following properties:

- the starting point, the end point of the curve and its direction tangent vectors at these points must be through at control point A and B , respectively;
- the curve $P(u)$ must be through at control point X and its shape will change according to the new position of the control point X (Figure 1).

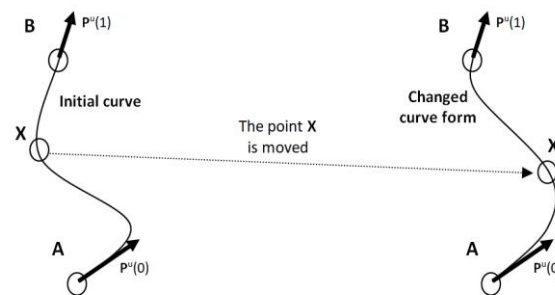


Figure 1: Draw a curve with data of two control points, two vectors and one moved point.

The step of evaluation of the curve which is defined by the boundary conditions, is described as follows. According to the data of the boundary conditions, it means that

$$P_0 = a_0 \quad (3)$$

$$P_x = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$= a_4 x^4 + a_3 x^3 + a_2 x^2 + P_0' x + P_0 \quad (4)$$

$$P_1 = a_4 + a_3 + a_2 + a_1 + a_0 = a_4 + a_3 + a_2 + P_0' + P_0 \quad (5)$$

$$P_0' = a_1 \quad (6)$$

$$P_1' = 4a_4 + 3a_3 + 2a_2 + a_1 + a_0 = 4a_4 + 3a_3 + 2a_2 + P_0' \quad (7)$$

Therefore, the coefficients $a_0 + a_1 + a_2 + a_3$ and a_4 can be written in the following forms

$$\begin{aligned}
\mathbf{a}_0 &= \mathbf{P}_0; \\
\mathbf{a}_1 &= \mathbf{P}_0^u; \\
\mathbf{a}_2 &= (b-3)\mathbf{P}_0 + c\mathbf{P}_x + (d+3)\mathbf{P}_1 + (e-2)\mathbf{P}_0^u + (f-1)\mathbf{P}_1^u; \\
\mathbf{a}_3 &= (2-2b)\mathbf{P}_0 - 2c\mathbf{P}_x - (2+2d)\mathbf{P}_1 + (1-2e)\mathbf{P}_0^u + (1-2f)\mathbf{P}_1^u; \\
\mathbf{a}_4 &= b\mathbf{P}_0 + c\mathbf{P}_x + d\mathbf{P}_1 + e\mathbf{P}_0^u + f\mathbf{P}_1^u
\end{aligned}$$

Where

$$\begin{aligned}
b &= -(2x^3 - 3x^2 + 1)/(x^4 - 2x^3 + x^2); \\
c &= 1/(x^4 - 2x^3 + x^2); \\
d &= (2x^3 - 3x^2)/(x^4 - 2x^3 + x^2); \\
e &= -(x^3 - 2x^2 + x)/(x^4 - 2x^3 + x^2); \\
f &= (-x^3 + x^2)/(x^4 - 2x^3 + x^2).
\end{aligned}$$

Thus the curve of equation (1) can be noted in the form

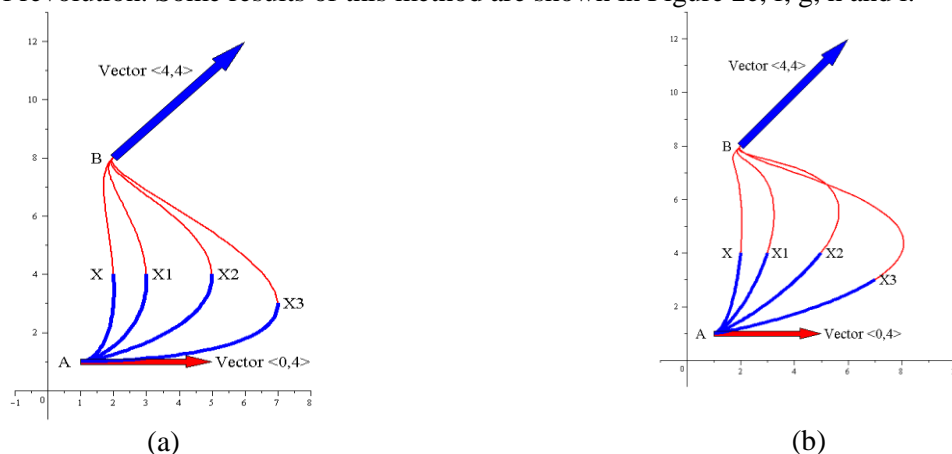
$$\begin{aligned}
\mathbf{P}(u) &= \mathbf{a}_4 u^4 + \mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 \\
&= \mathbf{P}_0 [b u^4 + (2-2b)u^3 + (b-3)u^2 + 1] + \\
&\quad \mathbf{P}_x [c u^4 - 2c u^3 + c u^2] + \\
&\quad \mathbf{P}_1 [d u^4 - (2+2d)u^3 + (d+3)u^2] + \\
&\quad \mathbf{P}_0^u [e u^4 + (1-2e)u^3 + (e-2)u^2 + u] + \\
&\quad \mathbf{P}_1^u [f u^4 + (1-2f)u^3 + (f-1)u^2].
\end{aligned} \tag{8a}$$

In general, we have equation

$$\begin{aligned}
\mathbf{P}(u) &= \mathbf{P}_0 [b u^4 + (2-2b)u^3 + (b-3)u^2 + 1] + \\
&\quad \mathbf{P}_x [c u^4 - 2c u^3 + c u^2] + \\
&\quad \mathbf{P}_1 [d u^4 - (2+2d)u^3 + (d+3)u^2] + \\
&\quad \alpha \cdot \mathbf{P}_0^u [e u^4 + (1-2e)u^3 + (e-2)u^2 + u] + \\
&\quad \beta \cdot \mathbf{P}_1^u [f u^4 + (1-2f)u^3 + (f-1)u^2].
\end{aligned} \tag{8b}$$

with α and β as fixed real scalar.

As a validation of equation (8a), let the value x be determined at $x = 0.5$. Then we will find the profile of curves graph that are defined by data $\mathbf{P}_0 = \langle 1, 1 \rangle$; $\mathbf{X} = \langle 2, 4 \rangle$; $\mathbf{P}_1 = \langle 2, 7 \rangle$; $\mathbf{P}_0^u = \langle 0, 4 \rangle$; $\mathbf{P}_1^u = \langle 4, 4 \rangle$ as shown in Figure 2a. On the other hand, if we change the position of the data $\mathbf{X} = \langle 2, 4 \rangle$ become $\mathbf{X}_1 = \langle 3, 4 \rangle$; $\mathbf{X}_2 = \langle 5, 4 \rangle$; $\mathbf{X}_3 = \langle 7, 3 \rangle$, then we will find the new curve graphs that are shown respectively in the second, third, and fourth curve of Figure 2a. In addition, when we change the value of the parameter x to be $x = 0.35, x = 0.75$ then we will obtain respectively the new shape graph as they are shown in Figure 2b and Figure 2c. Furthermore, if the tangent vectors are modified in different directions $\mathbf{P}_0^u = \langle 0, 8 \rangle$; $\mathbf{P}_1^u = \langle -7, 0 \rangle$, then we will obtain the new shape graph as they are shown in Figure 2d. So, to realize the various relief forms of marble vases in vertical direction, we can rotate the curves connection that are defined by equation (8) to the axes of revolution. Some results of this method are shown in Figure 2e, f, g, h and i.



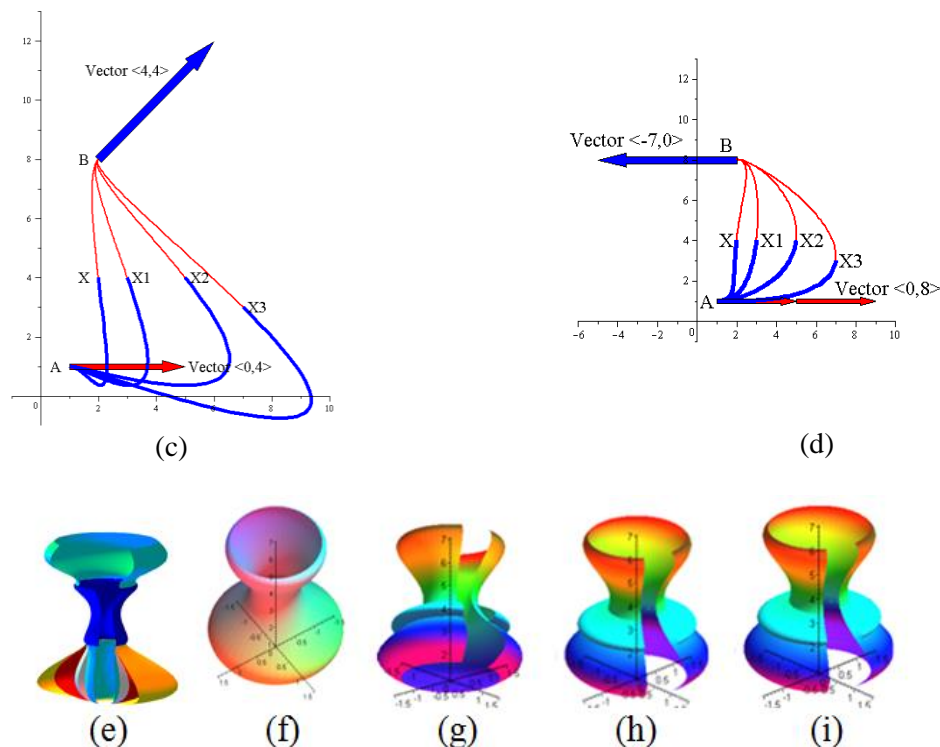


Figure 2: Some quartic curves (2a,b,c,d) and surface reliefs design (2e,f,g,h,i)

4. Construction of symmetrical reliefs with circles

In this section, we will construct the relief of the objects that are symmetrical to the axis of revolution. Then, we introduce the reliefs of the objects that are symmetrical center. The initial data to be used of the method is a circle. In the first step, we draft the reliefs shapes in two dimensions and the second step, we transform it to the reliefs of three dimensions.

4.1. Relief of the objects built from the axis symmetry

To build the object reliefs with its axis symmetry in vertical direction, can be made by using the quartic curve in the following ways.

- **Curves rotation technique**

1. In the meridian plane, determine an arbitrary (vertical) line g as an axis and set 3 points as a data point of curve that are a point between the end points of the curve and two end points of curve. Then determine 2 tangent vectors at the both end points of the curve to construct the quartic curve of equation (8);

2. Rotate the curve of step result (1) to the axe line \mathbf{g} to obtain the reliefs shape of the surface:

$$\mathbf{S}(u, v) = \langle P_x(u)\cos v, P_x(u)\sin v, P_z(u) \rangle \text{ with } 0 \leq u \leq 1 \text{ and } 0 \leq v \leq 2\pi.$$

in this case, if the order of three data points are positioned in the vertical direction and parallel to the axis of the rotation (Figure 2a), then we will obtain the reliefs form in the horizontal direction (Figure 2b), if not, the reliefs will tend in the vertical direction (Figure 2c).

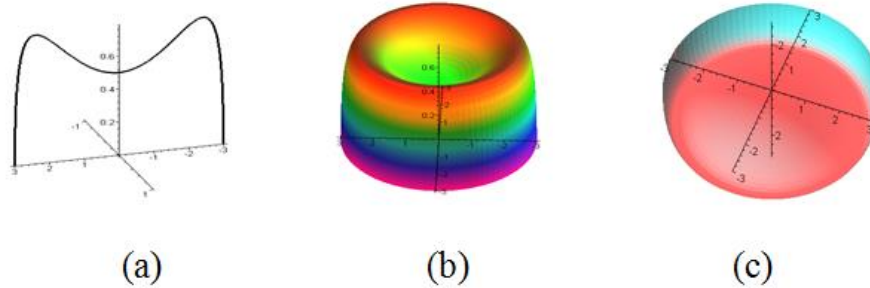


Figure 3. Designing reliefs using the revolution of quartic curves

• Curve shifting method

1. Determine an arbitrary line (vertical) \mathbf{g} as an axis of shifting.
2. Construct the symmetric quartic curve $\mathbf{P}(u)$ of equation (8) using two data initial points and one intermediate point symmetrically to the line \mathbf{g} and each tangent vector at the end points is equal each other but in the opposite direction.

3. To obtain the surface reliefs, move the result curve of step(2) using the formula:

$$\mathbf{S}(u, v) = \mathbf{P}(u) + v \mathbf{a} \text{ with } 0 \leq u \leq 1, v \text{ real scalar and } \mathbf{a} \text{ constant vector parallel to line } \mathbf{g}$$

(Figure 4).

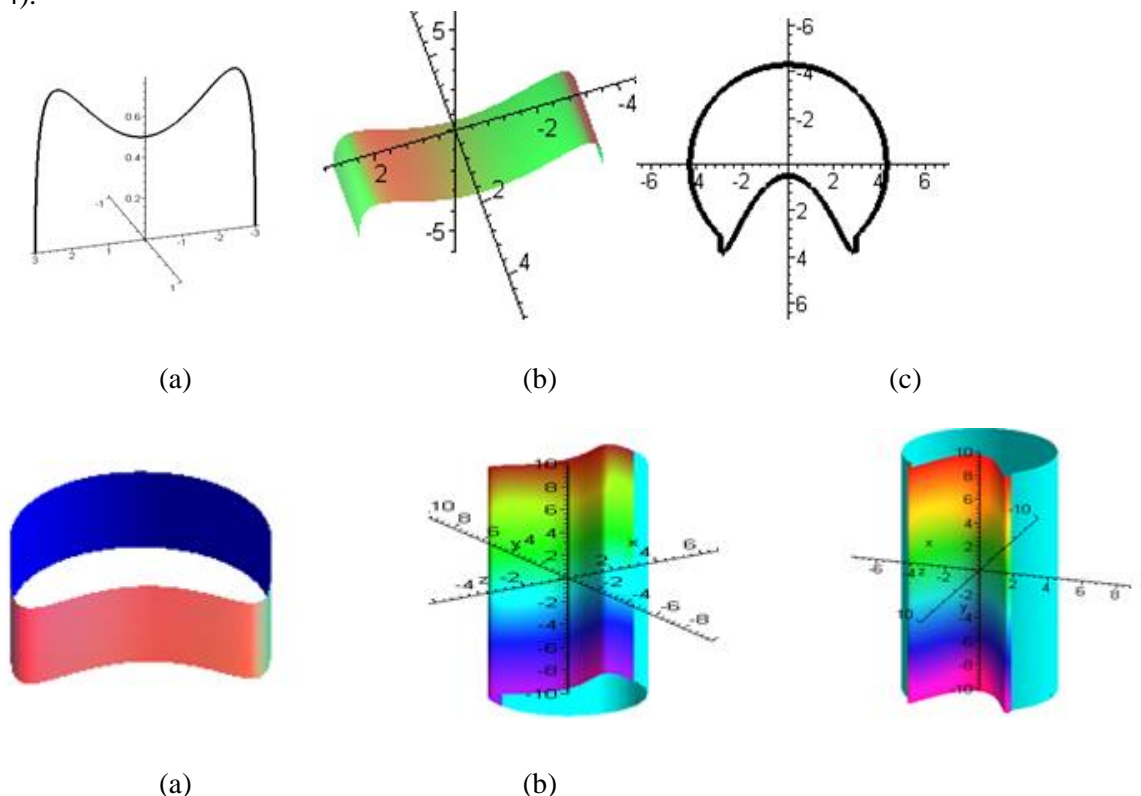


Figure 4. Reliefs design through shifting of curve

4.2. Object reliefs built from the center symmetry

To obtain an object of central symmetry can be effectively undertaken by rotating or reflecting each point towards the gravity point of the object. Base on the geometric properties and the application reasons, we discuss about the construction of object relief using a circle.

- **Construction of tangent circles on a circle**

Data : Circle of radius R in the form $\mathbf{L} = \langle R \cos u + P_x, R \sin u + P_y, P_z \rangle$ with $0 \leq u \leq 2\pi$.

Problem : Design n regular vertical circles with its center at the circle \mathbf{L} (Figure 5a).

Calculation:

1. Set a unit tangent vector on a circle \mathbf{L} in the form $\mathbf{e}_1 = (-\sin u)\mathbf{i} + (\cos u)\mathbf{j} = \langle -\sin u, \cos u, 0 \rangle$ with $0 \leq u \leq 2\pi$.
2. Determine the unit normal vector of circle on plane \mathbf{L} , that is $\mathbf{e}_2 = \langle 0, 0, 1 \rangle$.
3. Determine respectively n point $[P_1, P_2, P_3, \dots, P_n]$ as the center of tangential circles which are located on the circle \mathbf{L} of radius R in the form $\overrightarrow{OP_i} = \overrightarrow{OP} + \langle R \cos(k), R \sin(k), 0 \rangle$ with $i = 1, 2, 3, \dots, n$ and $k = (i/n) \times 6.3$ radian.
4. Design n tangential circles of radius r in the form of curves $\mathbf{L}_i(u) = \overrightarrow{OP} + \langle R \cos(k) + r \cos u (-\sin k), R \sin(k) + r \cos u (\cos k), r \sin u \rangle$ with $i = 1, 2, 3, \dots, n$. If $i = 1, 2, 3, \dots, 5$, we have $k = (i/5) \times 6.3$ radian and $0 \leq u \leq 2\pi$ (Figure 5b).

- **Construction of tangent cones on a cylinder**

Data : Circle $\mathbf{L} = \langle R \cos u + P_x, R \sin u + P_y, P_z \rangle$ with $0 \leq u \leq 2\pi$.

Problem : Design n cones touch a cylinder \mathbf{S} on circle \mathbf{L} .

Calculation:

1. Design a cylinder through circle \mathbf{L} in the form $\mathbf{S}_L(u, v) = \langle R \cos u + P_x, R \sin u + P_y, P_z + u \rangle$.
2. Compute the tangent circle \mathbf{S}_L as the base circle of tangent cone (Figure 6b);
3. Compute the direction vector of the summit cone in the form $\mathbf{n}_k = \lambda(\mathbf{e}_1 \wedge \mathbf{e}_2) = \langle \lambda \cos(k), \lambda \sin(k), 0 \rangle$.
4. Formulate the cone summit point at PK in the form $\overrightarrow{OP_k} = \langle R \cos u + \lambda \cos(k), R \sin u + \lambda \sin(k), P_z \rangle$.
5. Construct n tangent cones of cylinder $\mathbf{K}_1, \mathbf{K}_2, \mathbf{K}_3, \dots, \mathbf{K}_n$ of radius r respectively in the form

$$\mathbf{K}_i(u, v) = \langle R \cos(k) + (\lambda \cos(k))u + (\cos(k) + r \cos v (-\sin(k)))(1 - u), \\ R \sin(k) + (\lambda \sin(k))u + (\sin(k) + r \cos v \cdot \sin(k))(1 - u), \\ P_z \cdot u + (P_z + r \sin(v))(1 - u) \rangle$$

with $i = 1, 2, 3, \dots, n$. If $n = 5$, we have $k = (i/5) \times 6.3$ radian, $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$ (Figure 6c).

- **Construction of tangent pyramids on cylinder**

Data : Circle $\mathbf{L} = \langle R \cos u + P_x, R \sin u + P_y, P_z \rangle$

Problem : Design n pyramids touch a cylinder \mathbf{S} on circle \mathbf{L} .

Calculation:

1. Design the cylinder through circle \mathbf{L} .
2. Construct n regular polygons of the side n_1 as a base boundary of the pyramid in circle equation (19);
3. Formulate n summit points of tangent pyramids \mathbf{L}_s in equation (22);

4. Construct the tangent pyramids through the linear interpolation between each side of the base polygon to the summit points of the pyramid (Figure 6d).

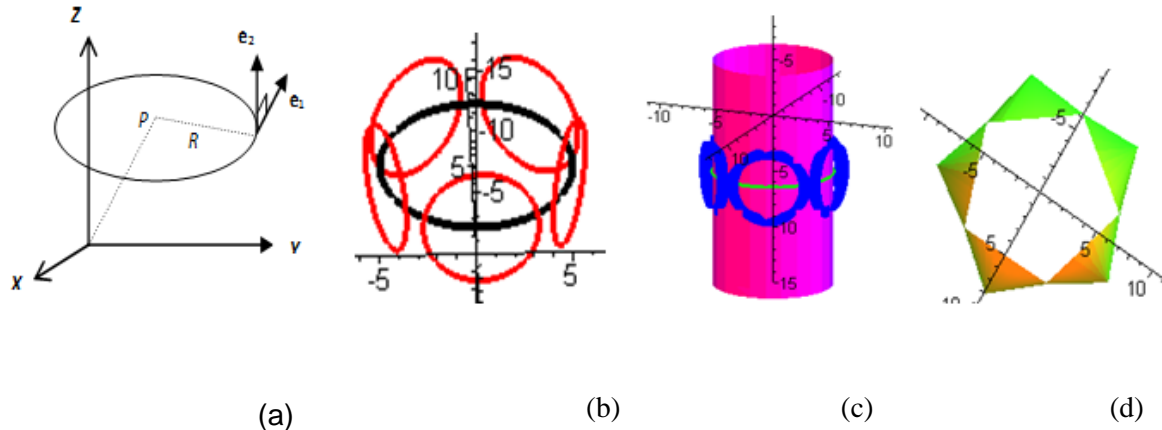


Figure 5. Tangent circles on circle (5a,b) and tangent cones/pyramids on cylinder (5c,d)

- **Construction of tangent cones on sphere**

Data : Sphere $\mathbf{B}(u, v) = \langle R \cdot \sin v \cdot \cos u + P_x, R \cdot \sin v \cdot \sin u + P_y, R \cdot \cos v + P_z \rangle$ with $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$.

Problem : Design n cones in horizontal direction with its base circles touch the sphere \mathbf{B} .
Calculation:

To design the tangent cones on sphere can be applied the construction method of tangent cones on cylinder. Especially for joining the cones on sphere, after determining a great circle \mathbf{L} on the sphere, first, we have to construct a regular polygon of n sides which its vertices lie on \mathbf{L} . Second, we build a concentric parallel circles in which its center lies respectively on the midpoints of the polygon sides and its radius r must be perpendicular to the circle center \mathbf{L} . After that, we calculate the position of the peak point of cone and then we build a conical shape along the circle \mathbf{L} (Figure 6).

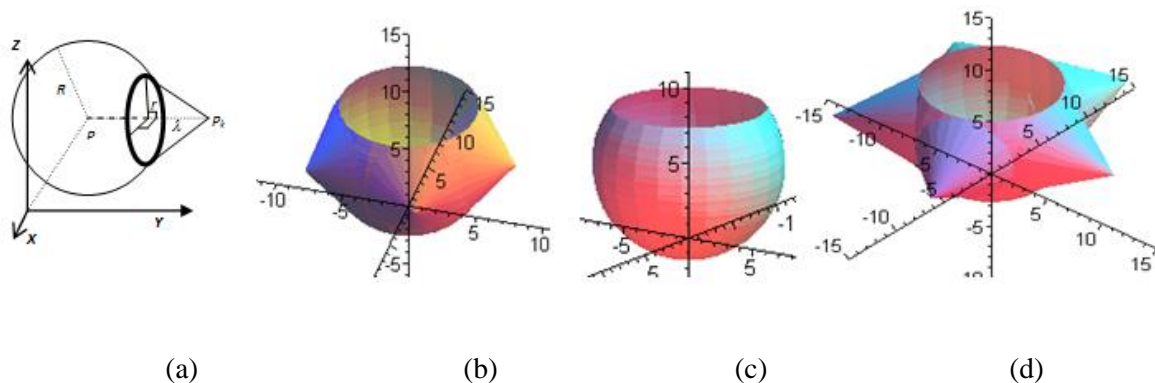


Figure 6. Tangent cones on sphere

5. Simulation and prototype design

We have introduced some formulations to design the different reliefs shape of the basic objects surface. In addition, it is supported by some parameters that can change the data points position, the vector values and their directions, size of the rotation radius, the angle of rotation or a parameter to modify the altitude. Based on the present vas models of the marketed marble (Figure 7), in Figure 8, we introduce some new models that are designed by these three steps. First, we identify some needed object components to be modeled. Second, we create many reliefs/profiles using the curve formula in the meridian plane and we rotate it to the axis of revolution. Finally, we join those components of the

objects to obtain a new whole objects in the form of marble handycrafts. When creating the reliefs, we can use: (a) quartic curve equation (8), (b) rotation or reflection operation of the curve to the symmetry axis of the objects and, (c) evaluation parameters of the curve/object. Some examples of fabrication results are shown in Figure 9.



Resource: <http://indonetwork.co.id/mailto.html> [5]

Figure 7. Some examples of the marketed marble objects

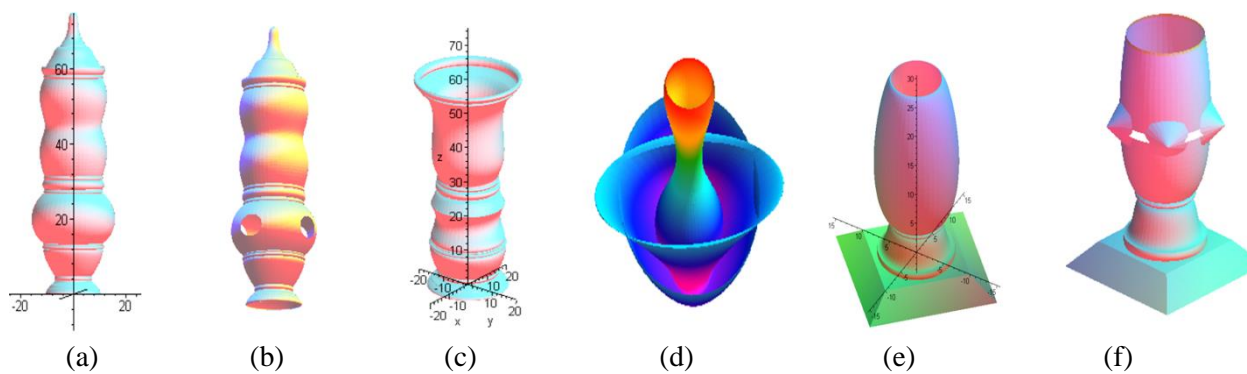
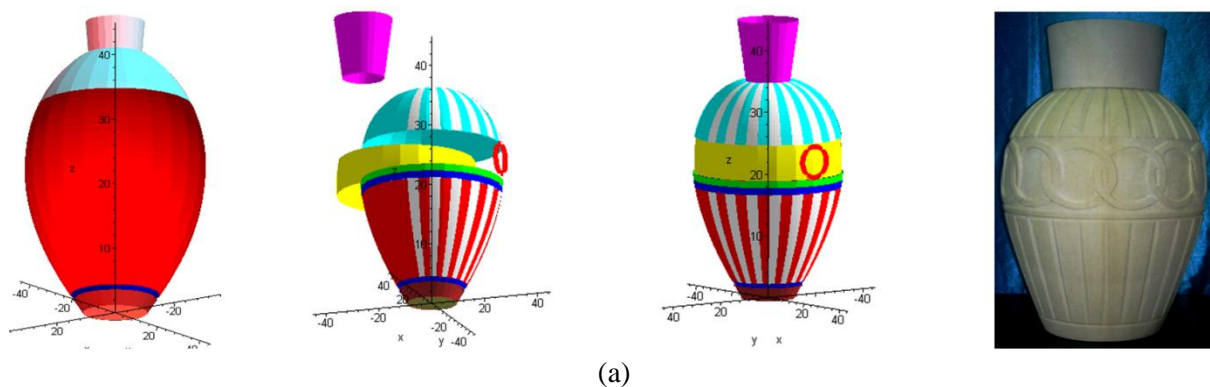
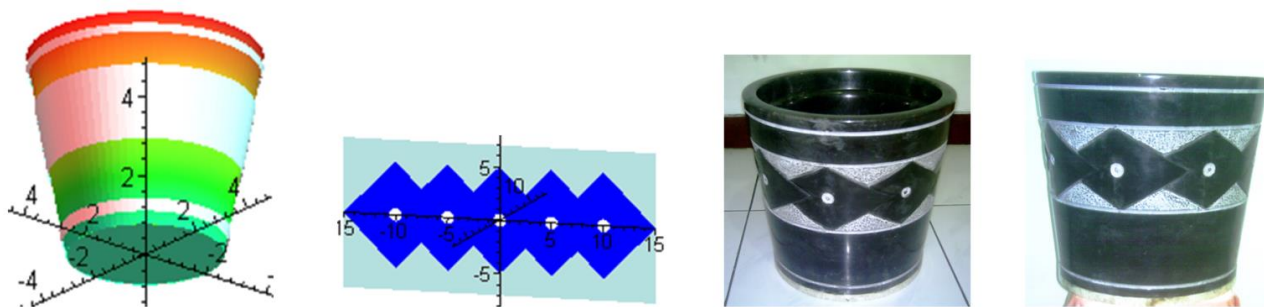


Figure 8. Prototype examples for the marble objects





(b)

Figure 9. Fabrication examples of the marble objects relief

6. Conclusion

The results of the discussion can be summarized as follows:

- Reliefs of marble objects can be designed by using quartic curve with the data: a starting point of the curve, one point between the starting and the end point of the curve, the end point of the curve and two direction vectors at the starting point and the end point of the curve.
- The reliefs of vas marble can be built in the following steps: first, construct the basic shapes of object that will be modeled. Second, add the components of the basic objects with some pieces of the other objects/surfaces by using a connection method. Third, design the surface reliefs of the objects through the equation (8) and some procedures which is introduced in section 4.

We can design the marble handicrafts by using the quartic curves and the circles. The interesting thing to discuss ahead is how to apply the curves if they are defined by some different formula types of high degrees.

7. Acknowledgement

We would like to thank and appreciate the officials Mathematics' Department of Jember University which provided us this research.

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