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To cite this article: J S Li 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **242** 062009

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Experience in study of constitutive relationship of unsaturated soils

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Abstract. Unsaturated soils refer to soils in which the pores are not fully filled with water, and its mechanical properties are obviously different from those of saturated soil. The constitutive relationship of unsaturated soil is vitally important to understand the properties of unsaturated soil. This paper introduces the problems encountered in process of studying and understanding the unsaturated soil mechanics, with emphases on the stress variables of unsaturated soils and the importance of observation of the basic concepts and principles in soil mechanics. It is meant to enlighten the beginners to understand the unsaturated soil mechanics.

1. Knowing about constitutive relationship

The author has known about *constitutive relationship* since he was a graduate student. It started with studying *Constitutive Equations for Engineering Materials*^[1], from the basic tensor, strength criterion, loading principle, performing derivation of the partial differential equation of the elastoplastic constitutive relationship over and over again. After some concrete experimental data were obtained, the author tried to modify and study the characteristics of yield surface and plastic potential equation in order to build a new constitutive relationship model. Although the model is very simple in simulation of some concrete experimental phenomena, the author felt proud of it. The programming language FORTRAN, seemed very simple, but it really took the author much time to use this language skillfully.

At that time, the postgraduate courses had already provided such courses as elastoplastic mechanics. Although the author just passed the exam, the author felt enthusiastic for this course. Remember that there were assignments required to conduct several sets of triaxial tests to verify the Modified-Cambridge model. There were more than a dozen students but only few were present in laboratory, and I worked hard from beginning to end. Through further studying the literatures, I knew some big people in the field of unsaturated soil mechanics, such as E.E Alonso, D.G. Fredlund and C.W.W. Ng. Alonso had proposed the famous BBM model for unsaturated soils^[2]. Fredlund had devoted himself to the application of stress variables in unsaturated soils and the practicality of unsaturated soil mechanics. His research team had put forward a large number of theoretical and practical results in unsaturated soils. Ng is one of outstanding Chinese soil mechanics experts at present. Although these achievements were known from textbooks or literature, they had a greater impact on the author. However, even though they have made many scientific achievements at present, these achievements do not completely cover all the characteristics of geotechnical engineering materials, but just only



partially affect and improve the engineering application practice. In addition, the constitutive relationship of soils seemed to have caused an upsurge at China and abroad.

2. Study of the constitutive relationship

It is no doubt that the constitutive relationship of soils has played a key role. However, it should be applied in practical engineering. Obviously the finite element method is effective and can promote the application of soil constitutive relationship models. The author still remembered the hard years of debugging programs in front of computers in order to apply Alonso model into a finite element code for simulation of some soaking characteristics of expansive soil. By programming the model, checking and compiling again and again, and selecting the appropriate processing tools for pre- and post-processing, the study process had a deep impression on the author. The biggest difficulty encountered was that there was no one who the author could have a discussion with. Due to tight time and other reasons, it was impossible to solve an extra linear equation added into the front method. Therefore the author did not complete a coupling program, which is still regrettable. Here the author should thank Owen and Hinton for providing all the source codes for elastic and elastoplastic finite element source code in their work.

At present, there are many commercial software codes about modeling rock and soil, and many ways to realize or apply soil constitutive relationships, including finite element and linear difference methods. The author studied several codes such as Flac3D, Plaxis, Geostudio, and deeply felt that these software were just a shell program though they provided various special means such as algorithm improvement and special unit processing as their own characteristics. The constitutive relation of the constitution is the key core of determining the final calculation results. However, limited to the engineers' cognitive levels, simple models such as ideal plastic models are still used for calculation. Although the unsaturated soil seepage is known widely, no distinction has been made between mechanical behaviors of saturated and unsaturated soils. On the other hand, application of advanced constitutive modes usually requires more complex experiment to calibrate model parameters.

Taking a calculation case as an example, a simple Mohr Coulomb model is used in the calculation of soil, regardless of whether the object is sand, gravel or clay. The resulted values may not be consistent with the project practice, but their development features and trends should be, which can be used as a qualitative reference. Since the soil is inherently complex and not homogeneous, it is acceptable the calculation results are not consistent in an accurate manner. Moreover, only a few of constitutive relationship models are provided in commercial software. New parameters should be determined by tests for introduction of complex models, which is not feasible for most projects.

3. Stress variables for unsaturated soils

Unsaturated soils are soils in which the pores are filled with air and water. The water cannot flow freely since it is hindered by the presence of air. The water in the pores exists in three states, i.e. bulk water and meniscus water and attached water.

Stress variables are controversial for unsaturated soils. For saturated soil, there is a definite effective stress principle. However for unsaturated soils, there is no widely accepted effective stress relationship. Nowadays, there are two kinds of stress relations for unsaturated soils, i.e. single variable and dual-stress variables. The former is represented by Bishop's stress^[3]:

$$s \phi = s - u_a + c(u_a - u_w) \quad (1)$$

where $s \phi$ is the effective stress for unsaturated soil, s is the applied total stress, u_a is the pore air stress u_w is the pore water stress and c is the parameter governing the formation of the unsaturated soils. Bishop^[3] believed that the value of c was not only mainly related to saturation, but depended on the soil structure, water absorption and dehydration cycles and the influence of saturation change. Jennings and Burland questioned Bishop's expression of effective stress and thought it could not reasonably explain the shrinkage of soil in water^[4]. They had conducted a series of consolidation tests, which show that the suction decreases during the process of water absorption and unsaturated soil

sample shrinks rather than expands in the water body, which does not conform to the changing trend of effective stress. Bishop studied the existing triaxial test shear data and suggested that both the c and stress path effects should be considered at the same time. But Burland^[5] believed that from a microscopic point of view, and they should not be combined together.

On the one hand, the development of effective stress for unsaturated soils was deeply influenced by Terzaghi's great success. As Shen^[6] and Fredlund^[7] said, single variable method was meant to extend the research results of saturated soils to unsaturated soils. In the process of exploration of stress variable for unsaturated soils, some new ideas also appeared, such as Bishop^[3] suggesting that the path of matric suction and net stress should be considered separately, which is closely related to the exploration of the constitutive relationship of unsaturated soil^[8]. Fredlund^[9] proposed the concept of independent stress state variables for unsaturated soils.

Fredlund^[8] deduced the quasi-static equilibrium relationship of the three-phase medium unsaturated soil element, and believed that any pair of the following three stress variables could be used as the stress state variables to describe unsaturated soil. This conclusion is based on the assumption that soil particles are incompressible (one of the basic assumptions used in saturated soil mechanics). Fredlund also defined three pairs of independent stress state variables, as he wrote:

“A suitable set of independent stress state variables are those that produce no distortion or volume change of an element when *the individual components of the stress state variables are modified but the stress state variables themselves are kept constant*. Thus the stress state variable for each phase should produce equilibrium in that phase when a stress point in space is considered.”

Compared with the Terzaghi's effective stress^[3]

“The stresses in any point of a section through a mass of soil can be computed from the *total principle stresses* s_1, s_2, s_3 which act in this point. If the voids of the soils are filled with water under a stress u , the total principal stresses consist of two parts. One part u , acts in the water *and* in the solid in every direction with equal intensity. It is called the *neutral stress* (or pore-water pressure). The balance $s_\phi = s_1 - u, s_\psi = s_2 - u$ and $s_\xi = s_3 - u$ represents an excess over the neutral stress u and it has its seat exclusively in the solid phase of the soil.”

“This fraction of the total principal stresses will be called the *effective principal stresses*... A change in the natural stress u produces practically no volume change and has practically no influence on the stress conditions for failure... Porous materials (such as sand, clay and concrete) react to a change of u as if they were incompressible and as if their internal friction were equal to zero. All the measurable effects of a change in stress, such as compression, distortion and a change of shearing resistance are exclusively due to changes in the effective stresses s_ϕ, s_ψ, s_ξ . Hence every investigation of the stability of a saturated body of soil requires the knowledge of both the total and the neutral stresses.

It can be found that Terzaghi's effective stress includes two points: firstly, the change of neutral stress (i.e. water pressure) will not cause body change and damage; secondly, the deformation and failure of soil is only related to effective stress. Therefore, Fredlund's independent stress state variable is equivalent to Terzaghi's effective stress, except Terzaghi's stress is called as *effective stress*, while Fredlund believed that at least two effective stresses were needed to describe unsaturated soil. Tarantino^[10] simply called the latter as *effective stress*.

The results of Houlsby^[11], Edgar^[12] and Romero^[13] showed that the linear combination of stresses could also be used as stress variables. In general, it is simple to take the degree of saturation as the c value in Bishop's stress. Houlsby provides a theoretical basis for using Bishop's effective stress variable. Zhao demonstrated the rationality of using Bishop effective stress, matrix suction and gas phase pressure to describe the elastoplastic constitutive relationship of unsaturated soil by using porous media and thermodynamic theory^[14]. In fact, the unsaturated soil mass is much more complicated than the simplified assumption used in the theoretical derivation above, such as Li's tensor considering the influence of microstructure. Edgar did not consider the influence of internal

water-gas interface changes, and considered this interaction to be internal, which Houlsby also ignored. Rojas^[15] considers the dual-mode structure of clay and supplements the results by Houlsby.

4. Observation of basic concepts and principles

The derivation of the constitutive relation often deals with some boring partial differential equations or tensors. Therefore it requires professional mathematical knowledge. For civil engineering students, it may be difficult to deal with these problems with simple high-level knowledge. The author has asked a scholar who has been deducing some formulas for many years in the field of constitutive relation theory. He admitted his limited mathematics knowledge had hindered the deduction of his theory. Furthermore, some scholars proposed wrong theory in their research due to confusion and unintentional misuse of some basic concepts or principles in soil mechanics. The author has encountered 3 cases. One took a sign convention used in finite element method by neglecting the sign of compression and expansion for soils and, resulting in the soil demonstrating expansion when compressed, even though the curves in the publication seemed reasonable and perfect. The other simply attributed the variation of degree of saturation to the change of soil moisture, neglecting the influence of variation of porosity, but the final resulted curves is very consistent and harmonious in the publication. These scholars are all famous experts on soil mechanics. On the one hand, this reflects the impetuosity of the current scientific researchers in their eagerness to produce results. These cases should serve as a warning to all constitutive researchers.

5. Conclusions

This paper has introduced the author's experience and process in studying and understanding the constitutive relationship in unsaturated soils. For the better understanding of properties of unsaturated soils, a cautious and serious attitude is necessary and there are many difficulties for us to resolve including the applicability of stress variables. Moreover we should always observe basic concepts or principle to obtain scientific and reasonable research result.

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