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A Method for Interpolating Digital Depth Model Based on Semiparametric Regression and Uncertainty Weighting

Shuaidong Jia¹, Qiu Chen² *, Lulu Tang¹ and Zeyuan Dai¹, Lihua Zhang¹

¹Department of Military Oceanography and Hydrography & Cartography, Dalian Naval Academy, Dalian, Liaoning, 116018, China

²Navigation Guarantee Bureau of the Naval Staff Department, Beijing, 100841, China

*Corresponding author's e-mail: 115665256@qq.com

Abstract: To address a limitation of DDM interpolation not considering the error distribution characteristics, a method for interpolating Digital Depth Model based on semiparametric regression and uncertainty weighting is proposed in this paper. Firstly, the observed error from the raw depths is considered to be random, and the weighting matrix is built for expressing this kind of random error. Secondly, the approximate error from the interpolation function is regarded to be systematic, and the semiparametric regression model is constructed for taking the part of systematic error into consideration. Finally, a new method for interpolating Digital Depth Model is proposed considering both the random and systematic error. The experimental results demonstrate that: (1) the proposed method has improved the quality of DDM; (2) the greater the uncertainties of the depths varies, the higher the proposed method has improved the quality of DDM; (3) the more complex the seafloor topography is, the higher the proposed method has improved the quality of DDM.

1.Introduction

Digital Depth Model is a model for representing the seafloor topographic surface, which is an important part in the marine spatial data infrastructure[1,2]. The DDM quality directly determines the accuracy of the seafloor representation and the safety of the ship navigation[3].

Interpolation is one of the core problems in the construction of digital depth model[4]. Usually, in the progress of the DDM interpolation, an appropriate two-element function needs to be established on a basis of the continuous smoothness of the true seafloor topography[1,4]. And then the depth value can be estimated according to the function above. Whichever form of interpolation function is selected, it is still an approximation to the true seafloor topography. Hu et al clearly pointed out that the error of interpolation function accords with the mathematical approximation theory, which belongs to the system error part of DDM[5]. On the other hand, the original bathymetric sampling points used in the interpolation model may be derived from the bathymetric data collected at different periods and equipment, and the quality of these depth points may vary greatly. Even for the depth data from the same source, with Multibeam bathymetric data as an example, the qualities would be different due to the difference of the survey line and beam angle[6].

For the system error in interpolation function, the semiparametric regression method can be used for analysis and processing. Semi-parametric regression analysis, as a mathematical model which contains both parametric and nonparametric components, is widely used in surveying data processing which contains systematic error and random error. On the other hand, for the random error in the



interpolation function, the errors of different depth sampling points can be analyzed and treated by the analysis of the uncertainties. Therefore, the measured errors of the raw depths and the approximate errors of the interpolation function are respectively considered as random and systematic errors, and then the semiparametric regression and uncertainty weighting method is utilized to analyze the influence mechanism of the two pairs of DDM interpolation, so that the quality of the digital depth model is improved.

2. Methodology

2.1 DDM interpolation

The basic idea of DDM interpolation is to select a reasonable mathematical model, use the information on the discrete point to find the undetermined parameters of the mathematical model, and then compute the depth value on the node[4]. With the development of depth data to high density and precision, some interpolation methods are used such as weighted averaging method and moving surface method[1]. Considering that the weighted averaging method is prone to common bovine eye effects[7], this paper adopts a more flexible and accurate moving surface method as the basic method of DDM interpolation[8].

The basic idea of moving surface method is to create the local actual surface by constructing an appropriate polynomial function as the interpolation function of the surface, so as to realize the digitized expression of the seabed surface. For an interpolated point M, the adjacent discrete points can be selected to fit a polynomial surface, and the general form of a surface fitting polynomial can be described as[4]:

$$z(x, y) = \sum_{0 \leq i+j \leq N} a_{ij} x^i y^j \quad (1)$$

where N represents the highest power of the fitted polynomial, (x, y, z) represents the coordinate of each discrete point, a_{ij} represents the undetermined parameters.

Taking the simplest linear interpolation function as an example, the fitting polynomial of linear surface is established, and the expression can be acquired that:

$$z = ax + by + c = [x \ y \ 1][a \ b \ c]^T \quad (2)$$

where $[a \ b \ c]^T$ represents a 3-D parameter vector to determine a linear surface.

Suppose that the coordinate of the interpolation point M is (x_m, y_m, z_m) and its adjacent discrete points are respectively (x_i, y_i, z_i) ($i = 1, 2, \dots, n$). Then establish the conditional adjustment model is establish that $L = B\hat{X} + \Delta$, where:

$$B = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix}, \quad L = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

According to the principle of least squares, it can be concluded that:

$$\hat{X} = [a \ b \ c]^T = (B^T P B)^{-1} B^T P L \quad (3)$$

where P represents the weight matrix of the observed vector L .

So the depth of the interpolated point M can be acquired that:

$$z_m = [x_m \ y_m \ 1] \quad (4)$$

2.2 DDM interpolation based on semiparametric regression and uncertainty weighting

2.2.1 Semi-parametric regression model of moving surface method

At present, the error of surface fitting is usually regarded as random error, but the approximation error of DDM interpolation function (i.e. the DDM of the seabed topography) is not considered to have statistic characteristics. Therefore, in this paper, the method of semiparametric regression is used for analyzing the DDM topography representing error as the systematic error, and reconstructing the fitting polynomial function of the seafloor surface, namely:

$$z(x, y) = \sum_{0 \leq i+j \leq N} a_{ij} x^i y^j + s \quad (5)$$

where s represents the error of DDM seabed topography representation. Corresponding to the expression (2), here the linear fitting polynomial parallel state is established:

$$L = B\hat{X} + S + \Delta \quad (6)$$

where $X = [a \ b \ c]^T$, $S = [s_1 \ s_2 \ \dots \ s_n]^T$.

According to the principle of the least squares, the solution to the semiparametric regression model can be acquired that[9]:

$$\begin{aligned} \hat{X} &= N^{-1} B^T D^{-1} L \\ \hat{S} &= \frac{1}{\alpha} R^{-1} D^{-1} (L - B\hat{X}) \end{aligned} \quad (7)$$

where $D = P^{-1} + \frac{1}{\alpha} R^{-1}$, $N = B^T D^{-1} B$.

2.2.2 the determination of P , R and α

When the semiparametric regression model is used, it is one of the key factors that the determination of the weighting matrix P , the proper normalized matrix R and the smoothing factor α .

Firstly, the weighting matrices needs to consider the differences of the measured data errors, which can be adopted as the following expression:

$$P = \begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 0 & p_{12} & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & p_{nn} \end{bmatrix} \quad (8)$$

where the element $p_{ii} = u_i$ represents the weight of the observed points depth i , u_i represents the vertical uncertainty of the raw depth point i .

Secondly, the regularization matrix R needs to be acquired. For the selection of the normalized matrix R , considering the variation of the DDM seafloor topographic description error has certain correlation with the plane position, namely the nearer two water depth point is, the smaller the difference between the nonparametric signal s_i and s_j , as shown in figure 1. Therefore, the basic method of constructing the normalized matrix by the distance method is that: (1) select the distance function $r_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2$, where the depth point is i and j ; (2) the inverse matrix R^{-1} of the normalized matrix R can be created and thus the normalized matrix R can acquired.

Thirdly, L curve method is adopted for the selection of smoothing factor α [10].

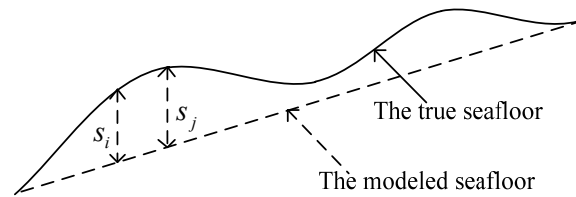


Figure 1. the variation law of DDM representing error

2.2.3 the estimating model for the node depth

According to the plane position of the interpolated point M , on a basis of the principle of the azimuth pick-up method, the discrete points $i(i=1,2,\dots,n)$ are selected and the nonparametric parameters s_M of the interpolation points M are estimated according to the nonparametric signals \hat{s}_i of the discrete points, such shown as following:

$$s_M = \sum_{i=1}^n \frac{\hat{s}_i}{d_{Mi}^2} / \sum_{i=1}^n \frac{1}{d_{Mi}^2} \quad (9)$$

The depth value of the interpolated point can be concluded that:

$$z_m = z'_m + s_m \quad (10)$$

3.Experiments and Analysis

3.1 Experimental Area

In order to compare the DDM quality of the two methods, the Multibeam sounding data in three sea area is selected. Some of the depth points are selected as checkpoints, and then the DDM is constructed by using the other of the depth points. Such as shown in figure 2, a three-dimensional diagram of the three sea area is performed. Sea area 1 is relatively flat, sea area 2 is relatively complex, and the sea area 3 is relatively complex. The depths in sea area 1 and 2 are from the single source, and the depths in the sea area 3 are from the multiple sources.

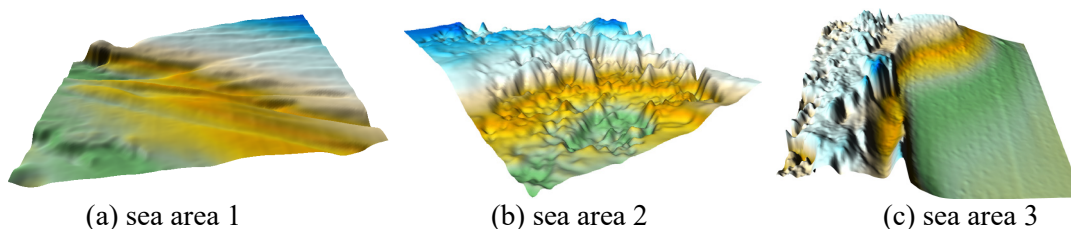


Figure 2. 3D display of the seafloor topographic surfaces inside the two sea areas

3.2 The comparison of DDM qualities

Through the previous set of checkpoints, the DDMs constructed with the two methods are carried out for compare their qualities. Supposing that The check point of the original observation depth value is $Z'_i (i=1,2,\dots,n)$, DDM interpolation depth value is Z_i , and then DDM quality can be evaluated by the following formula:

$$\sigma_{DDM} = \frac{1}{n} \sum_{i=1}^n (Z_i - Z'_i)^2 \quad (11)$$

Table 1 Comparison of DDM accuracy

Sea area	Interpolation method	DDM accuracy
1	The traditional	0.010

method		
2	The proposed method	0.006
	The traditional method	0.551
3	The proposed method	0.244
	The traditional method	0.787
	The proposed method	0.430

As shown in table 1, the interpolation quality of the two methods in the sea area 1 and 2 shows that the proposed method has a significant improvement on the DDM quality compared to the traditional methods.

Also, comparing the interpolation quality of the two methods in sea area 1 and 2, we can see that the greater the complexity of seabed topography in the sea area is, the greater the improvement of the interpolation quality with the proposed method.

On the one hand, for the sea area 1 and 2 from table 1, it can be seen that the overall mass of the two methods can achieve higher standard in the relatively flat area of the seabed, but in the relatively complex area of the seabed, the DDM overall quality with the proposed method is obviously better than that of the traditional method. Therefore, through this experiment, we can see that in the process of DDM interpolation, we should take into account the influence of DDM seabed topography description error on the construction quality of DDM, especially in the area with complicated seabed topography.

On the other hand, for the sea area 3 from the table 1, it can be seen that the quality of DDM constructed with the proposed method is higher than that of the traditional method. It shows that the proposed method can further improve the quality of the depth model through considering the difference of the uncertainties of depths from different sources.

4. Conclusion

Through analysis, calculation and experimental comparison, the conclusion is as follows:

(1) The errors from the measurement and the mathematical function approximation are effectively differentiated into the random and the systematic part in DDM error. The DDM interpolation model based on semiparametric regression and power weighting is established, which breaks through the limitations of the traditional interpolation method, and the precision of the interpolation depth is improved.

(2) The greater the degree of complexity of the seafloor topography is, the higher the quality of DDM constructed with the proposed method improves than that of the traditional method.

(3) The proposed method can further improve the interpolation accuracy when different sources of water depth uncertainty differ greatly.

Surely, limited by the data sources, only the water depth data of three sea areas are selected, and more regional bathymetric data are still awaiting further experimental analysis.

Acknowledgments

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