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The Electric Field Generated by Time-harmonic Vertical Electric Dipole in the Two-layer Model

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Abstract. The extremely low frequency electric fields generated by time-harmonic electric dipole can be used for telecommunication, geophysical exploration and electromagnetic information of earthquake precursor. There are two main purposes for the analytical solution of the electric fields with the electric dipole. One is that the field source can be described by the dipole, the other is that arbitrary field source can be obtained by the superposing of the electric dipole. By means of the image method, the general expression of electric fields generated by vertical time-harmonic electric dipole embedded in the second layer of stratified conducting half-space is derived. Firstly, with the boundary conditions, the magnetic vector potential is solved. Secondly, the expression of the electric field is derived from Maxwell's equation. Finally, using the partial differential operators, analytical solution can be obtained.

1. Introduction

Electric dipole is one of the most basic sources of electromagnetic field. It is a system consisting of two equal, different point charges. Electric dipole moment is used to describe the characteristics of electric dipole. Electromagnetic waves excited by the time-harmonic vertical electric dipole in layered media have received many concerns all the time. There are many methods to acquire the expressions of the electric dipole[1~4]. Assuming that the second half space is the uniform, linear and isotropic medium, the electric field generated by time-harmonic vertical electric dipole is acquired based on the image theory with the magnetic vector potential and the boundary conditions.

2. Vector magnetic potential of an electromagnetic field in a conducting medium

In a homogeneous, linear, isotropic medium, suppose the dielectric constant is ε, the permeability is μ, the conductivity is σ and angular frequency is ω. The magnitude of the field source varies sinusoidal with time. When the harmonic factor is $e^{j\omega t}$, the Maxwell's equations can be expressed as [5]:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_s + (\sigma + j\omega\varepsilon)\boldsymbol{E}$$
$$\nabla \times \boldsymbol{E} = -j\omega \boldsymbol{B}$$
$$\nabla \cdot \boldsymbol{D} = \rho$$

Where $\boldsymbol{H}$ is the magnetic field intensity, $\boldsymbol{J}_s$ is the current density of the external source, $\boldsymbol{E}$ is the electric field intensity, $\boldsymbol{D}$ is the electric displacement, $\boldsymbol{B}$ is the magnetic induction intensity, and ρ is the bulk density of the free charge. With Vector magnetic potential and scalar potential, the electric field intensity can be expressed as:

$$\boldsymbol{E} = -\nabla\varphi - \mathrm{j}\omega\boldsymbol{A}$$

Lorentz specification can be expressed as:

$$\nabla \cdot \boldsymbol{A} - \frac{k^2}{\mathrm{j}\omega}\varphi = 0$$

In the conductive medium, vector magnetic potential equation, vector magnetic potential of electric field intensity and magnetic induction intensity are as follows:

$$\nabla^2\boldsymbol{A} + k^2\boldsymbol{A} = -\mu\boldsymbol{J}_{\mathrm{s}}$$

$$\boldsymbol{E} = -\mathrm{j}\omega\left[\boldsymbol{A} + \frac{1}{k^2}\nabla\left(\nabla\cdot\boldsymbol{A}\right)\right] \tag{1}$$

In the definition, k is the propagation constant and can be obtained by the follows:

$$k^2 = -j\omega\mu\sigma + \omega^2\mu\varepsilon$$

It can be seen, as long as the vector magnetic potential is identified, the components of the electromagnetic field can be uniquely determined.

3. The expressions of electric field caused by the vertical electric dipole in the second medium

Assuming that the vertical dipole in the second layer medium is located at the coordinates $(0,0,z_0)$, and then the image of the dipole is located at $(0,0,-z_0)$ with the height is h. In the first layer, the dielectric constant is μ_0, the magnetic permeability is ε_0 and the conductivity is σ_0. In the second layer, the dielectric constant is μ_1, the magnetic permeability is ε_1 and the conductivity is σ_1. r is the distance from the projection on the xoy plane to central point. θ is the angle between the projection line and X-axis. The locations of the electric dipole and its image are shown in figure 1.

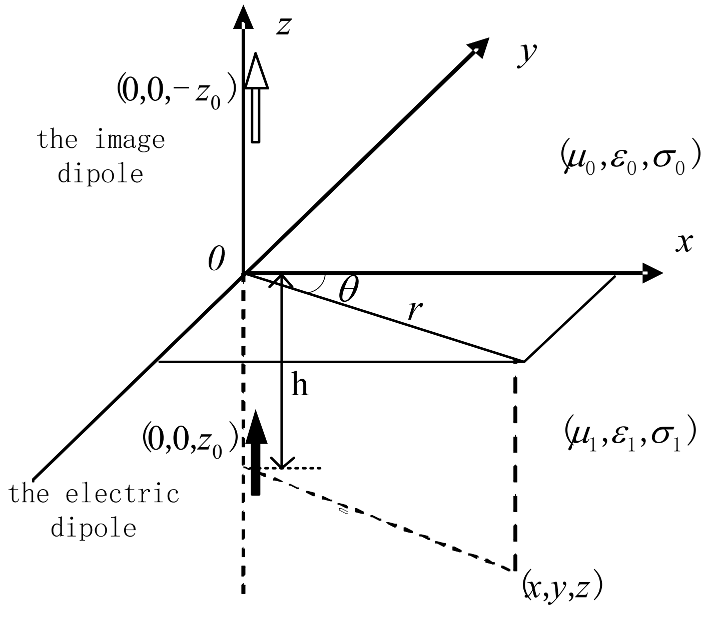

Figure 1. The locations of the electric dipole and its image.

If the whole space is overflowed by the sea water, the magnetic vector potential generated by the image in the sea water can be expressed as follows [6]:

$$A_{1z}^{i} = \frac{\mu_1 Il}{4\pi}\int_0^{\infty} h(\lambda)\frac{\lambda}{v_1}J_0\left(r\xi\right)e^{v_1(h-z)}d\xi$$

The magnetic vector potential generated by the real source in the sea water can be expressed as follows:

$$A_{1z}^{r}=\frac{\mu_1 Il}{4\pi}\int_0^{\infty}\frac{\lambda}{v_1}J_0\left(r\xi\right)e^{-\upsilon_1(h+z)}d\xi$$

Where, $J_0\left(\rho\xi\right)$ is the first kind Bessel function.

$$r=(\mathrm{x}^2+y^2)^{1/2}$$

$$\upsilon_1=(\xi^2-k_1^2)^{1/2}$$

According to the boundary condition of vector magnetic potential:

$$\lim_{z\to+0}\frac{1}{k_0^2}\frac{\partial A_{0z}}{\partial z}=\lim_{z\to-0}\frac{1}{k_1^2}\frac{\partial A_{1z}}{\partial z}$$

$$\lim_{z\to+0}\frac{A_{0z}}{\mu_0}=\lim_{z\to-0}\frac{A_{1z}}{\mu_1}$$

Then $h(\lambda)$ can be expressed as follows:

$$h(\lambda)=\frac{\mu_1 k_0^2 v_1-\mu_0 k_1^2\upsilon_0}{\mu_0 k_1^2\upsilon_0+\mu_1 k_0^2\upsilon_1}$$

The vertical magnetic vector potential in the second half of the space can be expressed as follows:

$$A_{1z}=A_{1z}^{i}+A_{1z}^{r}$$

Assume that the second half space is the uniform, linear and isotropic medium, and then k_0^2 is close to zero. The following formula can be obtained.

$$A_{1z}==\frac{\mu_0 Il}{4\pi}\left(H_1-H_2\right) \tag{2}$$

Where,

$$H_1=\frac{\mathrm{e}^{-\mathrm{j}k_1 R}}{R}$$

$$H_2=\frac{\mathrm{e}^{-\mathrm{j}k_1\bar{R}}}{\bar{R}}$$

$$R=\left[r^2+(z-h)^2\right]^{1/2}$$

$$\bar{R}=\left[r^2+(z+h)^2\right]^{1/2}$$

With the equation (1) and (2), the electric field intensity can be expressed as:

$$E_{1x}=\frac{Il}{4\pi\sigma_1}\frac{\partial^2}{\partial x\partial z}\left(H_1-H_2\right)$$

$$E_{1y}=\frac{Il}{4\pi\sigma_1}\frac{\partial}{\partial y\partial z}\left(H_1-H_2\right)$$

$$E_{1z}=\frac{Il}{4\pi\sigma_1}(k_2^2+\frac{\partial^2}{\partial z^2})\left(H_1-H_2\right)$$

Where, $k_2=(\frac{\omega\mu_0\sigma_2}{2})^{1/2}(1-j)$.

Using the following the partial differentialoperators, the analytical solutions of the electric field can be obtained.

$$\frac{\partial}{\partial x}[\quad]=\cos\theta\frac{\partial}{\partial r}[\quad]$$

$$\frac{\partial}{\partial y}[\ ] = \sin\theta \frac{\partial}{\partial r}[\ ]$$

$$\frac{\partial^2}{\partial x^2}[\ ] = \left(\cos^2\theta \frac{\partial^2}{\partial r^2} + \frac{\sin^2\theta}{r}\frac{\partial}{\partial r}\right)[\ ]$$

$$\frac{\partial^2}{\partial x \partial y}[\ ] = \sin\theta\cos\theta\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)[\ ]$$

Three component of the electric field can be expressed as:

$$E_{1x} = \frac{-\mathrm{j}\,\omega\mu_0 Il\cos\theta}{4\,\pi\,k_1^2}\frac{\partial^2}{\partial z \partial r}\left(H_1 - H_2\right)$$

$$E_{1y} = \frac{-\mathrm{j}\,\omega\mu_0 Il\sin\theta}{4\,\pi\,k_1^2}\frac{\partial^2}{\partial z \partial r}\left(H_1 - H_2\right)$$

$$E_{1z} = -\frac{\mathrm{j}\,\omega\mu_0 Il}{4\,\pi}\left[\left(H_1 - H_2\right) + \frac{1}{k_1^2}\frac{\partial^2}{\partial z^2}\left(H_1 - H_2\right)\right]$$

4. Computational example

In order to study the electric fields generated by extremely low frequency time-harmonic vertical electric dipole, the distribution of electric fields is calculated by the image method. Suppose that the vertical time-harmonic electric dipole is embedded in the sea water and the electromagnetic parameters of air and seawater are assumed respectively:

$$\mu_0 = 4\pi\times10^{-7}H/m\ ,\ \varepsilon_0 = (1/36\pi)\times10^{-9}F/m\ ,\ \sigma_0 = 0.$$

$$\mu_1 = \mu_0\ ,\ \varepsilon_1 = 80\varepsilon_0\ ,\ \sigma_1 = 4\Omega/m.$$

The frequency of the dipole which is located at (0, 0, 3m) is 2Hz and the magnitude is 5Am. The distributions of electric fields are shown in Figure2 to Figure7.

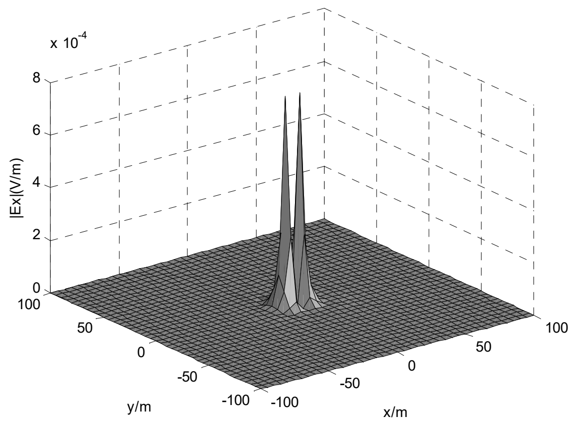

Figure 2. The three-dimensional spatial distributions in the x direction caused by the vertical dipole.

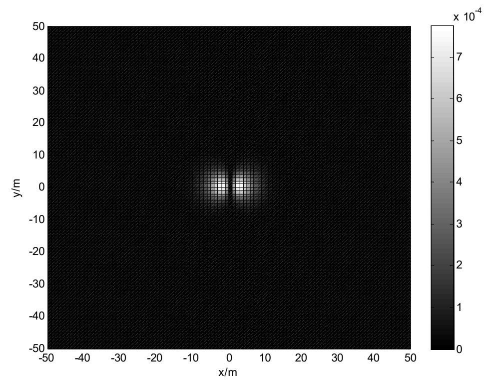

Figure 3. The two-dimensional spatial distributions in the x direction caused by the vertical dipole.

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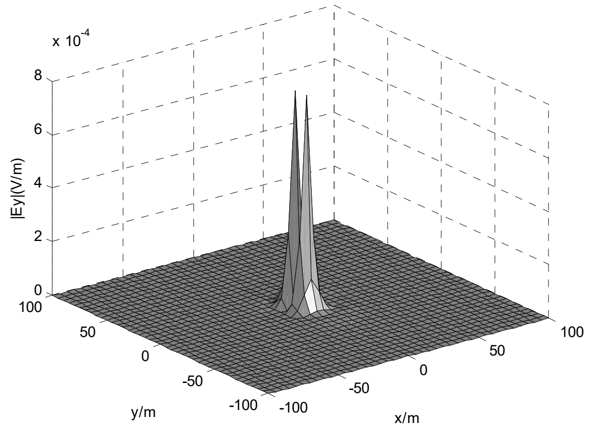

Figure 4. The three-dimensional spatial distributions in the y direction caused by the vertical dipole.

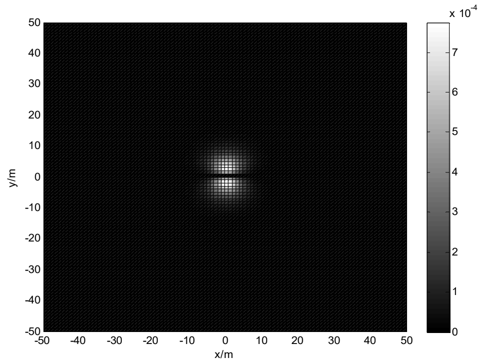

Figure 5. The two-dimensional spatial distributions in the y direction caused by the vertical dipole.

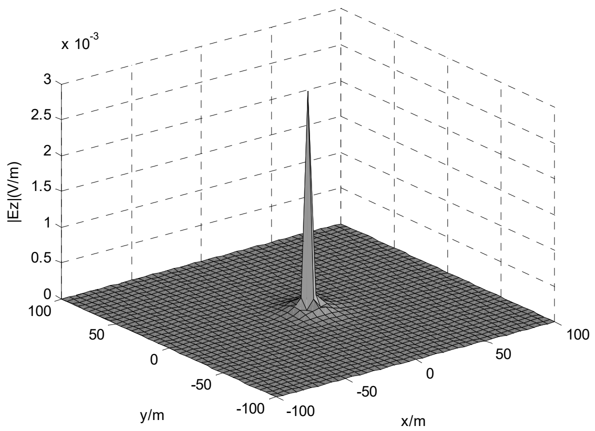

Figure 6. The three-dimensional spatial distributions in the z direction caused by the vertical dipole.

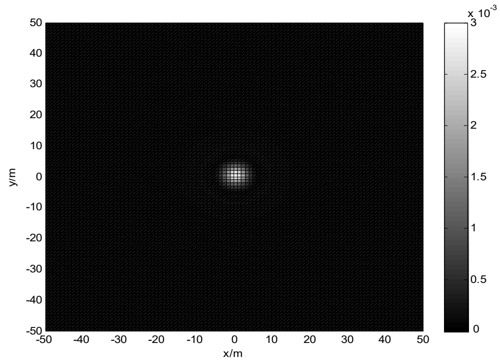

Figure 7. The two-dimensional spatial distributions in the z direction caused by the vertical dipole.

5. Conclusion

The expression of the electric field generated by time-harmonic vertical time-harmonic electric dipole embedded in deep sea water is deduced. After being simplified and using the partial differential operators, the analytical solutions can be obtained. However, the situation will not happen when the time-harmonic electric dipole is horizontal. The expression usually contains the Generalized Sommerfeld Integrals which can be calculated by using FFT based on Hankel transforms and the algorithm of optimized fast Hankel transform filters.

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