

PAPER • OPEN ACCESS

## Evaluation and prediction of reliability on liner based on time-dependent theory

To cite this article: Ruiyue Liu *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **242** 032058

View the [article online](#) for updates and enhancements.

# Evaluation and prediction of reliability on liner based on time-dependent theory

Ruiyue Liu, Boqiang Shi\*, Yanhua Shen and Guoguang Li

School of Mechanical Engineering, University of Science and Technology Beijing, Beijing, 100083, China

\*Corresponding author's e-mail: shiboqiang@ustb.edu.cn

**Abstract.** Considering the evolution of structure parameters of cone crusher, and based on the continuous-time and Ito Lemma, a model of evaluation and dynamic forecast on liner's reliability is studied in this paper. The performance of liner is fluctuant due to the uncertain factors (such as wear and adjustment of closed side setting) and gradually degraded. The degradation will influence the safe usage and reliability of liner. Degradation process of liner can be described by stochastic process and is modeled in geometric Brownian motions (GBM), and it is predicted with experimental data from a PYGB1821 cone crusher. The results show that the deviation between the predicted value and the experimental data is 2.62%. The model is valid on assessing the reliability of the liner, and the liner can be designed with the specified reliability value at the given time.

## 1. Introduction

Cone crushers are commonly used in the secondary and tertiary crushing of rock material. While the cone crusher is working, the head assembly exerts nutation and rotation motions. The compound movement of the head makes the rock material squeezed and crushed. The mantle is impacted by material at the moment of squeezing. The impact will lead to the wear of the mantle. The mantle wear directly affects not only the cone crusher performance and product quality, but also the reliability of frame and head [1-3]. The maintenance costs which is caused by replacing worn parts is high [4]. Moreover, the total estimated economic losses caused by wear in mineral mining are 210,000million Euros yearly [5]. Therefore, it is of great significance to analyse the reliability of liner and provide timely replacement.

Barabady [6] estimated the reliability of the crushing plant by using ReliaSoft's Weibull++6 software. A similar study was as well as conducted by Sinha [7-8]. They investigated the reliability of the crusher by using analysis of variance and Weibull model. However, for many engineering problems, the time-dependent characteristics caused by some factors, such as material deteriorate, loading variation in the service period, will cause the reliability of liner evolving with time and being dynamic process. Therefore, it is important to know the condition of the liner in order to change it timely or to extend the safe life before they are fully utilized [9]. Shi proposed a method to calculate the mechanical system reliability incorporating time varying and uncertainty. Uncertainty reliability analysis without considering the time effect will be the same as the traditional reliability analysis [10].

The liner will be affected by the uncertain factors such as wear and the adjustment of closed side setting which will cause the reliability of the liner to fluctuate. A model of evaluation and dynamic forecast on liner's reliability based on time-dependent theory is established from the angle of wear.



Combined with the study of liner wear under different conditions, the model can not only assess the reliability of liner in current time, but also dynamically predict that in future time.

## 2. Time-dependent theory

For many engineering problems, some attribute parameters will evolve with time due to the working environment, loading variation in the service period. When considering the service process of product over its lifetime, those parameters may follow a stochastic process. The evolution process can be described by the following stochastic process.

The stochastic process is defined on a probability space  $(\Omega, F, P)$ , where  $\Omega$  is a nonempty space,  $F$  is a  $\sigma$  field consisting of subsets of  $\Omega$ , and  $P$  is a probability measure [11]. It is assumed that the stochastic process  $x_t$  follows geometric Brownian motions (GBM), if it satisfies the following stochastic differential equation (SDE):

$$dx_t = \lambda x_t dt + \delta x_t dw_t \quad (1)$$

where  $w_t$  is a Wiener process (Brownian motion).  $t$  is continuous time, and the unit is second, hour or rotation times.  $\lambda$  (drift rate) and  $\delta$  (diffusion rate) are constants.

According to Ito's Lemma[10,11], SDE could be written as:

$$Y_t = \ln x_0 + \left( \lambda - \frac{1}{2} \delta^2 \right) t + \delta w(t) \quad (2)$$

Then exponent operation is operated on  $Y_t$ , we can get  $x_t$ :

$$x_t = x_0 \exp \left[ \left( \lambda - \frac{1}{2} \delta^2 \right) t + \delta w(t) \right] \quad (3)$$

Therefore,

$$\ln x_t \sim N \left( \ln \left( \ln(x_0) + \left( \lambda - \frac{1}{2} \delta^2 \right) t \right), \delta^2 t \right) \quad (4)$$

where, drift rate is used to model deterministic trends, while diffusion rate is often used to model a set of unpredictable events occurring during this motion. They can be obtained by related experiment data or working observed data. For the  $n+1$  observables value  $\{x_0, x_1, \dots, x_n\}$  which sampling  $x_t = x(t)$  at the time interval  $\Delta = T - t$ , the natural logarithm of observables value is  $\{\ln x_0, \ln x_1, \dots, \ln x_n\}$ . The value is analyzed by linear regression statistics and can be written as .Take the at the same time interval as the

observed value and obtain  $\{\ln \hat{x}_0, \ln \hat{x}_1, \dots, \ln \hat{x}_n\}$ . If  $q_i = (\ln x_i - \ln \hat{x}_i)(\Delta \cdot i)^{-1/2}$ , the mean and variance of the sampling data can be written as:

$$\bar{q} = \frac{1}{n} \sum_{j=1}^n q_j, s_q^2 = \frac{1}{n-1} \sum_{j=1}^n \left( q_j - \bar{q} \right)^2 \quad (5)$$

When  $n \rightarrow \infty$ , it will have  $\bar{q} \rightarrow E(q), s_q^2 \rightarrow \text{var}(q)$ , the drift rate  $\hat{\lambda}$  and diffusion rate  $\hat{\delta}$  of  $\ln x_t$  will be:

$$\hat{\lambda} = m + \frac{\hat{\delta}^2}{2}, \hat{\delta} = s_q \quad (6)$$

## 3. Influence factors analysis of reliability

The reliability of liner is mainly influenced by wear during the service, and the liner wear is decided by wear resistance coefficient, speed, closed side setting (CSS) and geometry of crushing chamber [12-14], as illustrated in figure 1.

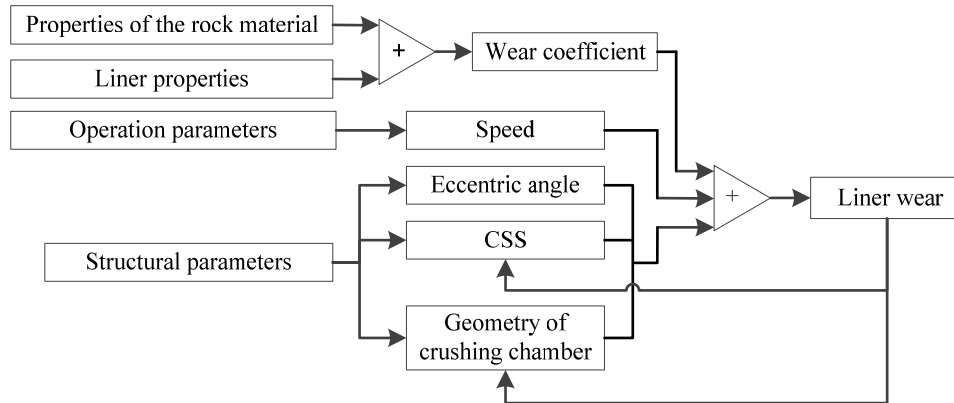


Figure 1. Influence factors of liner wear.

Wear resistance coefficient and speed basically remain unchanged, while CSS and geometry of crushing chamber are variables that evolve over time. CSS will increase with time. Geometry of crushing chamber is formed by bowl liner, mantle, eccentricity and location of the pivot point together. The radius of bowl liner will increase with time, and the radius of mantle and the eccentricity are just on the opposite, while the location of the pivot point remains unchanged. Thus the geometry of crushing chamber will be changed with time. Both geometry and CSS will influence the load on the liner, and load is one of the most important factor which influences wear rate. Therefore, the wear rate will be changed with time, so does reliability of liner.

#### 4. Reliability model of liner wear

##### 4.1. Reliability model based on time-dependent theory

The wear of liner is influenced by different time-dependent uncertain factors and follows a stochastic process. It is assumed that the wear process follows GBM. Since both the CSS and the geometry of crushing chamber will lead to the fluctuation of mean and variance, the logarithm of wear  $\ln\varpi(t)$

follows a normal distribution according to equation (3). Mean  $\hat{\mu}_{\ln\varpi}$  and variance  $\hat{\sigma}_{\ln\varpi}^2$  can be described as below:

$$\hat{\mu}_{\ln\varpi} = \ln\varpi(0) + \int_0^t \left[ \frac{\partial \ln\varpi}{\partial R_1} \lambda_{R_1} R_1 + \frac{\partial \ln\varpi}{\partial R_2} \lambda_{R_2} R_2 + \frac{1}{2} \left( \frac{\partial^2 \ln\varpi}{\partial R_1^2} \delta_{R_1}^2 R_1^2 + \frac{\partial^2 \ln\varpi}{\partial R_2^2} \delta_{R_2}^2 R_2^2 \right) \right] ds \quad (7)$$

$$\hat{\sigma}_{\ln\varpi}^2 = \left( \int_0^t \frac{\partial \ln\varpi}{\partial R_1} \delta_{R_1} R_1 dw_{1t} \right)^2 + \left( \int_0^t \frac{\partial \ln\varpi}{\partial R_2} \delta_{R_2} R_2 dw_{2t} \right)^2 \quad (8)$$

where  $R_1$  and  $R_2$  are the radius of mantle and bowl liner respectively.  $\lambda_{R1}$  and  $\delta_{R1}$  are the drift rate and diffusion rate of mantle radius.  $\lambda_{R2}$  and  $\delta_{R2}$  are the drift rate and diffusion rate of bowl liner radius. Drift rate and diffusion rate can be obtained according to equation (6).  $\ln\varpi(0)$  is the logarithm of wear at the first rotation and it can be calculated by equation (9)[15].

$$\varpi = \left( 1 + 50 \tan \frac{\alpha_0}{2} \right) \frac{ai^2 + bi}{W} \quad (9)$$

where  $\alpha_0$  is nip angle.  $a$  and  $b$  are coefficients of material properties<sup>[16]</sup>.  $i$  is the ratio of compression amount to the height before compression.

$$i = \frac{2k\gamma_0 \sqrt{R_1^2 + y^2}}{R_2 - R_1 + k\gamma_0 \sqrt{R_1^2 + y^2}} \quad (10)$$

where  $\gamma_0$  is the eccentric angle.  $k$  is a coefficient related to operating conditions and structure parameters of cone crusher [16].  $y$  is the distance between the pivot point and liner surface.

Then the logarithm of wear at  $t$  rotation  $\ln \varpi(t)$  can be described as:

$$\ln \varpi(t) \sim N \left( \ln \varpi(0) + \left( \lambda_{\varpi} - \frac{\delta_{\varpi}^2}{2} \right) t, \left( \delta_{\varpi} \sqrt{t} \right)^2 \right) \quad (11)$$

#### 4.2. Evaluation and prediction of reliability

In order to meet the performance of the liner, the inequality relationship between the wear  $\varpi(t)$  and allowable wear  $[\varpi(t)]$  should meet the input requirement described as follows:

$$\varpi(t) < [\varpi(t)] \quad (12)$$

The reliability of liner  $R(t)$  can be written as:

$$R(t) = P \{ \ln [\varpi(t)] - \ln \varpi(t) > 0 \} (\varpi(t) \neq 0) \quad (13)$$

For the wear of the liner following the Ito process, according to equation (1),  $\ln \varpi(t)$  will follow a normal distribution. The same situation to the allowable wear  $\ln [\varpi(t)]$ . Moreover,  $\ln \varpi(t)$  and  $\ln [\varpi(t)]$  are mutually independent. Therefore,  $Z = \ln [\varpi(t)] - \ln \varpi(t)$  will also follow a normal distribution. Mean  $\hat{\mu}_z$  and variance  $\hat{\sigma}_z$  can be described as below:

$$\hat{\mu}_z = \hat{\mu}_{\ln [\varpi]} - \hat{\mu}_{\ln \varpi(t)} \quad (14)$$

$$\hat{\sigma}_z = \left( \hat{\sigma}_{\ln [\varpi]}^2 + \hat{\sigma}_{\ln \varpi(t)}^2 \right)^{1/2} \quad (15)$$

Then the reliability of liner:

$$R(t) = \Phi \left( \frac{\hat{\mu}_z}{\hat{\sigma}_z} \right) = \Phi \left( \frac{\hat{\mu}_{\ln [\varpi]} - \hat{\mu}_{\ln \varpi(t)}}{\hat{\sigma}_{\ln \varpi(t)}} \right) \quad (16)$$

Finally, the value of reliability can be obtained by standard normal table.

### 5. Example

In order to study the reliability of liner, a series of experiments were conducted on a PYGB1821 crusher at Anshan Iron and Steel Group Mining Co., Ltd. Structural parameters of crushing chamber, liner wear and feed size distribution were measured. The coefficient of wear resistance was  $279 \text{ kNmm}^{-3}$  [12]. The parameters of cone crusher were as shown in table 1. The bowl assembly was moved upward every two hours in order to maintain the CSS.

Table 1. Parameters of cone crusher.

Coefficient	Value
Abscissa of initial point (mm)	187.8
Ordinate of initial point (mm)	333.3
Base angle of mantle (°)	50.5
Nip angle (°)	21
Eccentric angle (°)	2.5
CSS (mm)	19
Speed (rmin <sup>-1</sup> )	300

For the drift rate and diffusion rate of the radius of liner, it is necessary to conduct the measurement of liner wear. Then the value of drift rate and diffusion rate could be calculated according to equation (6). Because of the limitation of equipment and test conditions, the statistical data of liner radius were

obtained by Matlab simulation method. The relationship between the wear and rotation times was shown in figure 2 and figure 3.

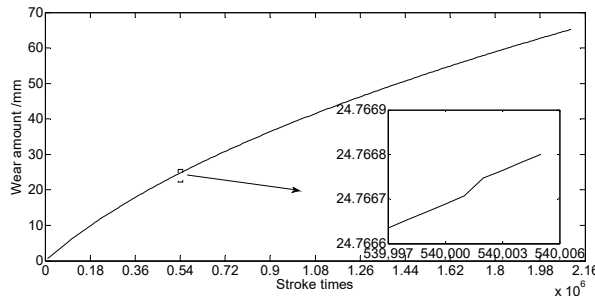


Figure 2. The relationship between wear amount and stroke times.

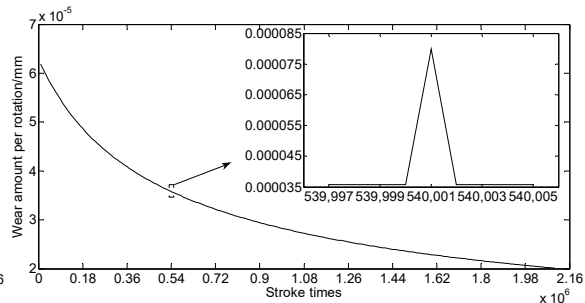


Figure 3. The relationship between wear amount per rotation and stroke times.

The drift rate and diffusion rate can be calculated based on the calculation results and are as follows:  $\lambda_{R1} = \lambda_{R2} = 0.0102$  and  $\delta_{R1} = \delta_{R2} = 0.0075$ . Then according to equations (1) and (9), we can get the partial derivative of a function  $\ln \sigma$  with respect to the variable  $R_1$  and  $R_2$ . Next the mean  $\hat{\mu}_{\ln \sigma}$  and variance  $\hat{\sigma}_{\ln \sigma}^2$  can be computed according to equations (7) and (8), they are -3.6669 and 0.0003 respectively.

Bowl liner and mantle must be replaced before they are worn through in order to prevent damage to crusher bowl or head. Liners should be changed no later than when the liner weight is consumed approximately 50%. For the liner of PYGB1821 cone crusher, the liner should be replaced before the maximum wear amount reaches 71.7mm. According to equation (16), the liner reliability can be calculated when the head rotates  $2.16 \times 10^6$  times, namely  $R(2.16 \times 10^6) = \Phi(2.11) = 98.26\%$ . The calculated wear amount is 66.85mm, and the measured value is 68.65mm. The relative error between the calculated value and the measured value of the maximum wear was 2.62%. Therefore, the model has the application value. Using the method described above, it is easy to analysis the reliability of liner at any rotation times.

## 6. Conclusion

A time-varying reliability calculation model of liner, which considers the evolution and uncertain characteristics, is established based on time-dependent theory. The performance of liner will fluctuate due to uncertain characteristics, such as wear and adjustment of closed side setting. The fluctuant of liner performance is expressed by the drift rate and diffusion rate of variables. These variables can be computed according to the experiment data or working observed data. The model was calibrated and validated based on a PYGB1821 crusher at Anshan Iron and Steel Group Mining Co. Ltd. Moreover, the model can predict the reliability of the liner at the time  $t$  and provide reference for the safety early warning of the crusher.

## Acknowledgments

This work was financially supported by National Natural Science Foundation of China (No. 51075029).

## References

- [1] Bengtsson, M., Asbjörnsson, G., Hulthén, E., Evertsson, M. (2017) Towards dynamical profit optimization of comminution circuits. *Minerals Engineering*, 103-104:14–24.
- [2] Peng, Y.X., Ni, X., Zhu, Z.C., Yu, Z.F., Yin, Z.X., Li, T.Q., Liu, S.Y., Zhao, L.L., Xu, J. (2017) Friction and wear of liner and grinding ball in iron ore ball mill. *Tribology International*, 115:506–517.

- [3] Cleary, P.W., Owen, P. (2015) Effect of liner design on performance of a HICOM® mill over the predicted liner life cycle. *International Journal of Mineral Processing*, 134:11–22.
- [4] Terva, J., Kuokkala, V.T., Valtonen, K., Siitonen, P. (2018) Effects of compression and sliding on the wear and energy consumption in mineral crushing. *Wear*, 398–399:116–126.
- [5] Holmberg, K., Kivikytö-Reponen, P., Härkisaari, P., Valtonen, K., Erdemir, A. (2017) Global energy consumption due to friction and wear in the mining industry. *Tribology International*, 115:116–139.
- [6] Barabady, J., Kumar, U. (2008) Reliability analysis of mining equipment: A case study of a crushing plant at Jajarm Bauxite Mine in Iran. *Reliability Engineering and System Safety*, 93:647–653.
- [7] Sinha, R.S., Mukhopadhyay, A.K. (2016) Failure analysis of jaw crusher and its components using ANOVA. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 38: 665–678.
- [8] Sinha, R.S., Mukhopadhyay, A.K. (2017) Failure rate analysis of jaw crusher using Weibull model. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of process mechanical engineering*, 231:760–772.
- [9] Shi, B.Q., Tang, G.T. (2014) Evaluation and dynamic forecast of reliability on vehicle brake system based on time-dependent theory. *Transactions of the Chinese Society of Agricultural Engineering*, 30:50–55.
- [10] Shi, B.Q., Shen, Y.H. (2018) Mechanical design with evolution-based uncertainty. Science Press, Beijing. (in Chinese)
- [11] Tsay, R.S. (2010) *Analysis of Financial Time Series*. Wiley.
- [12] Liu, R.Y., Shi, B.Q., Shen, Y.H., Li, G.G. (2018) Prediction model for liner wear considering the motion characteristics of material. *Mathematical Problems in Engineering*, 2018.
- [13] Tang, J.C., Shi, B.Q., Yu, H.J., Wang, R.J., Zhang, W.Y. (2017) Design of crusher liner based on time-varying uncertainty theory. In: 2017 2nd International Seminar on Advances in Materials Science and Engineering. Singapore.
- [14] Franke, J., Cleary, P.W., Sinnott, M.D. (2015) How to account for operating condition variability when predicting liner operating life with DEM—a case study. *Minerals Engineering*, 73:53–68.
- [15] Lindqvist, M., Evertsson, C. M. (2006) Development of wear model for cone crushers. *Wear*, 261:435–442.
- [16] Liu, R.Y., Shi, B.Q., Li, G.G., Yu, H.J. (2018) Influence of operating conditions and crushing chamber on energy consumption of cone crusher. *Energies*, 11:1102.