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Normal Mode Based MUSIC Method for DOA Estimation in Shallow Water Using a Single Vector Sensor

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Abstract. Sound field in the shallow water can be modeled as a superposition of multiple normal modes. Traditional underwater DOA estimation methods assume the sound wavefront to be plane, which will lead to relatively large estimation errors. To solve this problem, a normal mode based MUSIC (NM-MUSIC) method using a single vector hydrophone is presented. The orthogonality between the steering vector and the noise subspace in classic MUSIC method is replaced by that between the steering matrix and the noise subspace. Elements of the steering matrix are functions of the azimuth angles and the modal wavenumbers, and the azimuth angles can be estimated through minimal optimization searching. Computer simulation results verify that the proposed algorithm outperforms the traditional MUSIC method from the perspective of both estimation bias and root mean square error.

1. Introduction

The acoustic vector sensor (AVS) consists of an omnidirectional acoustic pressure receiver and a dipole-like directional particle velocity receiver [1]. AVS measures the three Cartesian components of the particle velocity as well as the scalar acoustic pressure in sound field synchronously and independently [2]. Acoustic vector sensors are endowed with the spatial filtering capability, which enables a single one to find the targets' azimuth angles. During the last two decades, lots of papers on underwater acoustic source DOA estimation using a single AVS have appeared in the literature, and several techniques have been proposed. For example, the histogram approach [3-4], multiple signal classification (MUSIC) [5], estimation of signal parameters via rotational invariance technique (ESPRIT) [6-7], the DOA matrix method [8], and the parallel factor (PARAFAC) algorithm [9].

For the convenience of signal processing, all of the DOA estimation techniques mentioned above assume that the sound waves received by the AVS are plane waves. However, in practical shallow water environment, the situation is more complicated. The plane wave assumption may cause serious estimation bias. The normal mode model is an appropriate instrument for computing the low frequency sound field in the shallow water [10]. Based on this theory, extensive and in-depth research on the DOA estimation problem in shallow water has been carried out. Unbiased estimates can be obtained through the matched field processing algorithm [11], but it costs enormous computing. Reference [12] presented the subspace intersection (SI) method, which provides a significant reduction in computational complexity. L. Zhang *et al.* proposed the normal mode based MUSIC (NM-MUSIC)



and normal mode based approximate maximum likelihood (NM-AML) algorithms in [13] and [14] respectively. All of techniques in [12], [13] and [14] are based on the uniform linear scalar sensor array, so only the azimuth angles in the range of $[0^\circ, 180^\circ]$ can be estimated, otherwise there will exist an ambiguity of 180° among the estimates.

This paper combines the single AVS DOA estimation problem with the NM-MUSIC algorithm. Compared with traditional plane-wave approximation based MUSIC algorithm, the proposed algorithm has higher estimation accuracy. Moreover, it only entails one dimension searching. An additional advantage of the proposed algorithm is that any prior knowledge of the environment is not required. A knowledge of the modal wavenumbers is sufficient.

2. Model

Consider a homogeneous water layer of constant depth h , density ρ , and sound speed c , overlying a homogeneous fluid half-space with density ρ_b and sound speed c_b . The surface of the water layer is a pressure release boundary. This shallow ocean model is known as the ideal Pekeris waveguide [15].

Let the AVS be located at depth z_0 ($0 \leq z_0 \leq h$). Let J mutually independent isotropic point sources be located at depths z_j and ranges r_j ($j = 1, \dots, J$) with respect to the AVS, and the AVS is assumed to be in the far-field of all sources. The j th source signal at the AVS is defined as

$$s_j(t) = \eta_j(t) \exp(-i\omega t) \quad (1)$$

where $\eta_j(t)$ is a zero mean random process, which denotes the slowly varying random amplitude of the j th source signal. And its variance $\sigma_j^2 = E[|\eta_j(t)|^2]$ denotes the power of the source signal at the AVS.

The acoustic pressure generated by the j th source at the AVS is expressed as

$$p(r_j, z_0, t) = p(r_j, z_0) s_j(t) \quad (2)$$

According to the normal mode theory, the complex amplitude of $p(r_j, z_0, t)$, which is $p(r_j, z_0)$, can be well approximated by a sum of M discrete normal modes by [10,15]

$$p(r_j, z_0) \approx \sum_{m=1}^M b_{mj} = \frac{i}{\rho \sqrt{8\pi}} e^{-i\pi/4} \sum_{m=1}^M \Psi_m(z_j) \Psi_m(z_0) \frac{e^{ik_m r_j}}{\sqrt{k_m r_j}} \quad (3)$$

where b_{mj} is the complex amplitude of the m th normal mode due to the j th source. $\Psi_m(z)$ and k_m denote the eigenfunction and the modal wavenumber of the m th normal mode respectively.

The acoustic pressure p and the particle velocity \mathbf{v} are related by the Euler Equation

$$\rho \frac{\partial \mathbf{v}(r, z, t)}{\partial t} = -\nabla p(r, z, t) \quad (4)$$

Using (3) and (4), the complex amplitudes of the radial and vertical component of \mathbf{v} at the AVS due to the j th source are given by

$$v_r(r_j, z_0) \approx \frac{1}{\rho \omega} \sum_{m=1}^M k_m b_{mj} \quad (5)$$

$$v_z(r_j, z_0) \approx \frac{-i}{\rho \omega} \sum_{m=1}^M \frac{\Psi'_m(z_0)}{\Psi_m(z_0)} b_{mj} \quad (6)$$

where $\Psi'_m(z) = d\Psi_m(z)/dz$. Let ϕ_j be the azimuth angle of the j th source, thus the complex amplitudes of x- and y-components of \mathbf{v} are given by

$$v_x(r_j, z_0) = \frac{\cos \varphi_j}{\rho \omega} \sum_{m=1}^M k_{rm} b_{mj} \quad (7)$$

$$v_y(r_j, z_0) = \frac{\sin \varphi_j}{\rho \omega} \sum_{m=1}^M k_{rm} b_{mj} \quad (8)$$

As the medium is homogeneous, under the isovelocity problem, solution of $\Psi_m(z)$ is given by $\Psi_m(z) = \sqrt{2\rho/h} \sin(\gamma_m z)$ [10], where γ_m is the vertical wavenumber of the m th normal mode. γ_m and k_m are related by

$$\gamma_m^2 + k_m^2 = k^2 \quad (9)$$

where k denotes the wavenumber in the medium, which is defined as $k = \omega / c$.

The noiseless output signal at time t is

$$\mathbf{x}(t) = \begin{bmatrix} \sum_{j=1}^J p(r_j, z_0) s_j(t) \\ \rho c \sum_{j=1}^J v_x(r_j, z_0) s_j(t) \\ \rho c \sum_{j=1}^J v_y(r_j, z_0) s_j(t) \\ \rho c \sum_{j=1}^J v_z(r_j, z_0) s_j(t) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^J \sum_{m=1}^M b_{mj} s_j(t) \\ \sum_{j=1}^J \sum_{m=1}^M \frac{\cos \varphi_j}{k} k_{rm} b_{mj} s_j(t) \\ \sum_{j=1}^J \sum_{m=1}^M \frac{\sin \varphi_j}{k} k_{rm} b_{mj} s_j(t) \\ \sum_{j=1}^J \sum_{m=1}^M \frac{-i\Psi'_m(z)}{k\Psi_m(z)} b_{mj} s_j(t) \end{bmatrix} \quad (10)$$

Assume the noise is Gaussian and temporally white. Take the noise into consideration, $\mathbf{x}(t)$ can further be written as

$$\mathbf{x}(t) = \Phi \Gamma \mathbf{s}(t) + \mathbf{n}(t) \quad (11)$$

where

$$\Phi(\varphi_j) = [\mathbf{A}(\varphi_1) \dots \mathbf{A}(\varphi_J)] \in \mathbb{R}^{4 \times MJ} \quad (12)$$

$$\mathbf{A}(\varphi_j) = [\mathbf{a}(\varphi_j, k_{r1}) \dots \mathbf{a}(\varphi_j, k_{rM})] \in \mathbb{R}^{4 \times M} \quad (13)$$

$$\mathbf{a}(\varphi_j, k_{rm}) = [1 \quad \frac{\cos \varphi_j}{k} k_{rm} \quad \frac{\sin \varphi_j}{k} k_{rm} \quad \frac{-i\Psi'_m(z)}{k\Psi_m(z)}]^T \in \mathbb{R}^{4 \times 1} \quad (14)$$

$$\Gamma(r_j, z_j) = \text{diag}(\mathbf{B}_1 \dots \mathbf{B}_J) \in \mathbb{R}^{MJ \times J} \quad (15)$$

$$\mathbf{B}_j = [b_{1j} \dots b_{Mj}]^T \in \mathbb{R}^{M \times 1} \quad (16)$$

$$\mathbf{s}(t) = [s_1(t) \dots s_J(t)]^T \in \mathbb{R}^{J \times 1} \quad (17)$$

$$\mathbf{n}(t) = [n_p(t) \quad \rho c n_x(t) \quad \rho c n_y(t) \quad \rho c n_z(t)]^T \in \mathbb{R}^{4 \times 1} \quad (18)$$

3. Algorithm

To facilitate the presentation, rewrite (11) as

$$\mathbf{x}(t) = \Phi \Psi(t) + \mathbf{n}(t) \quad (19)$$

where $\Psi(t) = \Gamma \mathbf{s}(t)$ is independent of the azimuth angles. Columns of $\mathbf{A}(\varphi_j)$ are the responses of the j th source's azimuth angle in the vector space of modal wavenumbers, and they span the signal subspace.

Compute the AVS output covariance matrix by

$$\mathbf{C} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \Phi E[\Psi\Psi^H]\Phi^H + \sigma^2 \mathbf{I} \quad (20)$$

where σ^2 denotes the noise power. Eigen-decompose \mathbf{C} , and four eigenvalues are obtained. Assume that $J < 4$. The eigenvectors that correspond to the 4- J smaller eigenvalues span the noise subspace \mathbf{U}_n . It has been proved that the signal subspace and the noise subspace are orthogonal. The distance between the noise subspace and an arbitrary vector space \mathbf{Y} is expressed as

$$d = \|\mathbf{Y}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{Y}\|_F \quad (21)$$

Search for the minimum distance between $\mathbf{A}(\varphi)$ and \mathbf{U}_n along φ . Due to the orthogonality between the signal subspace and the noise subspace, J angles that correspond to the minimum distances are the optimal estimates of the azimuth angles φ_j . That is to say,

$$\hat{\varphi}_{\text{NM-MUSIC}} = \arg \min_{\varphi} \|\mathbf{A}^H(\varphi) \mathbf{U}_n \mathbf{U}_n^H \mathbf{A}(\varphi)\|_F \quad (22)$$

The spatial spectrum is given by

$$P_{\text{NM-MUSIC}}(\varphi) = \frac{1}{\|\mathbf{A}^H(\varphi) \mathbf{U}_n \mathbf{U}_n^H \mathbf{A}(\varphi)\|_F^2} \quad (23)$$

4. Simulations

In this section, simulation results on the performance of the NM-MUSIC and the traditional MUSIC algorithms are presented for ideal Pekeris waveguide. The parameters are: $h=100$ m, $c=1500$ m/s, $c_b=1700$ m/s, $\rho=1000$ kg/m³, $\rho_b/\rho=1.5$. Depth of the AVS $z_0=100$ m. Assume there are two mutually uncorrelated sources, with the same center frequency $f_0=50$ Hz. $z_1=z_2=50$ m, $r_1=r_2=10$ km. $\varphi_1=-100^\circ$, $\varphi_2=30^\circ$. The geometry of the simulation is shown in figure 1.

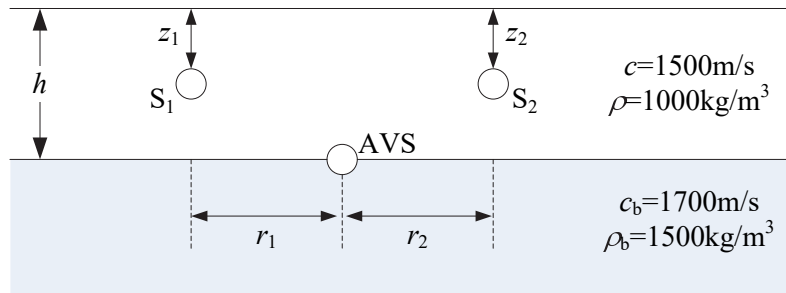


Figure 1. Geometry of the simulation.

Under the assumptions above, it can be obtained through the KRAKEN program [16] that $M=3$. Values of the modal wavenumbers can also be computed. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = 10 \log_{10} \frac{\sum_{j=1}^J \sigma_j^2 |p(r_j, z_0)|^2}{\sigma^2} \quad (24)$$

All of the results in the following figures are averaged from 100 times of Monte Carlo trials.

The spatial spectra computed through MUSIC and NM-MUSIC are shown in figure 2. The SNR is 10 dB. Figure 2 indicates that the peaks of the spectrum by NM-MUSIC diverge less from the true DOAs than the MUSIC algorithm.

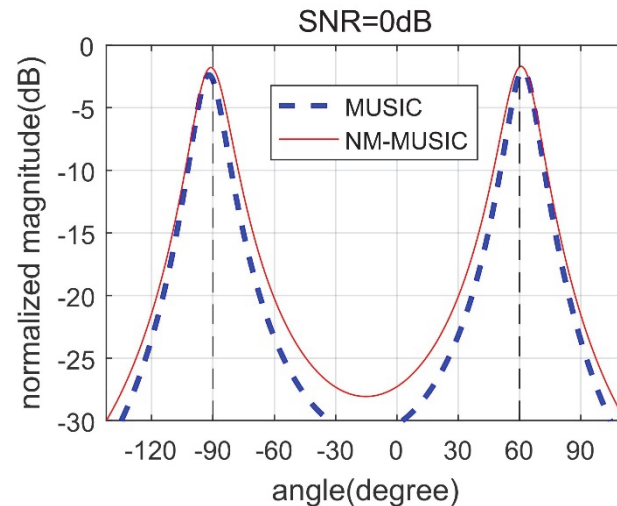


Figure 2. Spatial spectra computed through MUSIC and NM-MUSIC, SNR=10 dB

Figure 3 and figure 4 display the root mean square errors (RMSE) and the estimation biases of MUSIC and NM-MUSIC for different SNR respectively. It is obvious that the proposed algorithm performs better than the traditional method.

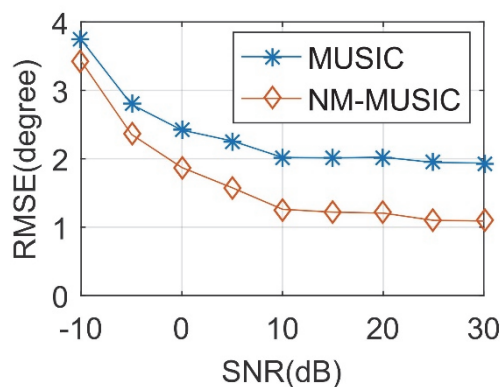


Figure 3. RMSE versus SNR.

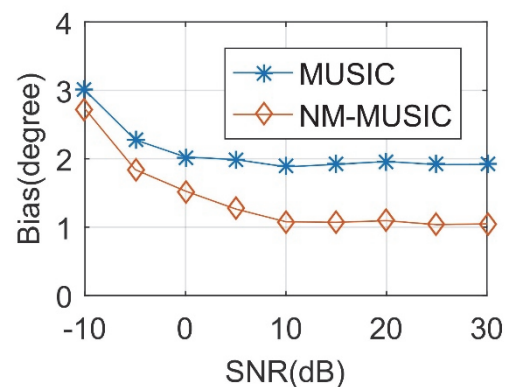


Figure 4. Estimation bias versus SNR.

5. Conclusions

In the traditional DOA estimation problems, the source signal is assumed to propagate as a plane wave. However in practical shallow water, the propagation occurs in the form of multiple normal modes. The output signal model of a single AVS is derived in this paper, and furthermore, a normal mode based MUSIC algorithm using a single AVS is proposed. The proposed algorithm only entails one dimension searching, and only a prior knowledge of the modal wavenumbers is required. Compared with the traditional MUSIC algorithm, the proposed method has higher estimation precision.

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