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An improved infinite composite-medium cylindrical source model for single U-tube ground heat exchanger

Changlong Wang, Huan Li, Zhijia Huang, Yuehong Lu, Gang Tang

School of Civil Engineering and Architecture, Anhui University of Technology,
Ma'anshan 243002, China

Email: clwang2017@163.com

Abstract. In this paper, a new heat transfer model for single U-tube ground heat exchanger is proposed. By considering the influence of U-tube heat capacity, an improved infinite composite-medium cylindrical source model is established to calculate the average fluid temperature, and then combined with a quasi-3D heat transfer model in borehole, the inlet and outlet fluid temperatures are derived as functions of average fluid temperature and the heat flow, therefore the inlet and outlet fluid temperatures of U-tube can be calculated. To verify the feasibility of the proposed model, it is applied to a sandbox experiment and compared to other models. The result shows that the inlet and outlet fluid temperatures calculated by the proposed model are accurate in the whole time period. Compared to other models, the inlet and outlet fluid temperatures calculated by the proposed model are in better agreement with the experimental data especially during short time, and the mean absolute percentage error, maximum absolute error, mean absolute error and root mean squared error are basically smaller. The mean absolute errors of the inlet and outlet fluid temperatures predicted are 0.11 °C and 0.09 °C, respectively, and the maximum absolute errors are less than 0.4 °C.

Keywords: ground heat exchanger; heat transfer; fluid temperature

1. Introduction

Ground-coupled heat pump is an energy-saving environment-friendly renewable energy technology. The core of this technology is the design of ground heat exchanger (GHE) (Fang et al. 2004). The rationality of its design determines the economy and reliability of ground-coupled heat pump system. The heat transfer model is the basis of the design of GHE, and thus has received extensive attention (Diao et al. 2006).

The early heat transfer models include the infinite line source (ILS) model and the infinite cylindrical source model and so on (Yang et al. 2010). However, these models ignore the influence of the borehole structure, and the calculated fluid temperatures have a large deviation. Lamarche (2015) simplified the single U-tube GHE as an equivalent pipe and considered the influence of fluid heat capacity, and established an infinite composite-medium cylindrical source (ICMCS) model, however, this model did not consider the thermal interference between the inlet and outlet pipes and assumed steady heat transfer in the equivalent pipe. Li et al. (2012) proposed an infinite composite-medium line source model and used the new g function to predict the fluid temperature. The results showed that the model was only applicable to short-term calculation. Zhang et al. (2016) introduced an improved



transient thermal resistance, and proposed a transient quasi-3D entire time scale line source model, but the fluid temperatures calculated by the model has a certain error. Yang et al. (2014) established a 2D numerical model by the finite volume method, which considers the heat capacity of circulating fluid by using transient boundary conditions on the inner wall of the tube, but the prediction value of the average fluid temperature had some errors. Rees et al. (2013) established a 3D numerical model to simulate the heat transfer of GHE and used the finite volume method for discrete solution, however, the calculation process of the model is complex and the calculation time is long.

Aiming at the deficiencies of the existing models of GHE, this paper establishes a new heat transfer model. The ICMCS model is improved by considering the influence of the U-tube heat capacity, and the average fluid temperature can be calculated, and then combined with a quasi-3D heat transfer model in borehole, the inlet and outlet fluid temperatures are derived as functions of average fluid temperature and the heat flow. In order to verify the feasibility of the proposed model, it is applied to a sandbox experiment (Beier et al. 2011), and the calculated inlet and outlet fluid temperatures are compared with the experimental values and the results of other models.

2. Heat transfer model of GHE

The single U-tube GHE is shown in Figure 1, and its section is shown in Figure 2.

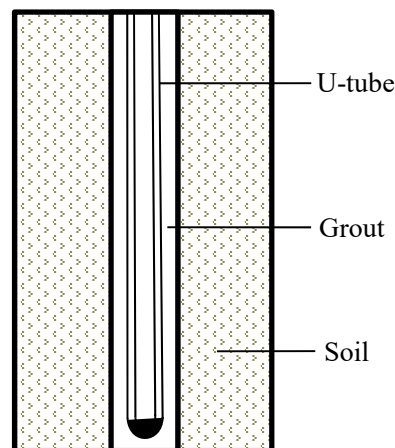


Figure 1. Schematic diagram of single U-tube GHE.

This paper simplifies the single U-tube as an equivalent pipe based on the ICMCS model. That is, Figure 2 is simplified to Figure 3. On the basis of the ICMCS model (Lamarche 2015), the influence of the U-tube heat capacity is considered, and an improved ICMCS model is established to calculate the average fluid temperature. Then, combined with a quasi-3D heat transfer model in borehole, the inlet and outlet fluid temperatures are derived as functions of average fluid temperature and the heat flow.

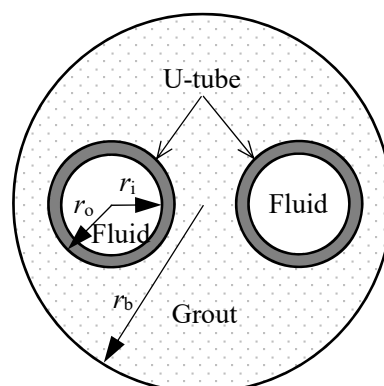
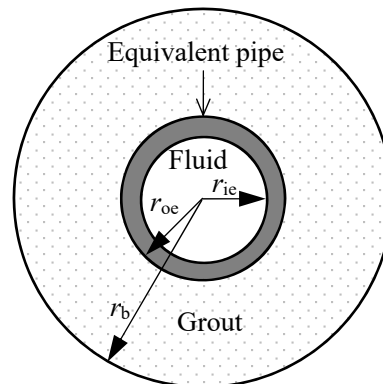


Figure 2. Single U-tube GHE section.**Figure 3.** Equivalent pipe section.

2.1. Improved ICMCS model to calculate the average fluid temperature

According to the ICMCS model, the average fluid temperature $T_f(t)$ is calculated as the following Eq. (1):

$$T_f(t) = T_0 + \frac{Q}{H k_g} \frac{8 k_0}{\pi^5 \delta^2} \int_0^\infty \frac{1 - \exp \left[-\beta^2 k_g t / (\rho c)_{ge} / r_b^2 \right]}{\beta^5 (\Phi^2 + \psi^2)} d\beta \quad (1)$$

where T_0 is the initial temperature, °C; Q is the heat flow, W; H is U-tube length, m; k_g is the thermal conductivity of the grout, $W \cdot m^{-1} \cdot K^{-1}$; β is the integration variable; t is the time, s; $(\rho c)_{ge}$ is the equivalent volumetric heat capacity of the grout, $J \cdot m^{-3} \cdot K^{-1}$; r_b is the borehole radius, m; k_0 , δ , Φ and ψ are intermediate variables:

$$k_0 = \frac{k_s}{k_g} \quad (2)$$

$$\delta = \frac{r_b}{r_{oe}} \quad (3)$$

$$\Phi = \xi_1 [Y_0(\beta \delta \gamma) J_1(\beta \delta) - Y_1(\beta \delta \gamma) J_0(\beta \delta) k_0 \gamma] - \xi_2 [Y_0(\beta \delta \gamma) Y_1(\beta \delta) - Y_1(\beta \delta \gamma) Y_0(\beta \delta) k_0 \gamma] \quad (4)$$

$$\psi = \xi_2 [J_0(\beta \delta \gamma) Y_1(\beta \delta) - J_1(\beta \delta \gamma) Y_0(\beta \delta) k_0 \gamma] - \xi_1 [J_0(\beta \delta \gamma) J_1(\beta \delta) - J_1(\beta \delta \gamma) J_0(\beta \delta) k_0 \gamma] \quad (5)$$

where k_s is the thermal conductivity of the soil, $W \cdot m^{-1} \cdot K^{-1}$; r_{oe} is the outer radius of the equivalent pipe, m; Y_0 is the zero-order Bessel function of the second kind; J_1 is the first-order Bessel function of the first kind; Y_1 is the first-order Bessel function of the second kind; J_0 is the zero-order Bessel function of the first kind; γ , ξ_1 and ξ_2 are intermediate variables:

$$\gamma = \sqrt{\frac{k_g (\rho c)_s}{k_s (\rho c)_{ge}}} \quad (6)$$

$$\xi_1 = (1 - 2 \pi k_g R_p \nu \beta^2) Y_1(\beta) - \nu \beta Y_0(\beta) \quad (7)$$

$$\xi_2 = (1 - 2 \pi k_g R_p \nu \beta^2) J_1(\beta) - \nu \beta J_0(\beta) \quad (8)$$

where $(\rho c)_s$ is the volumetric heat capacity of the soil, $J \cdot m^{-3} \cdot K^{-1}$; R_p is the thermal resistance between the fluid and the grout, $m \cdot K \cdot W^{-1}$; ν is intermediate variable. The formulas of R_p and ν are as follows:

$$R_p = \frac{1}{4 \pi r_i h} + \frac{\ln(r_o / r_i)}{4 \pi k_p} \quad (9)$$

$$\nu = \frac{r_{ie}^2 (\rho c)_{fe}}{2 r_{oe}^2 (\rho c)_{ge}} \quad (10)$$

where r_i is the inner radius of the U-tube, m; h is the convection heat transfer coefficient of the fluid, $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$; r_o is the outer radius of the U-tube, m; k_p is the thermal conductivity of the U-tube and the equivalent pipe, $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$. r_{ie} is the inner radius of the equivalent pipe, m.

On the basis of the ICMCS model, the influence of the U-tube heat capacity is considered, the intermediate variable ν can be modified, and Eq. (10) is modified to Eq. (11):

$$\nu = \frac{r_{ie}^2(\rho c)_{fe} + (r_{oe}^2 - r_{ie}^2)(\rho c)_{pe}}{2r_{oe}^2(\rho c)_{ge}} \quad (11)$$

where $(\rho c)_{fe}$ is the equivalent volumetric heat capacity of the fluid, $\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$; $(\rho c)_{pe}$ is the equivalent volumetric heat capacity of the pipe, $\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$.

The formulas of r_{oe} and r_{ie} are as follows:

$$r_{oe} = r_b \exp \left[-2\pi k_g (R_b - R_p) \right] \quad (12)$$

$$r_{ie} = r_{oe} \exp \left(-\frac{1}{2} \ln \frac{r_o}{r_i} \right) \quad (13)$$

where R_b is the borehole thermal resistance, $\text{m}\cdot\text{K}\cdot\text{W}^{-1}$. The formulas of $(\rho c)_{fe}$, $(\rho c)_{pe}$ and $(\rho c)_{ge}$ are as follows:

$$(\rho c)_{fe} = \frac{2r_i^2(\rho c)_f}{r_{ie}^2} \quad (14)$$

$$(\rho c)_{pe} = \frac{2(r_o^2 - r_i^2)(\rho c)_p}{r_{oe}^2 - r_{ie}^2} \quad (15)$$

$$(\rho c)_{ge} = \frac{(r_b^2 - 2r_o^2)(\rho c)_g}{r_b^2 - r_{oe}^2} \quad (16)$$

where $(\rho c)_f$ is the volumetric heat capacity of the fluid, $\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$; $(\rho c)_p$ is the volumetric heat capacity of the pipe, $\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$; $(\rho c)_g$ is the volumetric heat capacity of the grout, $\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$.

2.2. Derivation of inlet and outlet fluid temperatures

The single U-tube GHE is divided into S elements, and the dimensionless depth at the bottom of the m -th element is z_m , $z_m = m/S$, and $0 \leq m \leq S$. According to the quasi-3D heat transfer model proposed by Diao et al. (2006) and Zeng et al. (2003), the fluid temperatures in the inlet and outlet pipes at the depth of z_m are as follows:

$$T_{f1}(z_m, t) = T_b(t) + \Theta_1(z_m) \times [T_{in}(t) - T_b(t)] \quad (17)$$

$$T_{f2}(z_m, t) = T_b(t) + \Theta_2(z_m) \times [T_{in}(t) - T_b(t)] \quad (18)$$

where $T_{f1}(z_m, t)$, $T_{f2}(z_m, t)$ are the fluid temperatures in the inlet and outlet pipes at the depth of z_m , respectively, $^{\circ}\text{C}$; $\Theta_1(z_m)$ and $\Theta_2(z_m)$ are the dimensionless temperatures in the inlet and outlet pipes at the depth of z_m , respectively; $T_{in}(t)$ is the inlet fluid temperature, $^{\circ}\text{C}$; $T_b(t)$ is the borehole wall temperature, $^{\circ}\text{C}$.

Since Eqs. (17) and (18) are applicable to the fluid temperatures at different depths, the following equations can be obtained:

$$\sum_{m=0}^S T_{f1}(z_m, t) = (S+1)T_b(t) + \sum_{m=0}^S \Theta_1(z_m) [T_{in}(t) - T_b(t)] \quad (19)$$

$$\sum_{m=0}^S T_{f2}(z_m, t) = (S+1)T_b(t) + \sum_{m=0}^S \Theta_2(z_m) [T_{in}(t) - T_b(t)] \quad (20)$$

As the average fluid temperature $T_f(t)$ is equal to the average temperature of the fluid in the inlet and outlet pipes, then:

$$T_f(t) = \frac{1}{2(S+1)} \left[\sum_{m=0}^S T_{f1}(z_m, t) + \sum_{m=0}^S T_{f2}(z_m, t) \right] \quad (21)$$

By combining Eqs. (19), (20) and (21), we can obtain the following equation:

$$2(S+1)[T_f(t) - T_b(t)] = \sum_{m=0}^S [\Theta_1(z_m) + \Theta_2(z_m)] [T_{in}(t) - T_b(t)] \quad (22)$$

As the outlet fluid temperature $T_{\text{out}}(t)=T_{f2}(z_0,t)$ and $T_{f2}(z_0,t)$ satisfies Eq. (18), the following equation can be obtained:

$$T_{\text{out}}(t) = T_b(t) + \Theta_2(z_0)[T_{\text{in}}(t) - T_b(t)] \quad (23)$$

Since the outlet fluid returns to the inlet pipe after the influence of Q , the relationship between the inlet and outlet fluid temperatures is as follows:

$$M c_f [T_{\text{in}}(t) - T_{\text{out}}(t)] = Q \quad (24)$$

where M is the fluid mass flow rate, kg/s.

By combining Eqs. (22), (23) and (24), the inlet and outlet fluid temperatures can be calculated:

$$T_{\text{in}}(t) = T_f(t) + \frac{Q}{M c_f} \frac{2(S+1) - \sum_{m=0}^S [\Theta_1(z_m) + \Theta_2(z_m)]}{2(S+1)[1 - \Theta_2(z_0)]} \quad (25)$$

$$T_{\text{out}}(t) = T_f(t) + \frac{Q}{M c_f} \frac{2(S+1)\Theta_2(z_0) - \sum_{m=0}^S [\Theta_1(z_m) + \Theta_2(z_m)]}{2(S+1)[1 - \Theta_2(z_0)]} \quad (26)$$

3. Results and discussion

In order to verify the accuracy of the proposed model, it is applied to the sandbox experiment conducted by Beier et al. (2011), and the parameters are shown in Table 1.

Table1. Detail parameters for the sandbox experiment.

Parameter	Value
Borehole radius r_b/m	0.063
U-tube length H/m	18.32
Outer radius of the U-tube r_o/m	0.0167
Inner radius of the U-tube r_i/m	0.013665
Thermal conductivity of the soil $k_s/\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$	2.82
Volumetric heat capacity of the soil $(\rho c)_s/\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	2.0×10^6
Thermal conductivity of the grout $k_g/\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$	0.73
Volumetric heat capacity of the grout $(\rho c)_g/\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	3.8×10^6
Thermal conductivity of the U-tube $k_p/\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$	0.39
Volumetric heat capacity of the U-tube $(\rho c)_p/\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	2.2×10^6
Thermal conductivity of the fluid $k_f/\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$	0.618
Volumetric heat capacity of the fluid $(\rho c)_f/\text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$	4.17×10^6
Specific heat capacity of the fluid $c_f/\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	4180
Borehole thermal resistance $R_b/\text{m}\cdot\text{K}\cdot\text{W}^{-1}$	0.158
Fluid mass flow rate $M/\text{kg}\cdot\text{s}^{-1}$	0.2
Initial temperature $T_0/^\circ\text{C}$	22

The inlet and outlet fluid temperatures predicted by the proposed model are compared with experimental data and those predicted by the ILS model (Yang et al. 2010) and the ICMCS model (Lamarche 2015), as shown in Figure 5 and Figure 6. For the ILS model, the calculated fluid temperatures match the experimental data only when the time is long enough, and the temperature prediction errors are large in a short time; for the ICMCS model, the calculated fluid temperatures are not accurate enough in a short time and are in good agreement after a long time; the fluid temperatures predicted by the proposed model agree well with the experimental data over the whole time period.

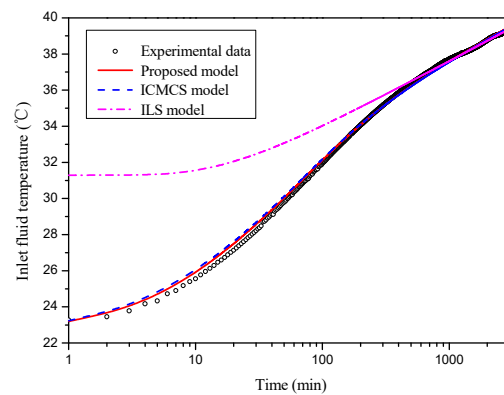


Figure 4. Comparison of inlet fluid temperatures predicted by each model with experimental data.

At the same time, the absolute errors of inlet and outlet fluid temperatures predicted by the proposed model are compared with that predicted by ICMCS model. The results are shown in Figure 6. It can be seen from the figure that the absolute errors of the inlet and outlet fluid temperatures predicted by the proposed model in the first two hours are basically smaller than that of the ICMCS model, and the differences between the proposed model and the ICMCS model are very small during the rest time, which indicates that the proposed model has a high prediction accuracy for the inlet and outlet fluid temperature in a short time. In addition, the absolute errors of the fluid temperatures predicted by the proposed model are within 0.4 °C for the entire period of time.

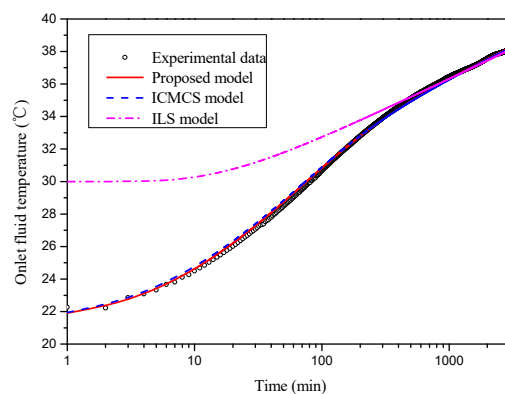


Figure 5. Comparison of outlet fluid temperatures predicted by each model with experimental data.

In the reference (Zhang et al. 2016), the calculation formulas of mean absolute percentage error (MAPE), mean absolute error (MAE), maximum absolute error (ME) and root mean squared error (RMSE) of inlet and outlet fluid temperatures were defined. Based on these calculation formulas, the four errors of the inlet and outlet fluid temperatures predicted by the proposed model are calculated and compared with the results of other models, as shown in Table 2. It can be seen that the errors of fluid temperatures predicted by the proposed model are basically smaller.

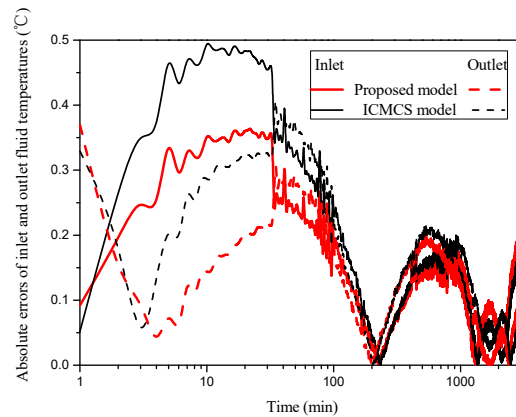


Figure 6. Absolute errors of the inlet and outlet fluid temperatures calculated by the proposed model and the ICMCS model.

Table 2. Errors of inlet and outlet fluid temperatures predicted by each model.

Model	MAE (°C)		ME (°C)		MAPE (%)		RMSE (°C)	
	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet
ILS model	0.25	0.27	8.00	7.77	3.48	6.18	0.78	0.77
ICMCS model	0.11	0.10	0.51	0.43	0.80	0.89	0.13	0.12
Proposed model	0.11	0.09	0.38	0.37	0.80	0.75	0.13	0.11

4. Conclusion

A new heat transfer model for single U-tube GHE is proposed. Based on ICMCS model, the influence of U-tube heat capacity is considered, and an improved ICMCS model is established to calculate the average fluid temperature, and then combined with the quasi-3D heat transfer model in borehole, the inlet and outlet fluid temperatures are derived as functions of average fluid temperature and the heat flow.

In order to verify the feasibility of the proposed model, it is applied to the sandbox experiment and compared with the ILS model and the ICMCS model. Compared with other models, the fluid temperatures predicted by the proposed model are in better agreement with the experimental data especially in a short time, besides, all kinds of the errors of the predicted fluid temperatures are basically smaller. The average absolute errors of the inlet and outlet fluid temperatures are 0.11 °C and 0.09 °C, respectively, and the maximum absolute errors are less than 0.4 °C. The model proposed in this paper has high precision.

Acknowledgments

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