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A new method for positioning a single transponder without sound velocity profile

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Abstract: The systematic error due to the spatial and temporal variation of sound velocity profile (SVP) is the main error source for positioning a single transponder. Difference techniques can eliminate systematic error of long period, while the short period terms will still retain. This paper focuses on improving the accuracy of the single transponder positioning when the systematic errors of short period exists. Based on the equivalent SVP method, a new method was put forward to calculate the effective sound velocity (ESV) which will be used to improve the traditional observation equation. Then, the three-dimensional coordinates of transponder will be determined by least squares adjustment. Simulation experiments shows that the new methods can greatly improved accuracy and efficiency for positioning a single transponder.

1.Introduction

Marine geodetic datum is an important component of global space datum. One of the key technologies is high-precision positioning the seafloor control network (transponders). Since the electromagnetic energy does not penetrate significantly into sea water, the seafloor control points positioning is usually carried out by kinematic GPS combined with underwater acoustic (Kussat et al.2005; Gagnon et al.2005; watanabe et al.2015). This techniques also widely applied in the seafloor geodetic deformation researchs (Spiess et al.1985; Spiess et al.1998; Chadwell et al.2008;). Precise GPS can provide real-time position for the survey ship and connect the local seafloor geodetic network to a global reference frame. Underwater acoustic can measure the distance between the transponder in the seafloor and the transducer under the survey ship. If multiple distance observations are measured, the three-dimensional coordinates of seafloor control point can be determined by intersection positioning model. The factors affecting the ranging accuracy mainly include the systematic error due to the time delay during re-transmitting the received signal from the transponder back to the transducer and the systematic error due to the spatial and temporal variation of SVP (Yamada et al. 2002). With the development of marine acoustic equipment, the time delay related to the transponder could be negligible. The most damaging factor is the variation of SVP. Since the transponder are arranged approximately in an isothermal layer, the differences of sound velocity between them are small. Therefore, the mutually measuring distances between underwater transponder can greatly improve the accuracy and efficiency of seafloor control network measurement. However, traditional method is still needed to determine the coordinates of absolute datum. The traditional method is that survey vessel



sail circling the transponder on a radial approximately equal to the depth while simultaneously collecting two-way acoustic ranges and GPS data. If the SVP of the survey region is measured, Constant gradient ray-tracing would be used to calculate the distances between the transducer and the transponder. The main factor that causes the ranging error is the replacement of a time dependent SVP with a fixed SVP. Another way to calculate the distance is to multiply the initial sound velocity by the propagation time. The errors of the initial sound velocity would be considered as a unknown parameter to be estimated by the least squares adjustment. Similarly, this method is also affected by the constantly changing of SVP. In order to reduce the positioning error caused by the uncertain sound velocity. Xu et al.(2005) has proposed a method which is difference ranging measurements between two consecutive ship positions for the single transponder positioning. The systematic errors of long period could be eliminated, while the systematic errors of short period will still remain. Zhao et al.(2016) consider the ranging error caused by the uncertain sound velocity as an unknown parameter to be estimated with the coordinates of underwater transponder. The experiment results show that the deviation of coordinates on the horizontal component is small while on the vertical component is relatively large. In this paper, we proposed a new method based on the sailing circle of survey ship around the underwater transponder. Different from traditional methods, a fixed sound velocity was replaced by an effective sound velocity (ESV). In the following section, this paper will discuss the calculation process of the ESV and the improved intersection positioning model in detail.

2. Mathematical model

2.1 The method to calculate the ESV at each acoustic ranging (epoch)

The ESV is the sound velocity multiplied by travel time between the transducer and the transponder, yields the geometric range. The geometric range is calculated based on the equivalent SVP method which is introduced by Geng et al.(1999). The deviation of beam footprint is mainly caused by the error of incidence angle. However, it exists a special situation that is starting incidence angle is zero, then the equivalent SVP method is equal to the ray-tracing algorithm. If the propagation time t_m which is that the sound waves transmit to the seafloor vertically is known, we can calculate the g_{eq} by the Eq(1).

$$t_m = \frac{1}{g_{eq}} \ln \left[\frac{c_0 + g_{eq} \cdot z_{B0}}{c_0} \right] \quad (1)$$

Where c_0 is the sound velocity at the sea surface, z_{B0} is the reference depth, it can be measured by other methods, such as a depth pressure sensor. If the t_m is difficult to acquired, it can be estimated with travel time multiplied by the sine of the starting grazing angle.

$$t_m = t_i \cdot \sin \alpha_0 \quad (2)$$

The equivalent SVP method always use a SVP with zero gradient as a reference SVP. The ε_s is the relative area difference between the actual SVP and the zero-gradient SVP c_0 - c_A . In the new method the ε_s can be expressed as

$$\varepsilon_s = \frac{z_{B0} \cdot g_{eq}}{2 \cdot c_0} \quad (3)$$

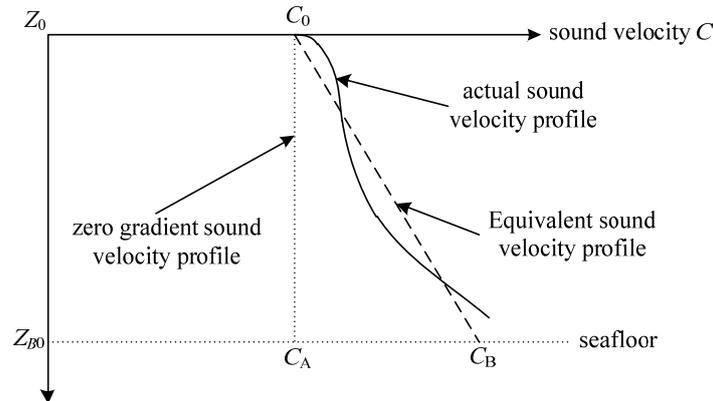


Figure 1 Using an equivalent SVP to represent an actual SVP

The radius of ray path on the equivalent SVP can be calculated by

$$R_{eq} = -\frac{C_0}{g_{eq} \cdot \cos \alpha_0} \quad (4)$$

If the starting depth is z_0 and horizontal position is x_0 , the depth and horizontal of beam footprint can be estimated by

$$z_B = z_0 + R \cdot (\cos \alpha_0 - (1 + 2\varepsilon_s) \cdot \cos \alpha_0) \quad (5)$$

$$x_B = x_0 + R_{eq} \cdot (\sqrt{1 - [(1 + 2\varepsilon_s) \cdot \cos \alpha_0]^2} - \sin \alpha_0) \quad (6)$$

Thus, the ESV can be calculated by

$$c_e = \frac{\sqrt{x_B^2 + z_B^2}}{t} \quad (7)$$

2.2 Intersection positioning model for a single transponder

In the process of single transponder positioning, the observation equation can be given in its simplest form as follows (Xu et al 2005):

$$p_i = f(X_i, X_o) + \delta p_{di} + \delta p_{vi} + \varepsilon_i \quad (8)$$

Where p_i is the slant range between the transponder on the seafloor and the transducer under the ship at the i th epoch, which can be calculated by the propagation time and the SVP or directly set as the product of the surface sound velocity and the propagation time. In the new method we use the ESV instead of the surface sound velocity; $f()$ is a function stand for the geometrical distance between transducer and transponder, X_o is the unknown position of the transponder. X_i is the position of the transducer at the i th epoch; δp_{di} is the systematic error due to the time delay in re-transmitting the received signal from the transponder back to the transducer; δp_{vi} is the systematic error due to the spatial and temporal variation of SVP; ε_i is the random error. With the modern transponder is concerned, δp_{di} is negligible. The most damaging factor is δp_{vi} .

The linearized version of Eq.(8) is given as

$$\bar{p}_i - p_0 = b_i \cdot dx + \varepsilon_i \quad (9)$$

Where $\bar{p}_i = p_i - \delta p_{vi}$ is the measured geometrical distance between transducer and transponder; p_0 is the approximate value of $f(X_i, X_o)$; X_o^0 is the approximate value of X_o ; b_i is a first partial derivatives of $f()$ with respect to X_o , dx is the unknown coordinate correction vector to be estimated.

If the multiple measurements are obtained, the observation equation in matrix form is

$$V = BX - L \quad (10)$$

Where $L = \bar{p}_i - p_0$, X is estimated by the least squares adjustment. Multiple iterations are required

until $\|dx\|$ is less than a given tolerance. The coordinate corrections can be depicted as:

$$dx = (B^T B)^{-1} B^T L \quad (11)$$

The absolute coordinates of transponder is

$$x_o = x_o^0 + dx \quad (12)$$

The positioning accuracy of the transponder can be express as

$$\hat{\sigma}_0^2 = \frac{V^T P V}{n-3} \quad (13a)$$

$$\hat{\sigma}_x = \hat{\sigma}_0 \sqrt{\text{tr}(B^T P B)^{-1}} \quad (13b)$$

Where n is the number of epoch.

2.3 Improved Intersection positioning model for a single transponder

Different from the traditional methods, the geometric distances will be used instead of actual propagation trajectory in the new method. The geometric distance is obtained by ESV multiplied by propagation time. Thus the δp_v in the observation equation would be eliminated, and the range error caused by the δp_d is also ignored. The new observation equation can be obtained as:

$$\bar{p}_i = f(X_i, X_o) + \varepsilon_i \quad (14)$$

Where $\bar{p}_i = c_i \cdot t_i$, c_i is the ESV at each epoch. The linearized version of (14) is given as follows:

$$c_i \cdot t_i - p_{0i} = b_i \cdot dx + \varepsilon_i \quad (15)$$

If the multiple measurements are obtained, the observation equation in matrix form can be depicted as Eq(10), The absolute coordinates of transponder is obtained as Eq(11).

Compared to the traditional intersection positioning model, the improved model does not need SVP in the date processing and simplifies the measurement of absolute datum transfer.

3. Simulated examples and results

We will design a number of simulated examples to test the accuracy and validity of new method. The water depth was decided to 3000 m and the SVP was shown in Figure 2. In order to simulate an approximate real oceanic world. The period and amplitude of the tide are respectively 12 h and 5 m and the period and amplitude of the wave are respectively 20 s and 2 m. The position of survey ship would be determined by GPS, the accuracy of GPS is assumed to be 5 cm in horizontally and 10 cm in vertically. The precision of timing is $5\mu\text{s}$ and the speed of survey ship is 4 knot (about 2m/s). The sampling period was 10 s. The simulation of system error is the same as Xu et al (2005), Including four types of effects: (i) a constant term;(ii) internal wave with a short period; (iii) diurnal or semi-diurnal tides; and (iv) factors of regional effect by using a Gaussian correlation function. Systematic errors was be simulated in centimeters as follows:

$$\delta p_v = 10 + 12 \sin\left(\frac{2(t-t_0)_m}{20} \pi\right) + 30 \sin\left(\frac{(t-t_0)_h}{12} \pi\right) + 2 \left[1 - \exp\left\{-\frac{1}{2} \|x-x'\|^2 / (2km)^2\right\} \right]$$

(16)

Where the $t-t_0$ is time spent in measuring the time units in the second and third terms on the right hand side are in minutes and hours, $\|x-x'\|$ is a geometric distance between the x and x' .

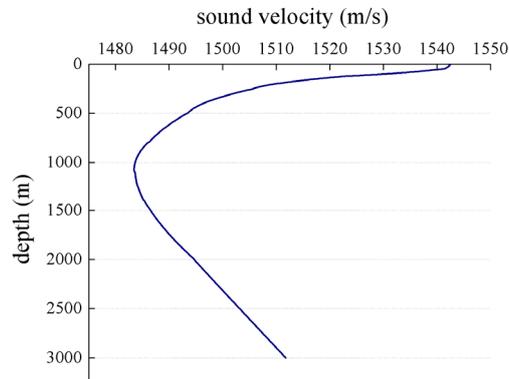


Figure 2. Sound velocity profile

This simulation mainly investigate the relationship between positioning accuracy due to variation of SVP or sampling interval on different methods. Three positioning model respectively are traditional method (method 1), difference techniques (method 2) and new method (method 3) was put forward in this paper. The sampling interval are respectively 8.73min, 1.57min, 0.311min, 0.17min. If the systematic error of short period is negligible. The results of different sampling interval is shown in table1 by three methods.

Table 1. The coordinates and deviation of transponder calculated by three methods with different sample intervals

sampling interval(min)	method	x/m	y/m	z/m	dx/m	dy/m	dz/m
8.73	1	1.151	5.959	-11.808	-1.849	-0.958	-13.808
	2	2.969	4.948	-40.179	-0.031	-0.052	-38.179
	3	2.986	4.991	-2.8680	-0.014	-0.009	-0.8678
1.57	1	3.014	5.111	-12.069	-0.014	-0.111	-14.069
	2	3.005	4.989	-16.230	-0.005	-0.011	-18.230
	3	2.991	5.002	-2.8820	-0.008	-0.002	-0.882
0.31	1	3.016	5.110	-12.073	-0.016	-0.110	-14.073
	2	2.986	5.030	-4.7150	-0.014	-0.030	-2.715
	3	2.997	5.000	-2.8920	-0.003	-0.000	-0.892
0.17	1	3.016	5.108	-12.074	-0.016	-0.108	-14.074
	2	3.002	5.006	-2.4400	-0.002	-0.006	-0.440
	3	2.998	4.998	-2.8790	-0.002	-0.002	-0.879

It can be seen from table1 that the positioning accuracy of three methods is related to the sampling interval and decreases with it. Affected by the spatial distribution of sampling points, the positioning accuracy in the horizontal components is higher than the vertical component. By comparing three methods, we can conclude that the new methods has the highest accuracy, followed by the difference techniques, and the traditional methods is the lowest. The positioning accuracy of the difference techniques in the vertical component is greatly affectedly by the sampling interval, while the new method is less affected by the sampling interval. When the sampling interval became small, the positioning accuracy of the difference techniques is higher than the new method. when the sampling interval became large, the positioning accuracy of new method is higher than the difference techniques. the positioning accuracy of the traditional method in the vertical component is less affected by the sampling interval, but the positioning accuracy is poor.

Another simulated experiments is designed to investigate the effect of internal wave on the single

transponder positioning by three methods. If the sampling interval is 0.17 min, and the measurement period exist a internal wave with a duration of 15 minutes. namely, systematic error include the long period and short period due to the variation of SVP. The experimental results are shown in table 2.

Table 2. The coordinates and deviation of transponder calculated by three methods with systematic error of short period

sampling interval(min)	method	x/m	y/m	z/m	dx/m	dy/m	dz/m
Internal waves	1	3.009	5.108	-12.078	-0.009	-0.108	14.078
	2	2.997	5.026	0.489	0.003	-0.026	1.511
	3	2.999	5.001	5.001	0.001	0.001	1.000
No internal waves	1	3.016	5.108	-12.074	-0.016	-0.108	14.074
	2	3.002	5.006	2.440	-0.002	-0.006	-0.006
	3	2.998	4.998	2.879	0.002	0.002	-0.879

It can be seen from the table 2 that the systematic error of short period due to the variation of SVP has a great influence on the positioning accuracy of difference techniques, but has little influence on the traditional method and new method. The deviations of positioning results achieved by difference techniques are -0.002 m in x direction, -0.006 m in y direction and -0.44 m in z direction when the internal wave does not exist, while those are 0.003, -0.026 and 1.511 m when the internal wave exist, respectively. The new method can realize sub-centimeter level of accuracy in the horizontal components and sub-meter level of accuracy in the vertical component. No matter whether there is an internal wave. Thus, we can draw the conclusion that the difference techniques can eliminate the systematic error of long period but without help to the systematic error of short period. The new method can eliminate the systematic error of long period and short period simultaneously.

4. Conclusions

With the development of underwater acoustic positioning technology and equipment, the mutual positioning information between transponders can be used to improve the positioning accuracy of the marine geodetic datum. However there still need traditional method to determine the absolute coordinate of the seafloor control point. The systematic error due to the variation of SVP can affect the positioning accuracy of single transponder significantly. The systematic error of sound velocity includes the fixed term and the change term. The traditional method can eliminate the constant term by considering it as an unknown parameter to be estimated with the coordinates of underwater transponder by the least squares adjustment. The difference techniques can eliminate the systemic error of long period but without help to the short period. Based on the above analysis, this paper improved the positioning observation equation of traditional method. Firstly, we proposed a new method to calculate the ESV at each epoch. Then, the coordinates of seafloor single transponder can be calculated by the least square adjustment. Simulation result shows that the positioning accuracy of the new method is better than the traditional method and difference techniques. Furthermore, the new method does not demand to measure the SVP which can greatly improve the operation efficiency.

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