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Numerical Analysis of Fracture Behavior of Multiple Cracks in Rock

Sun Yuelin[1], Xue Jianfeng[2*] and Shen Zhenzhong[3]

[1]China Institute of Water Resources & Hydropower Research, Beijing, 100038, China

[2]China Renewable Energy Engineering Institute, Beijing, 100120, China

[3]Hohai University, Nanjing, 210098, China

[*]Corresponding author's e-mail:5688047@qq.com

Abstract: Interaction between multiple cracks in rock mass makes the propagation process more different than that of a single crack. In the paper, the criteria of maximum circumferential shear stress was proposed to describe the crack branching and the rock bridge fracture between multiple cracks in experiment. Based on the theory of fracture mechanics, the process of multi-cracks propagation and connection was traced by element-free methods (EFM). The crack branching in experiment was simulated including the wing cracks appeared in initial cracking stage and the shear cracks in crack propagation stage. By contrasted with the single-crack model, the interaction mechanism in multiple cracks and its effect on cracks propagation and connection was analyzed. The results show that rock bridge fracture can be caused by the propagation and connection of shear cracks instead of initial wing cracks.

1. Introduction

Studies concerning multiple cracks in rock mass mainly focus on two major problems, namely, the interaction between cracks and the coherent mechanism of multiple cracks. Kachanov M[1] proposed a simplified pseudo-tension method-Kachanov method to calculate the interaction between multiple cracks. The literature[2] improved the Kachanov method in terms of accuracy. Li Yinping extended this method to closed compression shear cracks by combining Mohr-Coulomb's law, which provides a basis for further study of the propagation polymerization of compression shear cracks[3]. At present, there are many analytical methods for studying multi-cracks problems at home and abroad, including complex variable function theory, element free method, finite element method, boundary element method and numerical manifold method[4-8] .

Li Liyun, Che Faxing et al.[9-10] carried out uniaxial and biaxial loading tests on cement specimens with multiple cracks in the servo-control loading system. Using the stress intensity factor of crack tip of the specimens calculated by FEM, they proposed that for each compression-shear crack tip, there are a tensile stress extreme value, two tensile strain extreme values, and three shear stress extreme values; when the directions of the above-mentioned extreme values of the two crack tips can basically correspond, this crack tips will be connected in form of a straight line or a curve. Based on the tests mentioned above, the paper proposed the criteria of maximum circumferential shear stress. By using EFM, multi-cracks propagation and connection were simulated to explain the crack branching phenomenon, which is, in initial cracking stage the wing cracks appeared first, then following by the shear cracks in the cracking stage. The paper also analyzed the interaction mechanism of multiple cracks and its influence on multi-cracks propagation by comparing with the single-crack model.

2. Using element free method to simulate crack propagation

In EFM the field functions are fitted using the least squares method and the calculations are only done on discrete points and the geometric boundary of the domain, which breaks the constraint of the element [11-12]. At the same time, movable finer nodes can be arranged around the crack tip, making EFM significantly advantageous in tracing crack propagation.

The basis of EFM is the least square method, which uses a weighted least squares method to approximate the field function using the values of several uncorrelated known points. When it comes to the calculation of the field function, it is needed to solve a linear equation group, which means the shape of the coefficient matrix should be good. When the orthogonal basis is used as the basis function, the obtained equation is neither ill nor singular, and the calculation speed and precision are also high. The approximation field function [13] can be expressed as:

$$Gu(\boldsymbol{x}) = \sum_{i=1}^{n} n_i(\boldsymbol{x}) u_i \tag{1}$$

$$n_i(\boldsymbol{x}) = w_i(\boldsymbol{x}) \sum_{j=1}^{m} c_{ji}(\boldsymbol{x}) \tag{2}$$

$$c_{ji}(\boldsymbol{x}) = \frac{q_j(\boldsymbol{x}, \boldsymbol{x}) q_j(\boldsymbol{x}_i, \boldsymbol{x})}{b_j(\boldsymbol{x})} \tag{3}$$

$$b_j(\boldsymbol{x}) = \sum_{i=1}^{n} w_i(\boldsymbol{x}) q_j^2(\boldsymbol{x}_i, \boldsymbol{x}) \tag{4}$$

In which, u_i is the field function value of the known node *i*; $w_i(\boldsymbol{x})$ is the value of the node *i* at the point $\boldsymbol{x} = (x, y)^{\mathrm{T}}$; q_j is the basis function of the node *j*.

3. Multi-cracks shear propagation criteria

3.1 Related tests of multi-cracks interaction [9-10]

Two existing typical tests were selected as reference, including a uniaxial compression test with collinear double-cracks specimen and a biaxial compression test with triple-cracks specimen. The size and shape of the specimens and the applied load are shown in Figure 1 and Figure 2. Collinear double-cracks specimen is uniaxial compressed with gradually increasing compressive stress σ_1. The triple-cracks specimen is biaxial compressed with the gradually increasing compressive stress σ_1 and the unchanged lateral compressive stress σ_2.

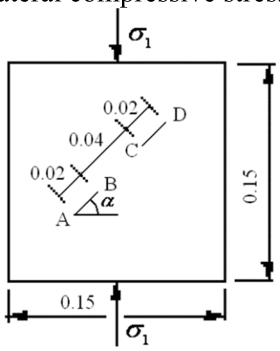


Figure 1. Double-cracks specimen (m)

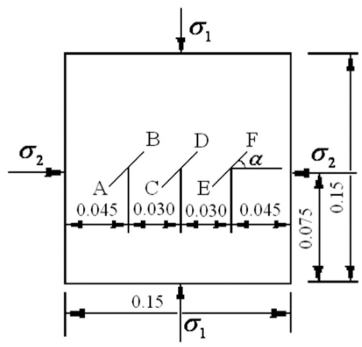
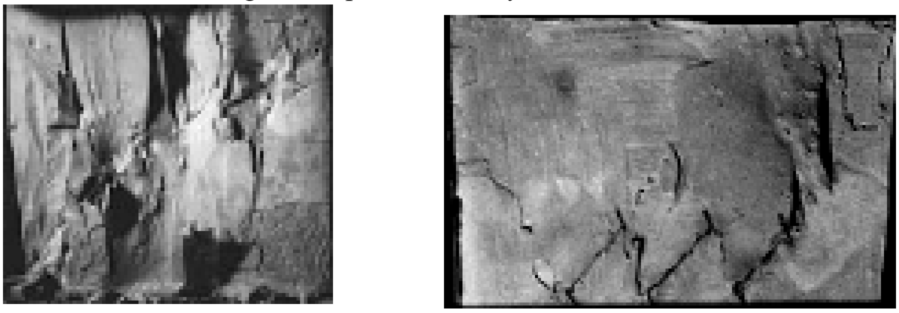


Figure 2. Triple-cracks specimen (m)

Figure 3 shows the crack traces from initiation to propagation, and then to connection. The wing cracks expanded from the crack tips at initial cracking stage. As the stress at the crack tips is fully released, the wing cracks stop propagation. With the increase of the compressive stress σ_1, the tips at the proximal ends of the cracks begin to expand until they are connected.



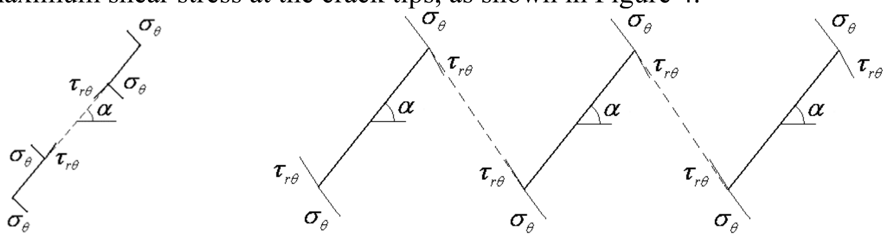
(a) Double-cracks specimen (b) Triple-cracks specimen

Figure 3. Crack propagation trace of multi-cracks specimens

3.2 Maximum circumferential shear stress criteria

The above two tests show that the multi-cracks propagation mechanism is more complicated than that of single crack. The wing cracks appeared at the initial cracking stage is consistent with the criteria of maximum circumferential normal stress and the dominant rule of tension type I crack. However, under the interaction of cracks, crack tips that are close to each other will expand departing from the initial cracking direction, approach other crack tips nearby, and eventually form a connection crack, which directly causes structural damage.

Related calculation analysis found that [10], the cracks in the above two specimens are gradually connected due to shear slip, that is, the connection trace of the cracks is basically consistent with the direction of maximum shear stress at the crack tips, as shown in Figure 4.



(a) Double cracks (b) Triple cracks

Figure 4. Cracking direction of multiple cracks

Based on the test results, the paper proposed the maximum circumferential shear stress criterion for multiple cracks, which is similar to the criterion of maximum circumferential normal stress. This criterion is based on the following two assumptions.

(1) The shear propagation of the crack is along the direction of the shear stress $\tau_{\gamma\theta}$ with maximum absolute value.

It is well known that the shear stress can do damage to the structure no matter if it is positive or negative, so it is necessary to find the direction in which the absolute value of the shear stress is largest. Since the shear stress at the crack tip is close to infinity, the crack angle θ_0 can only be determined by comparing $\tau_{\gamma\theta}$ at every point with the same tiny distance from the tip of the crack and locating the position of the extreme value. θ can be searched within the range of $(-\pi,\ \pi)$ to find the crack angle θ_0 when the absolute value of the shear stress is largest.

(2) When the maximum absolute value of the shear stress $\tau_{\gamma\theta}$ at the tip of multi cracks reaches the critical value, shear cracking occurs:

$$\left(\tau_{\gamma\theta}\right)_{max} = \tau_{\gamma\theta\text{cri}} \tag{5}$$

In which

$$(\tau_{r\theta})_{max} = \frac{1}{2\sqrt{2\pi r_0}}\cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\sin\theta_0 + K_{\mathrm{II}}\left(3\cos\theta_0 - 1\right)\right] \tag{6}$$

$\tau_{\gamma\theta\mathrm{cri}}$ can be determined by K_{IIC}, the crack fracture toughness of type II:

For type I cracks, $K_{\mathrm{II}} = K_{\mathrm{IIC}}$, $K_{\mathrm{I}} = 0$. Substitute them into equation (6)

$$(\tau_{r\theta})_{max} = \frac{K_{\mathrm{IIC}}}{2\sqrt{2\pi r_0}}(3\cos\theta_0 - 1)\cos\frac{\theta_0}{2}{}_0 \tag{7}$$

$\tau_{\gamma\theta\mathrm{cri}}$ is the maximum value of $\left(\tau_{\gamma\theta}\right)_{max}$ in equation (7). When $\theta_0 = 0^{\circ}$, that is $\cos\theta_0 = 1$ and $\cos\frac{\theta_0}{2} = 1$, $\left(\tau_{\gamma\theta}\right)_{max}$ reaches the maximum, so:

$$\tau_{\gamma\theta\mathrm{cri}} = \frac{K_{\mathrm{IIC}}}{\sqrt{2\pi r_0}} \tag{8}$$

Substitute equations (6) and (8) into equation (5), and get a cracking criterion of

$$K_{r\theta} = \frac{1}{2}\cos\frac{\theta_0}{2}\left[K_{\mathrm{I}}\sin\theta_0 + K_{\mathrm{II}}\left(3\cos\theta_0 - 1\right)\right] = K_{\mathrm{IIC}} \tag{9}$$

$K_{\gamma\theta}$ is called the effective stress intensity factor of shear type.

4. Analysis of the connection mechanism of multiple cracks

This paper simulated the above two tests using EFM to verify the proposed maximum shear stress criterion for multiple cracks. By comparing with the simulation result of the propagation trace of single crack, the paper analyzed the multi-cracks connection mechanism and the influence of the interaction among multiple cracks on the whole process from initial cracking to connection of the cracks.

Two EFM models for multi-cracks propagation were established based on Figure 1 and Figure 2, as shown in Figure 5 and Figure 6, wherein 289 uniformly distributed nodes are adopted in the square plate and 50 addition nodes are arranged at each crack tip. The fracture toughness of the material is defined as $K_{\mathrm{IC}} = 0.025\mathrm{MPa}\cdot\mathrm{m}^{1/2}$ and $K_{\mathrm{IIC}} = 0.027\mathrm{MPa}\cdot\mathrm{m}^{1/2}$. The loading process in Figure 1 and Figure 2 was simulated until the cracks began cracking and kept expanding, wherein σ_1 increased by 0.01MPa each step from 0, while σ_2 in the triple-cracks model maintained at 0.01MPa. In order to analyze the interaction of multiple cracks, the propagation of the single crack models was also simulated, that is, the model only containing crack AB in Figure 5 and the model only containing crack CD in Figure 6.

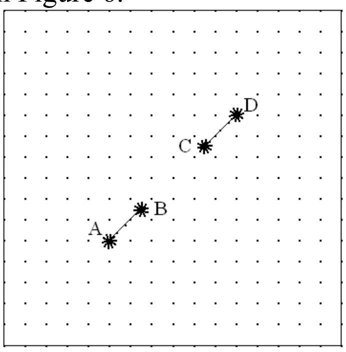


Figure 5. Nodes distribution of double cracks

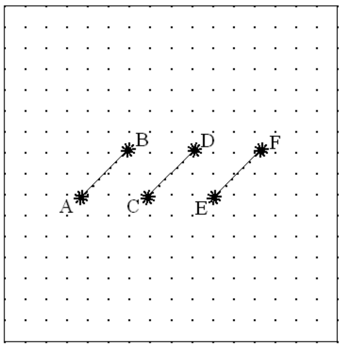
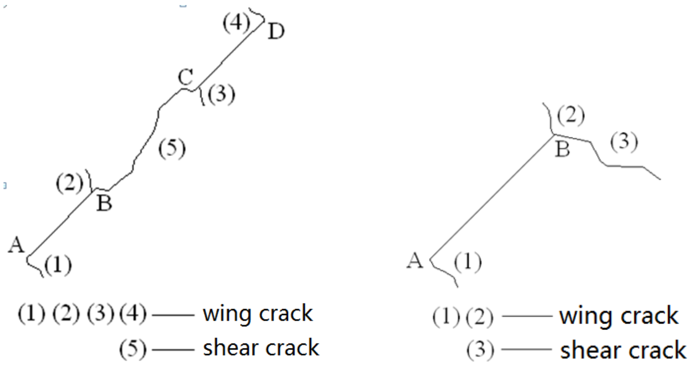


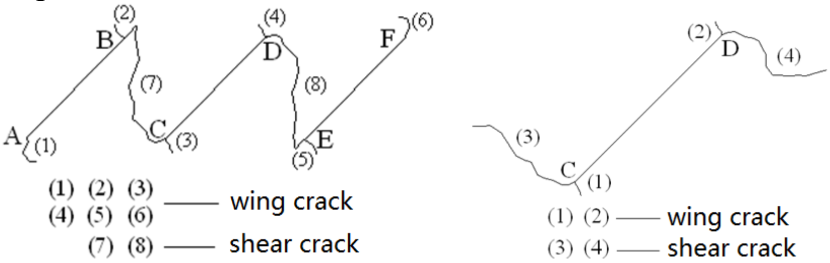
Figure 6. Nodes distribution of triple cracks

According to the maximum shear stress criterion proposed in the paper and the maximum circumferential normal stress criterion, the crack propagation traces were calculated as shown in Figure 7 and Figure 8.



(a) Double cracks (b) Single crack

Figure 7. Formation and connection of shear cracks of double cracks



(a) Triple cracks (b) Single crack

Figure 8. Propagation and connection trace of triple cracks

It can be seen from Figure 7 and Figure 8 that the connection process of the multiple cracks is closely related to the formation of the shear cracks and the interaction between cracks. $K_{\gamma\theta}$ at the crack tips (crack tip B and crack tip C) of multiple cracks increases in the final stage of propagation until the crack tip B and the crack tip C are connected, forming shear-connection crack in Figure 7(a) and Figure 8(a). $K_{\gamma\theta}$ at the crack tips in the case of a single crack is reduced to less than K_{IIC} after several iterations and the crack stops, forming the shear cracks in Figure 7(b) and Figure 8(b), which is significantly different from the shear cracks in Figure 7(a) and Figure 8(a).

Numerical propagation simulation of these two multi-cracks models are basically consistent with the experimental results shown in Figure 3, indicating that the maximum shear stress criterion proposed in the paper can fairly reflect the multi-cracks connection mechanism. By combining the maximum circumferential normal stress criterion, it can accurately trace the whole process of multiple cracks from initial cracking to propagation. Meantime, multiple cracks can be connected by the shear cracks which are sufficiently expanded instead of the wing cracks. In this case, the interaction between the cracks makes the cracks easier to crack and to expand to the direction of connection.

5. Conclusion

The paper proposed the maximum circumferential shear stress criterion based on the relevant experimental results and simulated the process of the multi-cracks propagation and connection using EFM. Meantime, a reasonable explanation was made for the crack branching phenomenon in the test and the interaction mechanism analysed deeply of between multiple cracks. The main conclusions are as follows:

1) The propagation mechanism of multiple cracks is more complicated than that of single crack. The connection between cracks is not necessarily along the initial cracking direction. That is, the maximum circumferential normal stress criterion is mostly used to find the initial cracking direction. In the case where the wing cracks cannot be connected, the shear cracks may be sufficiently extended to form a connection.

2) When the maximum absolute value of the shear stress $\tau_{\gamma\theta}$ near the crack tip reaches the critical value, shear crack propagation will occur and is along the direction of the maximum shear stress. This is the maximum circumferential shear stress criterion proposed in the paper.

3) The interaction among multiple cracks influences the cracking, making the cracks more easily to expand in the direction of connection. This kind of interaction grows gradually during the crack propagation process.

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