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On-line Estimation Algorithms for Mathematical Model Parameters of Lithium Batteries

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Abstract. In the state model of potassium batteries, the discharge rate proportional coefficient and temperature proportional coefficient compensated by second-order fitting polynomial are proposed. The mathematical model is improved and the concrete steps to determine the above two coefficients are given. The parameter estimation technology of lithium battery observation model is studied. The algorithm and experimental results of parameter estimation of mathematical model based on the algorithm and sampling point *Kalman* filter algorithm are given. The experimental results show that the improved lithium battery model and parameter estimation method proposed in this paper can accurately describe the discharge characteristics of lithium battery. The online estimation algorithm of model parameters based on *Kalman* filter at sampling points can lay a good foundation for the embedded implementation of the subsequent estimation system.

1. Introduction

In order to estimate the residual capacity of lithium batteries better, first of all, lithium batteries must be modeled to obtain more accurate model eucalyptus, and according to the obtained model, the final estimation of the residual capacity of lithium batteries must be completed. Generally speaking, the physical quantities which are closely related to the residual capacity of lithium battery include the battery terminal voltage, the charging and discharging current of the battery, the working temperature of the battery, the internal resistance of the battery and so on. Modeling of batteries means using certain models to directly or indirectly describe the relationship between these physical quantities and the remaining battery capacity. On the other hand, the appropriate battery model must accurately reflect the working mechanism of the battery, reflect the main parameters affecting the remaining electricity, and accurately match the charging and discharging characteristics of the battery.

2. Estimation of Model Parameters for Lithium Batteries

The undetermined parameters of lithium battery mathematical model mainly come from the parameters in the observation model [1]:

$$p = [K_0 \ R \ K_1 \ K_2 \ K_3 \ K_4]^T$$

Generally speaking, there are two main methods to determine this parameter: off-line method and on-line method. Off-line method is to obtain a set of test data through preview, and estimate the parameters offline, while on-line method is to synchronously complete the parameter estimation while acquiring the test data.



2.1 *LS -based model parameter estimation algorithm*

The most commonly used offline parameter estimation algorithm is Least Square (*LS* algorithm [2]). *LS* algorithm searches for the best function matching of data by minimizing the square of error.

Therefore, the least square method can be used to obtain unknown data conveniently, and the error between the unknown data and the actual data can be equalized. The sum of squares is the smallest.

The steps of parameter estimation of lithium battery mathematical (observation) model based on *LS* method are as follows [3]:

1) At room temperature of 25 degrees, the charged batteries are discharged at constant current with a discharge rate of 1/30 until the power consumption is exhausted.

2) In the discharge process, the terminal voltage y_s of the battery at s time is measured with time interval Δt . $s = 0, 1, 2, \dots, M$, Among them, $s = 0$ corresponds to the initial discharge time after the battery is charged, while $s = M$ corresponds to the termination time of the battery's power consumption after discharge.

3) Calculating the Residual Electricity at s -time, $z_s = 1 - s/M$.

$$4) \quad Y = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_M \end{bmatrix}, \quad H = \begin{bmatrix} 1 - \frac{C}{30} - \frac{1}{z_0} - z_0 & \ln z_0 & \ln(1 - z_0) \\ 1 - \frac{C}{30} - \frac{1}{z_1} - z_1 & \ln z_1 & \ln(1 - z_1) \\ \vdots & \vdots & \vdots \\ 1 - \frac{C}{30} - \frac{1}{z_M} - z_M & \ln z_M & \ln(1 - z_M) \end{bmatrix}, \quad p = \begin{bmatrix} K_0 \\ R \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix}.$$

Then, according to *LS* algorithm, the optimal estimation results of model parameters can be obtained as follows: $p = (H^T H)^{-1} H^T Y$.

The main problem of *LS* algorithm is that it needs a large amount of storage space to store all data of a complete discharge process. Because of the need for multiplication and inversion of large matrix, the computation complexity of *LS* algorithm is high, which is not conducive to the implementation of embedded system or single chip computer. For this reason, for the same type or specification of batteries, the parameters based on *LS* model often need only to be determined once. Once the parameters are determined, they will be directly used as known constants in the modeling of the same type or specification of batteries. However, it is difficult to apply to the aging treatment of batteries.

2.2 *Model Parameter Estimation Based on Sampling Point Kalman Filter*

On-line model parameter estimation algorithm completes the step-by-step estimation process of model parameters by iteration algorithm. For example, the classical *RLS* algorithm is one of the online estimation algorithms. In this paper, we apply the Sample Point *Kalman* Filter (*SPKF*) algorithm to the estimation of battery model parameters.

Consider the following discrete-time nonlinear systems:

$$x_{k+1} = f(x_k, u_k) + \omega_k \quad (1)$$

$$y_k = g(x_k, u_k) + v_k \quad (2)$$

Among them, x_k is the L-dimensional state vector of the k-time system, u_k is the control input vector, y_k is the measurement vector of the system, at least one of the two functions of f and g is non-linear, while ω_k and v_k deal with noise and measurement noise respectively.

Because the functions of f and g are non-linear, it is impossible to estimate the state of battery system directly using *Kalman* filter. An alternative mathematical method is to expand them with *Taylor* series and take their first-order linear terms. After linear approximation of the original function, *Kalman* filter algorithm is used to estimate the battery state. 1) When the *Taylor* expansion of the nonlinear function can not be ignored, the simple linearization process will lead to large errors in the system estimation, and even make the filter not convergent and unstable.

2) The *Jacobian* matrix needs to be calculated repeatedly in each filtering cycle. Therefore, for complex high-order systems, the computation amount of filtering estimation will be very large, and it will be difficult for ordinary *ARM* processor or single chip computer to complete real-time. *SPKF* is another iteration type of least mean square error estimator. Like *EKF* . it still belongs to the framework of the optimal Gaussian approximation *Kalman* filter [4]. Row approximation. In *SPKF* , a $2L+1$ sampling point is constructed from the optimal estimation of the state according to a certain distribution, and the corresponding weights of each sampling point are given, as shown in Figure 1.

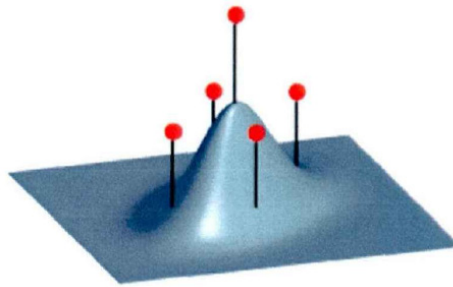


Figure 1. Weighted sampling points under Gaussian distribution

These sampling points fully possess the mean and covariance of the state variables. The state estimation and observation estimation of the system are the weighted structure of these sampling points propagating through the actual non-linear system, so this process is called Unscented Transform (*UT*) [4]. The diagram of *UT* transformation is shown in Figure 2. The mean and variance of *UT* transform are used to update the state estimation and observation estimation respectively

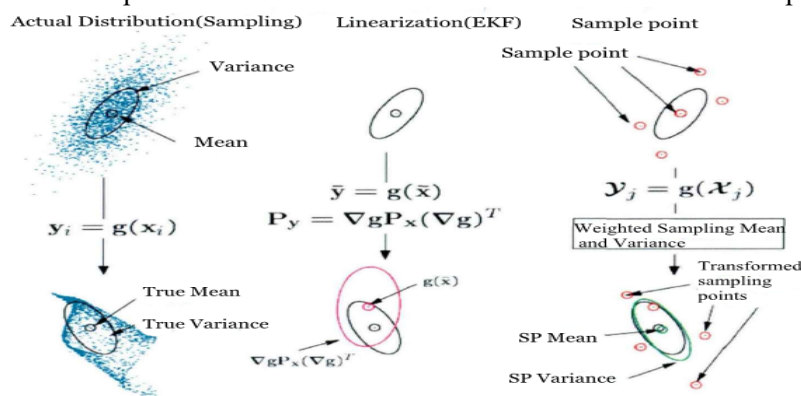


Figure 2. Mean and Variance Propagation Processes in *EKF* and *UT*

According to the *Kalman* filtering process, and finally the whole filtering process is completed. S. J. Julier et al. have shown that the posterior mean and covariance of any nonlinear system can be

accurately reduced to the second order of the *Taylor* series expansion by using this process, so that the error only exists in the third order and above.

In fact, because of its excellent performance, *SPKF* has been widely used in the state estimation of non-linear systems [5-8]. In addition, *SPKF* is also applied to model parameter estimation. Its basic idea is to take model parameters as state vectors to be estimated, and then use *SPKF* state estimation algorithm to directly estimate model parameters.

For parameter estimation of lithium battery model, in order to estimate its parameter $p = [K_0 \ R \ K_1 \ K_2 \ K_3 \ K_4]^T$, the following equation of state is used:

$$p_{k+1} = p_k + \omega_k \quad (3)$$

1) At room temperature of 25 degrees Celsius, the charged battery is discharged at a constant current of 1/30 times the rated current until the power consumption is exhausted.

2) In the discharge process, the terminal voltage y_s of the battery at s time is measured with time interval Δt . $s = 0, 1, 2, \dots, M$. Among them, $s = 0$ corresponds to the initial discharge time after the battery is charged, while $s = M$ corresponds to the termination time of the battery's power consumption after discharge.

3) Calculating the Residual Electricity at s -time, $z_s = 1 - s/M$.

4) Perform the initialization process:

① Select any initial model parameters:

$$p = p_0 \quad (4)$$

② Square Root Mean Variance Matrix for Setting Initial Model Parameters S_{p_0} :

$$S_{p_0} = I_6 \quad (5)$$

I_6 is the unit matrix of 6×6 .

③ Select proportional constant h .

$$h > 1 \quad (6)$$

④ Set the observation noise variance R_r :

$$R_r = \sqrt{10^{-3}} I_6 \quad (7)$$

⑤ Set weighting coefficients $W_0^{(m)}$:

$$W_0^{(m)} = \frac{h^2 - 7}{h^2}, W_i^{(m)} = \frac{1}{2h^2}, W_i^{(c1)} = \frac{1}{2h}, W_i^{(c2)} = \frac{\sqrt{h^2 - 1}}{2h^2}, i = 1, 2, \dots, 12 \quad (8)$$

For $s = 1, 2, \dots, M$, iterate successively according to the following steps (5) ~ (7):

5) Computing time domain updates:

① Estimates of the parameters of the computational model p_s^- :

$$p_s^- = p_{s-1} \quad (9)$$

② Estimation of Square Root Mean Variance Matrix for Calculating Model Parameters $S_{p_s}^-$:

$$S_{p_s}^- = S_{p_{s-1}} + D_{r_{s-1}} \quad (10)$$

$$D_{r_{s-1}} = -diag\{S_{p_{s-1}}\} + \sqrt{diag\{S_{p_{s-1}}\}^2 + diag\{R_r\}} \quad (11)$$

$diag\{\cdot\}$ is a column vector composed of diagonal elements of the corresponding matrix.

- 6) Sampling Point Sequence for Calculating p_s^- : $\mathbf{x}_{s|s-1}^p = [p_s^- \quad p_s^- + hS_{p_s}^- \quad p_s^- - hS_{p_s}^-]^T$:

Among them, p_s^- is a 6×1 column vector, $S_{p_s}^-$ is a 6×6 matrix, so $\mathbf{x}_{s|s-1}^p$ is a 6×13 matrix.

- 7) Calculate and update the measurements according to the following formulas:

- ① Calculating the observation sequence of sampling points $\mathbf{R}_{s|s-1}$, $\mathbf{R}_{s|s-1}$ is a 6×13 matrix:

$$\mathbf{R}_{s|s-1} = g(\mathbf{x}_{s|s-1}^p, i_s, z_s) \quad (12)$$

- ② Calculating the estimated value \hat{d}_s^- of observation sequence $\mathbf{R}_{s|s-1}$, and $\mathbf{R}_{i|s-1}$ is column i of $\mathbf{R}_{s|s-1}$

$$\hat{d}_s^- = \sum_{i=0}^{12} W_i^{(m)} \mathbf{R}_{i,s|s-1} \quad (13)$$

- ③ Calculating the Square Root Mean Variance Matrix $S_{d_s^-}$ of the Observation Sequence $\mathbf{R}_{s|s-1}$:

$$S_{d_s^-} = qr \left\{ \sqrt{W_1^{(c1)}} (\mathbf{R}_{1:6,s|s-1} - \mathbf{R}_{7:12,s|s-1}) \sqrt{W_1^{(c2)}} (\mathbf{R}_{1:6,s|s-1} - \mathbf{R}_{7:12,s|s-1} - 2\mathbf{R}_{0,s|s-1}) \right\} \quad (14)$$

- ④ Calculating the variance matrix $P_{p_s d_s}$:

$$P_{p_s d_s} = \sqrt{W_1^{(c1)}} S_{p_s}^- [\mathbf{R}_{1:6,s|s-1} - \mathbf{R}_{7:12,s|s-1}]^T \quad (15)$$

- ⑤ Calculating *kalman* Gain K_s :

$$K_s = (P_{p_s d_s} / S_{d_s}^T) / S_{d_s}^- \quad (16)$$

- ⑥ Update of calculation parameters p_s :

$$p_s = p_s^- + K_s (y_s - \hat{d}_s^-) \quad (17)$$

- ⑦ Calculating temporary variables U :

$$U = K_s S_{d_s}^- \quad (18)$$

- ⑧ Update of Square Root Mean Variance Matrix for Calculating Model Parameters S_{p_s} :

$$S_{p_s} = cholupdate\{S_{p_s}^-, U, -1\} \quad (19)$$

$qr\{\cdot\}$ represents the orthogonal triangular decomposition of the matrix and returns the upper triangular matrix. $(\cdot)^T$ is a matrix transposition operation; $cholupdate\{S_{p_s}^-, U, -1\}$ represents the *Cholesky* decomposition of the matrix $(S_{p_s}^-)^T * S_{p_s}^- - U * U^T$.

3. Mathematical Model Estimation of Lithium Battery Experiments

We have simulated the estimation algorithm of the lithium battery mathematical model proposed in this paper. All the experimental data are obtained through the lithium battery charging and discharging experimental platform, which is mainly composed of high and low temperature programmable test box, battery internal resistance tester, programmable DC electronic load, lithium battery data acquisition module, screen display module, communication module, machine, DC power supply and so on. The batteries used in the test are all lithium iron phosphate batteries produced by Zhejiang Wanxiang Electric Vehicle Co., Ltd. with nominal voltage of 3.2V and nominal capacity of 15.5Ah.

3.1 Discharge Rate Proportional Coefficient Model

Under the condition of 25 degrees Celsius at room temperature, using single lithium batteries as experimental objects, the full discharge of Li-ion batteries was carried out with different discharge currents from 1A, 2A,... And 15A. After fitting the optimal curve with formula (1), the ratio coefficient of discharge rate was obtained as follows:

$$[a, b, c] = [3.9291, -123.46, 15027] \quad (20)$$

3.2 Temperature Proportional Coefficient Model

Similar to the above-mentioned discharge rate proportional coefficient model, single lithium batteries were placed in a constant temperature environment of -20°C , 0°C , 20°C , 25°C , 40°C , 60°C under fully charged condition. The total discharge capacity of batteries at different ambient temperatures was obtained by discharging the batteries at standard C/30 rate. After curve fitting, the final parameters of the model are as follows:

$$[p, q, s] = [-0.001, 0.0075, 0.8255] \quad (21)$$

3.3 Parameter Estimation Results of Observation Model

Through the calculation of *MATLAB*, the parameters of the observation model obtained by *LS* algorithm are as follows:

$$p = [3.374 \quad 0.005 \quad -3.74e^{-5} \quad 0.119 \quad 0.093 \quad -0.0198]^T \quad (22)$$

The fitting degree between the obtained model and the original data is shown in Figure. 3. In the figure, the fitting of the observed data based on the observed model obtained by the algorithm is given when the complete discharge process, the data of smaller part ($SOC < 10\%$) of SOC and the data of larger part $SOC > 90\%$ of SOC are neglected respectively. Condition.

$$p = [3.36 \quad 0.0039 \quad -5.87e^{-0.5} \quad 0.092 \quad -0.013]^T \quad (23)$$

The comparison formula (23) and formula (22) show that the observation models obtained by the two algorithms are basically the same. Compared with *LS* algorithm, the advantage of sampled-point *Kalman* filter algorithm is that the parameters of the model can still be calculated online without storing a large number of measurements and digging. The algorithm has low complexity and is easy to be implemented by an embedded processor.

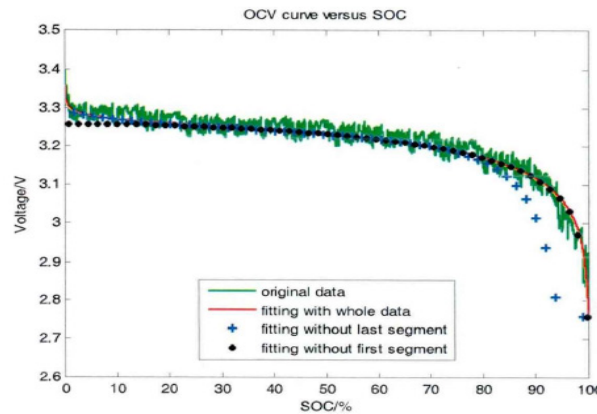


Figure 3. *LS* algorithm fits the actual data of different parts of the curve

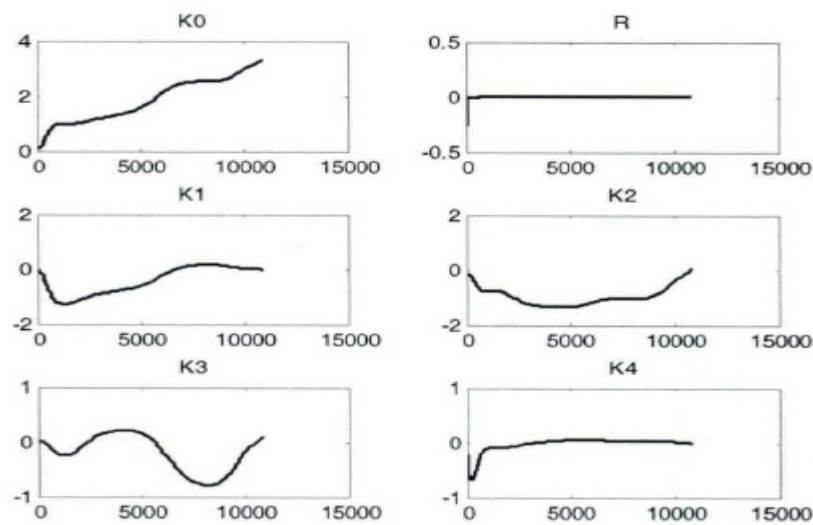


Figure 4. Parameter Estimation of Observation Model Based on Sampling Point *Kalman* Filter

3.4 Dynamic updating of Model Parameters

Because the sampling point *Kalman* filter algorithm can be carried out online, the dynamic updating of lithium battery observation model is carried out by using the strategy shown in Figure. 5 in practical application. That is to say, after a certain number of cycles have been accumulated for new or intermediate batteries, the parameters of primary battery model are re-estimated by using the above online estimation algorithm. Using this strategy, a more accurate lithium battery model can be obtained, and the influence of model parameters changes on estimation due to battery aging or cell differences can be reduced.

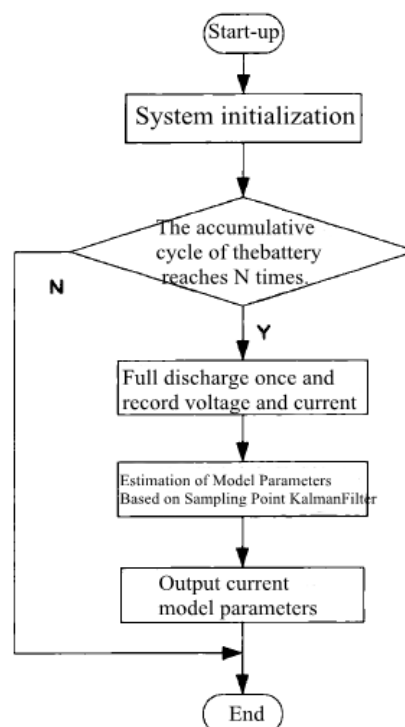


Figure 5. Estimation Strategy of Model Parameters Considering Battery Aging

4. Conclusion

Based on the analysis of the working mechanism of lithium batteries, this paper summarizes the battery parameters related to SOC lithium batteries, and then briefly summarizes the current commonly used models of potassium batteries. On this basis, a mathematical model based on open-circuit voltage, discharge current and other external characteristics of batteries is emphatically studied. In order to describe the effect of discharge rate and operating temperature on the residual capacity of potassium batteries, the mathematical model was improved by using second-order fitting polynomial compensation, and the specific steps to determine the two coefficients were given. The parameter estimation technology of lithium battery observation model is studied. The algorithm and experimental results of parameter estimation of mathematical model based on the algorithm and sampling point *Kalman* filter algorithm are given. The experimental results show that the improved lithium battery model and parameter estimation method proposed in this paper can accurately describe the discharge characteristics of lithium battery. The online estimation algorithm of model parameters based on *Kalman* filter at sampling points can lay a good foundation for the embedded implementation of the subsequent estimation system.

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