

PAPER • OPEN ACCESS

A new method to calculate hydraulic transients in HDPE pipes using the standard solid model to represent the HDPE viscoelastic behaviour

To cite this article: R B Carmona-Paredes *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **240** 052020

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the **collection** - download the first chapter of every title for free.

A new method to calculate hydraulic transients in HDPE pipes using the standard solid model to represent the HDPE viscoelastic behaviour

R B Carmona-Paredes¹, R Autrique², E Rodal-Canales³ and D Paniagua-Lovera⁴

¹Academic Researcher, Instituto de Ingeniería UNAM, México

²Policonductos, México

³Academic, Instituto de Ingeniería UNAM, México

⁴Postgraduate Student, Instituto de Ingeniería UNAM, México

rcarmonap@ii.unam.mx

Abstract. In this paper, a method for the calculation of hydraulic transients for pressurized flow in high density polyethylene (HDPE) pipes is developed. The proposed method incorporates the Standard Solid Model in the Method of Characteristics (MOC) to represent the viscoelastic behaviour of the pipe material. The Standard Solid Model is able to represent both a short-term elastic response and a long-term viscoelastic response. This model only requires the calibration of two parameters in a test facility, what is an advantage over other methods reported in the literature. The paper present how these parameters are calibrated. Keeping constant the wave velocity and the friction factor, the method estimates with high precision both the extreme values and the damping time of the pressure due to a water hammer in HDPE pipes. The results are compared against the data gathered over tubes with PPI 4710 resin (equivalent to PE100) in the test facility of a HDPE pipe producing plant, located in San Luis Potosí, México. The same procedure can be used to collect data of pipes made with other viscoelastic material. The main contribution of the paper is to avoid the need to perform calibration measurements during the start-up of the hydraulic systems.

1. Introduction

The numerical method to analyse hydraulic transients in pressurized pipes is chosen according to the desired quality of the results and how difficult its application is. The more refined the model, the greater the information required of the system to be studied.

The simplest solutions for estimating water hammer in elastic pipes are: first, the Joukowsky equation to calculate the maximum and minimum pressures produced by an instantaneous change in the flow and, second, the Allievi Chains that allow to estimate the time evolution of the pressure produced by slow changes in the flow conditions.

The method of characteristics (MOC) is the most commonly used numerical procedure to simulate hydraulic transients in pressurized pipes. The MOC gives a simple numerical treatment to the hyperbolic partial differential equations that models the hydraulic transient in pipes. The MOC is widely reported in literature ([1] and [2]). This method can be applied to study hydraulic transients in systems formed with pipes of different diameters and characteristics, containing pumping stations, throttling valves, transient control devices and relief valves, among many other devices.



For a wide variety of manoeuvres that modify the conditions of a permanent flow, these systems calculate with very high precision the hydraulic transients produced in elastic pipes. Therefore, computer-based simulation systems using the MOC are used as tools to support the design of pressurized water pipes [3].

Otherwise, the development of plastic materials and their application to the handling of pressurized fluids generates important benefits in manufacturing and installation costs, as well as in durability of water transport systems due to their high resistance to corrosion.

Nowadays, high density polyethylene (HDPE) is one of the most widely used viscoelastic materials for pipe manufacturing. The importance of calculating the hydraulic transients for the pressurized flow in HDPE pipes is due to its high energy dissipation capacity compared to rigid wall pipes (steel and concrete). Therefore, the development of mathematical models that can be easily incorporated in computer-based simulation systems by using the MOC has become a great challenge.

1.1. Background

The literature review shows that different authors have proposed to represent the viscoelastic behaviour of the HDPE using the modified and generalized Kelvin-Voigt model (Figure 1). The response to a stress acting on the material is modelled including an elastic element that can represent the short-term or instantaneous response, plus a series of Kelvin-Voigt elements proposed to describe the long-term response composed by the stages of creep, relaxation and recovery.

According to literature, many efforts have been made to calibrate the values of the parameters of the Kelvin-Voigt model (Figure 1). However, it has not been possible to calibrate unique values for the application of this model to any system in general.

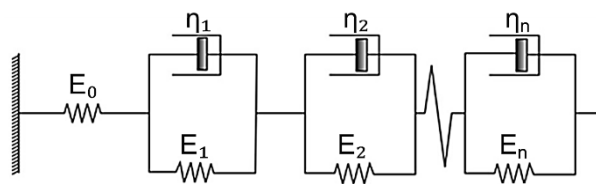


Figure 1. Modified and generalized Kelvin-Voigt model.

The first obstacle lies in defining the number of Kelvin-Voigt elements needed to accurately represent the transient evolution of the pressure along the HDPE pipe [4]. Some works on this subject ([5] and [6]) conclude that the best results are achieved using 4 to 5 Kelvin-Voigt elements in the generalized model. However, other papers show that it is possible to obtain same results using different number of Kelvin-Voigt elements with the appropriate calibrated values of their parameters ([7] and [6]). This problem is known as equifinality, which means, there are many ways of achieving the desired state.

A second obstacle is caused by the fact that the model in Figure 1 generates a mathematical expression that reproduce casuistically the results of different experimental tests. There is not a physical correlation between the model and the HDPE [4]. The correlation between the parameters of the model and the HDPE dynamic response has been sought through different paths. One is to perform mechanical tests on the material [5], but the results are not useful because the dynamic response of the materials is measured with long time scales comparing with time scales from water hammer hydraulic transients [6]. The second way consists in the calibration of the parameters against data obtained from transient pressure registers, which allows to reach excellent numerical results ([4], [5], [6] and [7]). However, it has not been possible to find a matchless relationship between the parameters of the numerical model and the physical characteristics of the HDPE used to build the pipe [6]. Then, it has not been possible to generalize the application of the Kelvin-Voigt model to the analysis of hydraulic transients in viscoelastic pipes.

In addition to the dynamic response of the pipe material, there are other phenomena that influence the development of hydraulic transients by water hammer in HDPE pipes. Many authors agree that the mechanical response of the pipes is the main factor influencing pressure transient evolution, so this paper aims to develop a simple model capable of reproducing with high precision both the extreme values and the damping of the pressures transient by water hammer. The model contains only three parameters to represent the viscoelastic response of HDPE pipe and considers that both wave velocity and friction are constant. It means, the good precision of the results of the model proposed in this paper is achieved without considering other physical factors that affect the development of hydraulic transients. In literature, some papers present some physical factors such as: first, the changes in the flow conditions that generate disturbances in the fluid, which also produce a wave train in the pipe wall, so that, in addition to the dissipative effect, a dispersive effect of the transient phenomena appears too [5]; second, due to local and convective effects caused by abrupt changes in flow conditions, friction is not stationary [8]; third, the temperature and the application of water hammer loads affect increasingly during time the HDPE wall pipe, because of the effect on the material caused by the considerable deformations to which it is exposed during each hydraulic transient [9].

2. Mathematical development

In this paper, the standard solid model (Figure 2) represents both the short-term and the long-term responses of a HDPE element when it is submitted to an external stress σ . The model also represents the gradual return of the element to its original state after withdrawing such stress.

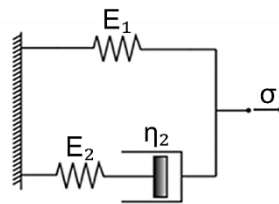


Figure 2. Standard solid model.

Figure 2 shows that the model has two branches acting in parallel. The Maxwell branch contains two elements connected in series, the elastic element (E_2) and the damping element (η). The other branch only has one elastic element (E_1).

When a suddenly stress is applied, E_1 and E_2 allow to represent the short-term response of the material. The long-term response is represented by the slow deformation of the damping element. A final condition is reached when the external stress is equilibrated by the deformation of the pure elastic branch. In other words, E_1 represents the long-term elastic modulus and can be easily measured. At the final condition, the damping element deformation allows that the E_2 had returned to relaxation state.

When the external stress is withdrawing the pure elastic branch acts over the Maxwell branch producing an instantaneous compression of the E_2 . After that, both E_1 and E_2 act on the damping element forcing a long-term response to slowly return the complete model to its initial condition.

Equation (1) represents the physical response of the model to external stress σ . When the model is used to represent the deformation of a HDPE pipe, σ is the circumferential stress produced by the pressure of the fluid and ε is the unitary deformation of pipe perimeter.

$$\frac{d\sigma}{dt} + \frac{E_2}{\eta} \sigma = (E_1 + E_2) \frac{d\varepsilon}{dt} + \frac{E_1 E_2}{\eta} \varepsilon \quad (1)$$

In the next section, Equation (1) is combined with the MOC to model the creep, relaxation and recovery stages of the HDPE pipe subject to a water hammer transient. Hence, a new calculation method is developed with only 3 elements (E_1 , E_2 and η). The procedure to estimate each element is simple, so the application of the new method can be easily generalized.

E_1 values are directly obtained from manufacturers sheet specifications, while E_2 and η values are calibrated against experimental data. Therefore, the model presented in this paper can be a designer support tool. It is important to note that the model cannot consider plastic deformations of the material.

2.1. Application of the model of solid standard through the method of characteristics

As it is well known, the equations that represent the non-permanent flow in deformable wall pipes are the mass conservation equation:

$$\left(\frac{\partial \rho}{\partial p} + \frac{\rho}{A} \frac{\partial A}{\partial p}\right) \left(\frac{\partial p}{\partial t} + V \frac{\partial p}{\partial x}\right) + \rho \frac{\partial V}{\partial x} = 0 \quad (2)$$

And the equation of conservation of momentum or dynamic equation:

$$\frac{\partial H}{\partial x} + \frac{1}{g} \frac{\partial V}{\partial t} + \frac{fV|V|}{2gD} = 0 \quad (3)$$

where H is the piezometric head [m], V is the flow velocity [ms^{-1}], ρ is the water density [kgm^{-3}], p is the water pressure [Pa], D is the pipe diameter [m], A is the pipe cross sectional area [m^2], f is the Darcy-Weisbach friction factor [-], g is the gravity acceleration [ms^{-2}], t is the time [s] and x is the position variable along the pipeline axis [m].

The proposed mathematical model for hydraulic transients by water hammer in viscoelastic wall pipes is formed by four equations: the constitutive equation of pipe material (Equation 1), the mass conservation equation (Equation 2), the dynamic equation (Equation 3) and the constitutive equation of water (Equation 4):

$$\frac{\partial \rho}{\partial p} = \frac{\rho_0}{K} \quad (4)$$

where K is the modulus of elasticity of water [Pa].

For a thin-walled pipe, Equation (5) assumes that the external stress σ is directly proportional to the fluid pressure and the diameter of the pipe and inversely proportional to the thickness of its wall

$$\sigma = \frac{\gamma(H-z)D_0(1+\varepsilon)}{2e} \quad (5)$$

where z is the elevation of the pipe element [m] and e is the thickness of the pipe wall [m].

The hydraulic transients by water hammer in viscoelastic wall pipes mathematical model (Equation 1 to Equation 4) can be reduced to a system of two equations by neglecting the transmission of stresses between contiguous sections of the pipeline, as well as the action of convective phenomena in the fluid [10]:

$$L_1: g \frac{\partial H}{\partial t} + a^2 \frac{\partial V}{\partial x} + \frac{2a^2 \frac{E_2}{\eta} \left[\frac{\gamma(H-z)D_0}{e} - E_1 \frac{\varepsilon}{1+\varepsilon} \right]}{E_1 + E_2 - \frac{\gamma(H-z)D_0}{2e}} = 0 \quad (6)$$

$$L_2: \frac{\partial V}{\partial t} + g \frac{\partial H}{\partial x} + \frac{fV|V|}{2D} = 0 \quad (7)$$

where γ is the specific weight of the water [Nm^{-3}] and a is the propagating wave velocity in the water [ms^{-1}]. It is assumed that a is constant and expressed by the following approximation:

$$\frac{1}{a^2} = \rho_0 \left(\frac{1}{K} + \frac{\frac{D_0}{e}}{E_1 + E_2 - \frac{\gamma(H-z)D_0}{2e}} \right) \approx \rho_0 \left(\frac{1}{K} + \frac{\frac{D_0}{e}}{E_1 + E_2} \right) \quad (8)$$

Equation (8) shows that the calculation of the celerity a depends on the parameters E_1 and E_2 of the standard solid model. This is a basic modification of the classical expression to calculate the celerity a for pipelines with elastic behaviour.

The MOC allows to rewrite the hyperbolic partial differential Equation (6) and Equation (7) as a set of two simultaneous ordinary differential equations. The method combines both equations linearly by writing them as: $L = L_2 + \lambda L_1$.

The first equation in total derivatives is obtained choosing $\frac{dx}{dt} = a$:

$$\frac{dV}{dt} + \frac{g}{a} \frac{dH}{dt} + \frac{fV|V|}{2D} + \frac{2a \frac{E_2}{\eta} \left[\frac{\gamma(H-z)D_0}{2e} - E_1 \frac{\varepsilon}{1+\varepsilon} \right]}{E_1 + E_2 - \frac{\gamma(H-z)D_0}{2e}} = 0 \quad (9)$$

The second equation is obtained choosing $\frac{dx}{dt} = -a$:

$$\frac{dV}{dt} - \frac{g}{a} \frac{dH}{dt} + \frac{fV|V|}{2D} - \frac{2a \frac{E_2}{\eta} \left[\frac{\gamma(H-z)D_0}{2e} - E_1 \frac{\varepsilon}{1+\varepsilon} \right]}{E_1 + E_2 - \frac{\gamma(H-z)D_0}{2e}} = 0 \quad (10)$$

Equation (1), Equation (9) and Equation (10) must be satisfied at any time t and for all x values representing a pipeline point.

A numerical solution of the differential Equation (9) and Equation (10) is obtained using the method of finite differences whose premise is to replace the differentials by differences and increments. That means, a discrete-time solution is obtained at every time t for every point x of the MOC mesh presented in Figure 3, and the algebraic system of equations is written as:

$$(V_P - V_A) + \frac{g}{a}(H_P - H_A) + F_A = 0 \quad (11)$$

$$(V_P - V_B) - \frac{g}{a}(H_P - H_B) + F_B = 0 \quad (12)$$

In the MOC scheme, F_A (Equation 11) and F_B (Equation 12) are known as the positive and negative characteristics. F_A and F_B must be evaluated using Equation (13) and Equation (14).

$$F_A = \left[\frac{f_A V_A |V_A|}{2D_0(1+\varepsilon_A)} + \frac{2a \frac{E_2}{\eta} \left[\frac{\gamma(H_A - z_A)D_0}{2e} - E_1 \frac{\varepsilon_A}{1+\varepsilon_A} \right]}{E_1 + E_2 - \frac{\gamma(H_A - z_A)D_0}{2e}} \right] \Delta t \quad (13)$$

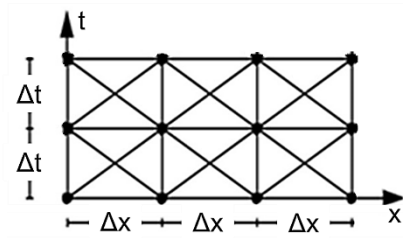


Figure 3. Method of characteristics mesh.

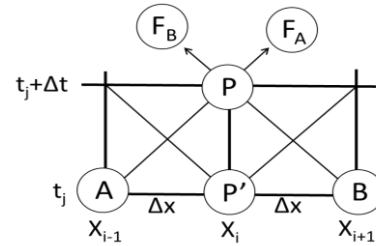


Figure 4. Characteristics lines for the finite differences method.

$$F_B = \left[\frac{f_B V_B |V_B|}{2D_0(1+\varepsilon_B)} - \frac{2a \frac{E_2}{\eta} \left[\frac{\gamma(H_B - z_B)D_0}{2e} - E_1 \frac{\varepsilon_B}{1+\varepsilon_B} \right]}{E_1 + E_2 - \frac{\gamma(H_B - z_B)D_0}{2e}} \right] \Delta t \quad (14)$$

A numerical solution for Equation (1) can be written as:

$$\varepsilon_p = \varepsilon_{p'} + \frac{\frac{\gamma(H_p - H_{p'})D_0}{2e} + \frac{E_2}{\eta} \left[\frac{\gamma(H_{p'} - z_{p'})D_0}{2e} (1+\varepsilon_{p'}) - E_1 \varepsilon_{p'} \right] \Delta t}{E_1 + E_2 - \frac{\gamma(H_{p'} - z_{p'})D_0}{2e}} \quad (15)$$

The subscripts A, B, P and P' indicate the discrete-time and the point of the pipe where each of the variables must be evaluated, as it is sketched in Figure 4.

3. Experimental calibration and results

3.1. Test facility

Experimental data is collected in the test facility of a HDPE pipe production plant located in San Luis Potosi, Mexico. PPI 4710 resin pipes are tested using tap-water without considering the air content in the water. The PPI 4710 is equivalent to PE100. Figure 5 shows a scheme of the test circuit. A pump with 11.2 kW of power allows to establish different combinations of piezometric head and flow. The head can reach up to 90 m, while the maximum flow can be 0.02 m³/s. In all experiments, flow conditions are chosen before the water hammer is provoked to avoid water column separation and to mitigate effects of air content in water. At the end of the HDPE pipe, 0.30 m downstream point T4 (Figure 6), a controlled shut-off valve close the flow faster than the period of time the pressure wave returns from the upstream end. The air chamber at the upstream end of the HDPE pipe reflects the pressure waves caused by the water hammer.

Experimental data are recorded using a high frequency data acquisition system at T1, T2, T3 and T4 points (Figure 5). Table 1 shows the lengths of the HDPE pipes under test and the distance between the data recorder points (a, b, c and d). For all tests, T2 and T3 are at positions of $\frac{1}{3}$ and $\frac{2}{3}$ of the length of the pipe.

Table 1. Test lengths and distance between pressure transducers.

Test length (m)	a (m)	b (m)	c (m)	d (m)
60	2.6	17.3	19.9	19.9
113	2.6	35	37.5	37.6
150	2.6	47.3	49.9	49.9
300	2.6	97.3	99.9	99.9

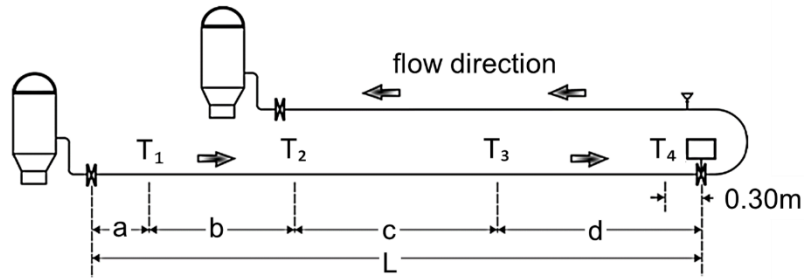


Figure 5. Representation of the experimental facility.



Figure 6. General overview of the test facility.

Experimental data are recorded on testing pipes of different external diameter-wall thickness (RD) ratio and under different initial load and flow conditions. [11] show the details of 37 tests and conclude that the wave celerity in HDPE PPI 4710 pipes is determined with Equation (16) (Figure 7).

$$a = 1423.6RD^{-0.503} \quad (16)$$

where

$$RD = \frac{D_{\text{ext}}}{e} \quad (17)$$

3.2. Numerical results

To apply the new method to calculate hydraulic transients in HDPE pipes using the standard solid model to represent the HDPE viscoelastic behaviour, the value for the long-term modulus of elasticity, E_1 , is obtained from the specifications of the resin used, so according to Polypipe, 2009, $E_1 = 32000$ (psi) = 220.6×10^6 (Pa).

The estimation of values for the Maxwell branch elements (Figure 2) is a main contribution of this work. In literature, there are not values for E_2 and η because these parameters can only be estimated by comparing numerical results against experimental data recorded during water hammer transients.

Once the wave velocity is known, it is possible to determine the value of parameter E_2 . Solving Equation (8), E_2 expressed in Pa is obtained with:

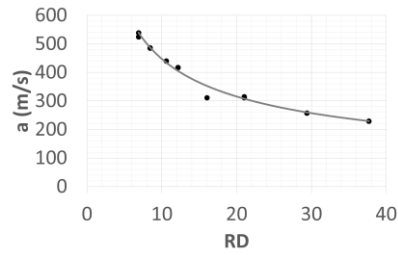


Figure 7. Wave celerity a as a function of the RD ratio. (Extracted from [11]).

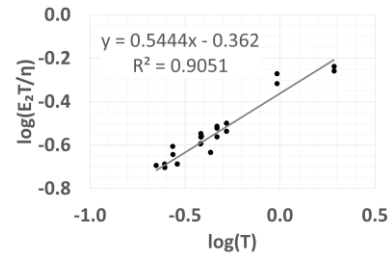


Figure 8. Relationship between the parameter E_2T/η and the characteristic time T .

$$E_2 = \frac{D_0}{e} \frac{K\rho_0 a^2}{K - \rho_0 a^2} - E_1 \quad (18)$$

The last parameter of the standard solid model is η and must be expressed in Pa.s. The pipe response is purely elastic as η grows up infinitely. The energy dissipation due to the pipe deformation depends on η and the period of time the pipe keep deformed due to the water hammer phenomena. The characteristic time of the pipe T measures the time during which a section of the pipe is subject to the action of the water hammer. T depends on the wave velocity and the distance the wave must travel.

Therefore, to estimate η values, a combination of η with E_2 and T are used. Figure 8 shows for the 37 tests performed at the test facility, the relationship between the parameter E_2T/η [-] and the characteristic time T [s]. The correlation between these variables is high, so it is proposed to determine η using the potential equation:

$$\frac{E_2T}{\eta} = 0.4345T^{0.5444} \quad (19)$$

where

$$T = \frac{2L}{a} \quad (20)$$

L is the pipe length [m]. Solving Equation (20) results

$$\eta = 2.3014E_2T^{0.4556} \quad (21)$$

The standard solid model can be easily calibrated due to the accuracy of the proposed method. Table 2 shows the calibration of the three parameters (E_1 , E_2 and η). Therefore, the proposed method can be generalized and it can be used as a supporting tool for the design of HDPE pipes water systems.

Table 2. Test conditions and values obtained for E_2 and η parameters.

Test length (m)	D_{ext} (m)	e (m)	RD	Q (m ³ /s)	a (m/s)	E_1 (Pa)	E_2 (Pa)	η (Pa s)
60	0.1145	0.0108	10.6	0.0094	438.93	2.206(10 ⁸)	1.6(10 ⁹)	2.24(10 ⁹)
150	0.0598	0.0037	16.0	0.0058	311.03	2.206(10 ⁸)	1.2(10 ⁹)	2.16(10 ⁹)

Figure 9 to Figure 12 show how high is the accuracy of the proposed method for calculating transient pressures in HDPE pipes. The experimental data recorded at points T2 and T4 are presented with dashed lines for the tests listed in Table 2. The hydraulic transients in HDPE pipes calculated with the proposed method are shown in black lines. The results without the standard solid model are

shown in grey lines. The damping of pressure waves can be noticed by comparing the black with the grey lines.

The results indicate that the proposed method based on the standard solid model is capable to reproduce with accuracy the damping of the transient phenomena over the entire length of the HDPE pipeline (Figure 9 to Figure12).

4. Conclusions

The proposed new method of calculation assumes that the wave celerity remains constant, reason why it is deduced that the variations in the diameter of the pipe do not significantly affect the development of the transient phenomena. Also, this is the reason why the conventional mesh of the MOC can be kept in use.

It is confirmed that the speed of the wave propagation can be calculated with the classic expression as a function of the dimension relationship of the pipe.

It is important to realize that the proposed method only works in the elastic deformation limits of the pipe walls.

The results obtained in this paper are calibrated for HDPE 4710 pipes. Nevertheless, no obstacles are identified to extend the application of the new method for pipes manufactured with other resins, or even for pipes manufactured with other viscoelastic materials.

Even though the results indicate that the proposed method is highly accurate, the precision can be increased for points far from the section where the transient occurs if two phenomena are considered in the proposed model. The first phenomenon to consider is the stress transmission between a pipe section and its adjacent sections. The second phenomenon to consider is the propagation of wave trains traveling in the pipe wall, induced by the hydraulic transient traveling in the water. These two phenomena cannot be modelled using the standard solid model to represent the viscoelastic material because this is a one-dimensional model. The introduction of these two phenomena into the proposed model is a future work.

Finally, another future work is to explorer if the liquid column separation phenomenon can be simulated with the proposed method.

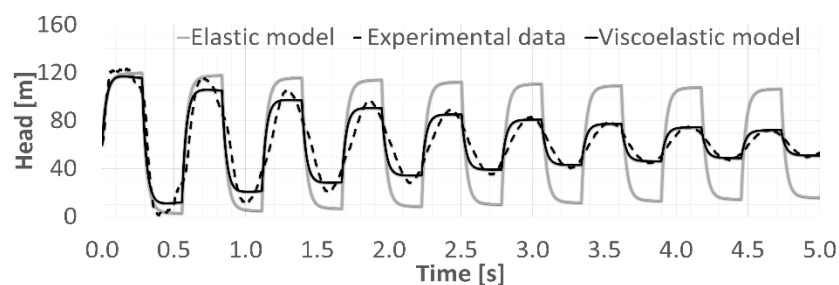


Figure 9. Data recorded and numerical results for point T4, and $L=60$ m.

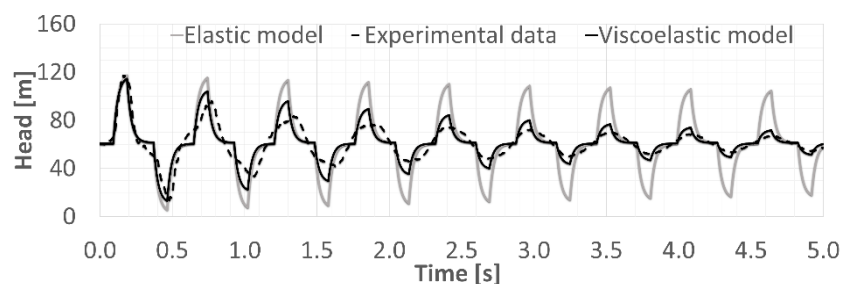


Figure 10. Data recorded and numerical results for point T2, and $L=60$ m.

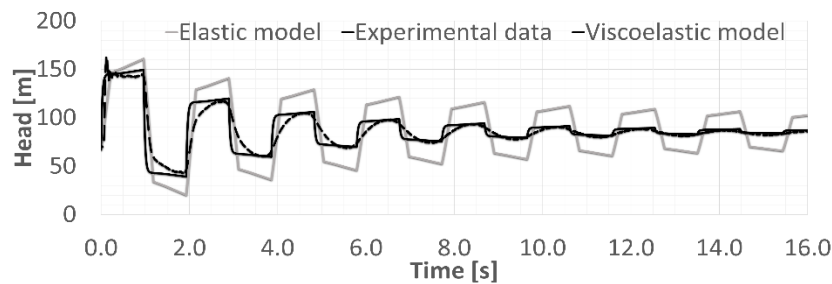


Figure 11. Data recorded and numerical results for point T4, and $L=150$ m.

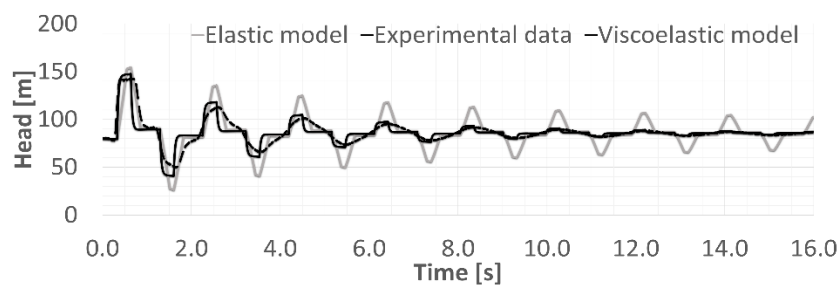


Figure 12. Data recorded and numerical results for point T2, and $L=150$ m.

References

- [1] Chaudhry M H 1979 *Applied Hydraulic Transients* (New York: Van Nostrand Reinhold Company)
- [2] Wylie E B and Streeter V L 1993 *Fluid Transientes in Systems* (NI, Englewood Cliffs: Prentice Hall)
- [3] Carmona-Paredes L and Carmona-Paredes R 2013 *Manual versión 2010.1.0 del sistema de simulación de transitorios hidráulicos en tuberías a presión Trans.* (México : Instituto de Ingeniería, UNAM)
- [4] Weinerowska-Bords K 2006 Viscoelastic model of waterhammer in single pipeline problems and questions *Archives of Hydro-Engineering and Environmental Mechanics* **53** 331–351
- [5] Covas D., Stoianov I, Mano J F, Ramos H, Graham N and Maksimovic C 2005 The dynamic effect of pipe-wall viscoelasticity in hydraulic transients. Part II-model development, calibration and verification *Journal of Hydraulic Research* **43** 56–70
- [6] Carmona L, Autrique R and Rodal E 2014 Comparación del transitorio hidráulico medido en tuberías de polietileno de alta densidad con resultados numéricos *Congreso Nacional de Hidráulica XXVI*
- [7] Soares A, Covas D and Reis L 2008 Analysis of pvc pipe-wall viscoelasticity during water hammer *Journal of Hydraulic Engineering* 1389–1394
- [8] Ramos H, Covas D, Borga A and Loureiro D 2004. Water hammer in pressurized polyethylene pipes: conceptual model and experimental analysis. *Urban Water Journal* **1** 177–197
- [9] Covas D, Stoianov I, Ramos H, Graham N, and Maksimovic C 2004 The dynamic effect of pipe-wall viscoelasticity in hydraulic transients. Part I-experimental analysis and creep characterization *Journal of Hydraulic Research* **42** 517–531
- [10] Paniagua-Lovera D 2017 *Golpe de ariete en tuberías de pared con comportamiento viscoelástico* Master's thesis (México : Posgrado de Ingeniería, UNAM)
- [11] Autrique R and Rodal E 2014 Medición experimental de celeridades de ondas de presión en tuberías de polietileno de alta densidad. *Congreso Latinoamericano de Hidráulica XXVI*