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Finding the optimal route of wood transportation

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Abstract. This article describes an algorithm for finding the optimal routes and volumes of cargo transportation along the routes at minimal transportation cost of a given cargo' volume from the starting to the final point. The algorithm, based on Dijkstra's approach (shortest path search) and dynamic programming method, is used for the calculation of the optimal route of wood transportation from the cutting area. The peculiarity of forest roads are different transport costs and carrying capacities on the forest road network sections influenced by the natural and climatic conditions of the route section. The cost of delivery includes both costs, depending on the volume of cargo (transportation), and not depending on it (construction and restoration of roads). The peculiarity of the task is that the costs of restoration (construction), transportation along the section included in the route may vary depending on the transported cargo volume.

1. Introduction

Many algorithms have been developed in order to find optimal routes for the transportation of various cargos [1-3]. There are approaches aimed to determine the shortest path [4], while, the others are to pass the route in the shortest time [5, 6]. However, nowadays the actual issue is still to find the cargo transportation route of the minimal cost [7]. The basic data for solving such problems includes the following indicators: the unit cost of cargo transportation and the length of the path.

The lumbering industry is developing actively in many countries around the world, but the problem of transportation of wood products remains relevant in most regions. In this regard, the problem of finding the optimal route of timber transportation becomes the subject of scientific research of many scholars. A number of algorithms have been developed to solve the problems of timber transportation from the cutting area to the consumer [8, 9]. Usually these algorithms do not take into account all the costs that affect the choice of the optimal transportation route. In most cases, the problem is solved considering just the cost of wood delivery (transportation). Costs that do not depend on the volume of timber removal (construction and restoration of roads, etc.) are less often taken into account.

The importance of finding the correct route of timber removal from the cutting area is substantiated in the paper [10]. The authors find a direct connection between the optimization of the wood transportation process and the productivity of timber industry. Also, authors point out that the low level of development of forest road infrastructure adversely affects the ecological state of the forest. The main hypothesis of research is that there are differences between the geometric and the actual distance of wood transportation. The geometric distance is the length of the path marked on the map.



On this way it is planned to transport wood from the cutting area. As a rule, the geometric distance is a straight line. However, in real conditions roads are laid considering the terrain specifics (mountains and ravines, as well as water objects). This is called the actual wood transportation distance. The actual distance is greater than the geometric one. Based on this, the aim of study is to determine a simple and effective method for calculating the optimal wood transportation route taking into account different reliefs.

Digital technologies are often used to solve the problem of finding the optimal route for timber transportation. K Sterenczak and T Moskalik [11] dealt with analytical procedure to determine the best way to transport wood using data from onboard laser scanner and digital forest maps. They created a procedure for optimizing the network of logging roads, fully based on GIS and ALS data, assessed the length of the road network and the volume of woody biomass before and after the wood harvesting process, and finally, they compared the received results with the plan. However, besides the reducing transportation costs, this technology will reduce the number of trees that need to be cut down for the construction of roads in the forest.

The article [12] describes a study to identify the optimal density (or thickness) of the road network. In this paper the authors tried to find an optimal balance between the costs for forest roads creation and the economic benefits of these roads. For this purpose, the software NETWORK 2000 is used. During the implementation of this project, scientists are looking for the optimal ways to transport wood from the cutting area. In this case, the optimality criterion is the economic effect of their use.

The paper [13] is also devoted to the study of the optimality of wood transportation. However, authors approached the issue from the other side. They defined the conditions under which transportation equipment reaches the highest level of productivity and, therefore, provides the most profitable economic results. The study objects were: characteristics of loads on wood and the skidding distance, characteristics of the working time structure, characteristics of the duration of the skidding cycle, determining the performance of the skidder and the development a mathematical model of work efficiency. Thus, the authors of this study have built a relationship between the optimal performance characteristics of transportation equipment and the best option for transporting wood from the cutting area. Based on the data obtained, it is possible to build an optimal route for timber transportation.

This problem is considered from the strategic planning point of view in the paper [14]. In this study, the finding of the optimal wood transportation route is the first step for the implementation of forest roads operational planning. In total, the authors identifies three levels of forest road planning: strategic planning (planning at the state or regional level), tactical planning (planning of forest transport infrastructure at the economic unit level) and operational planning (planning of specific forest roads). The authors believe that the finding of the optimal wood transportation route is the key to the successful finance investment in the development of forest road infrastructure.

In this paper, the transport network used for the wood transportation is presented in the form of a graph. A graph is an abstract mathematical object that represents a set of vertices and a set of edges or arc (connections between pairs of vertices). Transportation costs may depend directly on the volume of goods transported, or they may not be related to the volume of cargo transported. The construction (or restoration) and maintenance of transport routes can be attributed to the costs that do not depend on the volume of cargo transported. It is also important to consider that the value of transport costs on each arc (or on the edge of the graph) depends on the environmental and climatic conditions in the area. The most significant environmental and climatic conditions are the type of soil, the magnitude of the road slope (terrain), the presence of water barriers [15]. The limited capacity of certain sections of transport routes is also a peculiarity of wood transportation [16]. This characteristic will also depend on the climatic conditions of the area.

The aim of the presented study is as follows: it is necessary to deliver the entire volume of harvested wood from the forest area to the delivery point with minimal costs. There are a number of intermediate points between the forest area and the delivery point, which are connected with each other by a network of roads with different capacity. In addition, there are no roads between some of points, but they can be built. Among other things, natural and climatic conditions have an impact on

the cost of timber removal and construction (or repair) of roads. The harvested wood can be transported from the starting point to the final one by different routes.

Thus, the aim of the described study is to solve the problem of finding the route of harvested wood transportation from the cutting area to the consumer with minimal total cost. The peculiarity of the task is that the algorithm takes into account the costs that vary depending on the cargo volume transported along this route.

2. Problem solving and discussion of the results

We denote:

1. v_0 – a starting point (cutting area), v_n – a final point (delivery point), $v_1, \dots, v_{n-1}$ – intermediate points.
2. Q – the volume of wood in the starting point which should be taken out.
3. u_{ij} – the road capacity between the points v_i and v_j (the maximum amount of goods which can be transported).
4. c_{trij} – the cost of transporting 1 m³ of goods between the points v_i and v_j , construction (restoration) costs – as c_{rij} .

The points and connecting their existing and projected roads are represented in the form of a connected oriented graph (network) $G=(V, E)$, where V the set of vertices v_i ($i=0, \dots, n$), E is the set of arcs (v_i, v_j) . Vertex v_0 corresponds to the starting point, v_n is the final one, vertices v_i ($i=1, \dots, n-1$) are to intermediate points. Arc (v_i, v_j) corresponds to the road between points v_i and v_j .

Each arc is assigned three numbers: road capacity, cost of hauling 1m³ of wood and cost of construction (restoration) of the road.

The following **mathematical model of the problem** can be elaborated.

Let x_{ijk} be a volume of timber transportation on the section (v_i, v_j) along the k -th route.

Total costs transportation of all volume of wood and construction (restoration) of the roads should be minimal:

$$F = \sum_{(v_i, v_j) \in E} \left(z_{ij} c_{rij} + c_{trij} \left(\sum_{k=1}^m x_{ijk} \right) \right) \rightarrow \min, \quad (1)$$

where c_{rij} – costs of construction (restoration) of the section (v_i, v_j) ; c_{trij} – cost of transporting 1 m³ of goods on the arc (v_i, v_j) ; m – a quantity of the routes used for transportation; $z_{ij}=1$, if goods is transported on the arc (v_i, v_j) ; $z_{ij}=0$ in other cases.

The whole volume of wood should be transported from the starting point to the final one:

$$\sum_{k=1}^m \sum_{(v_0, v_j) \in E} x_{0jk} = \sum_{k=1}^m \sum_{(v_i, v_n) \in E} x_{ink} = Q. \quad (2)$$

The volume of wood delivered to the intermediate point v_i , should be transported from there respectively:

$$\sum_{k=1}^m \sum_{(v_i, v_j) \in E} x_{ijk} = \sum_{k=1}^m \sum_{(v_j, v_i) \in E} x_{jik}, j = 1, \dots, n-1. \quad (3)$$

The volume of wood transported on the arc (v_i, v_j) of the k -th route, is not a negative figure and should not exceed the arc capacity:

$$0 \leq x_{ijk} \leq u_{ij}, \forall (v_i, v_j) \in E. \quad (4)$$

The algorithm for solving the problem is to find the optimal routes and volumes of cargo transportation on these routes, in which the delivery cost of a given volume of cargo from the starting point to the final one is minimal. The algorithm is based on Dijkstra's algorithm (shortest path search) and dynamic programming method.

The 1st stage. We need to perform the check:

1. If $\sum_{(v_0, v_j) \in E} u_{0j} < Q$, then the problem has no solutions.
2. If $\sum_{(v_0, v_j) \in E} u_{0j} \geq Q$, then we make the assumption that $k=1$.

The 2nd stage. We determine k -th route of the minimal cost from the vertex v_0 to the vertex v_n and the quantity of goods being transported along it.

Step 1. We need to assign initial labels $[l(v_i); Q_i]$ to the graph vertices: $l(v_i)$ is a minimal adduced cost of goods transportation from the vertex v_0 to the vertex v_i , Q_i is a cargo volume in the vertex v_i .

The label of v_0 is assumed to be equal to $[0; Q_0]$. When we find the first route, we also assume $Q_0=Q$. The labels $[\infty; 0]$ are assigned to all the other vertices, we will consider them to be temporary.

Step 2. We need to determine the minimal value of $l(v_i)$ in the labels of vertices: $l(v_0)=0$. We will consider the label of v_0 to be constant.

Step 3. Now, we need to change the labels of vertices v_j , which belong to the set $\Gamma(v_0)$ ($\Gamma(v_0)$ - the set of vertices to which a directed arc from vertex v_0) in the following way: $[l(v_j); Q_j; v_0]$, where $l(v_j)$ is a minimal cost of transportation of Q_j units of goods to the vertex v_j from the vertex v_0 :

$$l(v_j) = \min\{l(v_j), l(v_0) + c_{0j}\}, \quad (5)$$

Q_j – quantity of goods, which are transported to the vertex v_j from the vertex v_0 :

$$Q_j = u_{0j}. \quad (6)$$

c_{0j} - construction and transportation costs for the segment (v_0, v_j) :

$$c_{0j} = \frac{c_{tr0j} \cdot u_{0j} + c_{r0j}}{u_{0j}}. \quad (7)$$

If the capacity of the arc (v_0, v_j) is equal to 0, then the label of v_j does not change. The labels obtained are considered to be temporary. It should be noted that several temporary labels can be assigned to all the vertices, except for the initial and the final one, when we proceed with the next steps.

Step 4. 1. If the labels of all the vertices except for v_n are constant, then we consider the label of this vertex to be constant as well and we can move to step 6.

2. If any temporary label's value $l(v_j)$ is higher than $l(v_n)$, it can be excluded from investigation. The vertex v_i^* should be chosen from all the temporary labeled vertices, where $l(v_i^*) = \min\{l(v_i)\}$. If there are several vertices with such values, any of them can be chosen.

Step 5. The vertices v_j which belong to the set $\Gamma(v_i^*)$, and for which $u_{i^*j} \neq 0$ should be determined.

If the set $\Gamma(v_i^*)$ is empty, then the label of the vertex v_i^* becomes constant. If the vertex v_j is the third indicator in the label of the vertex v_i^* , then it can be excluded from the set $\Gamma(v_i^*)$, due to the fact that this case relates to the situation when transportation is made to the vertex from which it was made during the previous step which is obviously unprofitable from the point of view of transportation costs.

Now, we need to compare the capacities of the arcs (v_i^*, v_j) and Q_{i^*} in the label of the vertex v_i^* .

1. If only one vertex v_j belongs to the set $\Gamma(v_i^*)$, then the following cases are possible:

1.1. If $u_{i^*j} < Q_{i^*}$, then we need to calculate a new weight of the arc between the vertex v_0 and the vertex $v_i \in \Gamma(v_0)$, which can be determined, sequentially moving in the opposite direction from the vertex v_i^* to the vertex v_0 in the arcs, the end of which is indicated by the third indicator in the labels:

$$c_{0i} = \frac{c_{tr0i} \cdot u_{i^*j}^* + c_{r0i}}{u_{i^*j}^*}. \quad (8)$$

A new value of $l(v_i)$ should be calculated:

$$l(v_i) = l(v_0) + c_{0i}. \quad (9)$$

The label of the vertex v_i is replaced with a new label $[l(v_i); u_{i^*j}^*; v_0]$. Similar labels will be added to all the vertices determined when moving from the vertex v_i^* to the vertex v_0 . We get back to the step 4.

1.2. If $u_{i^*j}^* \geq Q_j$, then the weight of the arc c_{i^*j} and the value of $l(v_j)$ should be calculated:

$$c_{i^*j} = \frac{c_{tri^*j} \cdot Q_{i^*} + c_{ri^*j}}{Q_{i^*j}}, \quad (10)$$

$$l(v_j) = \min\{l(v_j), l(v_i^*) + c_{i^*j}\} \quad (11)$$

The vertex v_j has a new label assigned:

$$[l(v_j); Q_{i^*}; v_i^*] \quad (12)$$

The label of v_i^* will be considered to be constant. We get back to the step 4.

2. If several vertices v_j ($j=1, \dots, l$) belong to the set $\Gamma(v_i^*)$, then the following situations are possible:

2.1. For one of the vertices v_j $u_{i^*j}^* < Q_{i^*}$, this case is similar to 1.1, but the difference is that there is a new label $[l(v_i); u_{i^*j}^*; v_0]$ for the vertex v_i . We get back to the step 4. By doing so, when comparing $l(v_i)$, it is necessary to take into account all the labels, those which already exist and those which were added back, and then when comparing the capacities of the arcs (v_i^*, v_j) with Q_{i^*} we need to consider only the vertices from the set $\Gamma(v_i^*)$, where $u_{i^*j}^* \geq Q_{i^*}$.

2.2. $u_{i^*j}^* < Q_{i^*}$ for all the vertices v_j ($j=1, \dots, l$), then two cases are possible:

a) all the $u_{i^*j}^*$ are the same, this case is similar to 1.1;

b) there are different values of $u_{i^*j}^*$ and for all the $u_{i^*j}^*$ new values of the weight of the arc are calculated using the formula (8), the arc is between the vertex v_0 and the vertex $v_i \in \Gamma(v_0)$, which can be determined sequentially moving in the opposite direction from the vertex v_i^* to the vertex v_0 of the arcs, and their end can be determined as the third indicator in the labels.

Now, we should calculate $l(v_i)$ for each new value of c_{0i} respectively.

For the value v_i new labels are added which correspond to various values $u_{i^*j}^*$ and, consequently, different values $l(v_i)$.

We get back to the step 4. By doing so, when comparing $l(v_i)$, it is necessary to consider all the labels, those which already exist and those which were added back.

2.3. For all the vertices v_j ($j=1, \dots, l$) $u_{i^*j}^* \geq Q_{i^*}$, then these vertices will have the label (12), where c_{i^*j} is calculated using the formula (10), $l(v_j)$ – using the formula (11). The label of the vertex v_i^* is considered to be constant. We get back to the step 4.

Step 6. We need to determine the route which has a minimal cost of transportation of Q_n units of goods from the vertex v_0 to the vertex v_n (minimal adduced cost is $l(v_n)$). Q_n is the second indicator in the label assigned to the vertex v_n . In order to determine the optimal route, a move in the opposite direction from the vertex v_n to the vertex v_0 should be performed: v_n is the last vertex of the route and a vertex preceding to v_n is the third indicator in the label of the vertex v_n . Then, we need to move to that

vertex, which has a label where the preceding vertex can be found, and so on – until we reach the vertex v_0 , which is the starting point of the route.

Thus, as a result, we will determine the optimal route $v_0 \rightarrow v_{i_1} \rightarrow v_{i_2} \rightarrow v_{i_3} \rightarrow \dots \rightarrow v_{i_n}$, where s is a quantity of intermediate points of the route between v_0 and v_n .

The 3d stage. Optimal values of variables should be determined: $x_{0i_1k}^* = x_{i_1i_2k}^* = x_{i_2i_3k}^* = \dots = x_{i_nk}^* = Q_n$.

The 4th stage. The value of Q_0 should be changed: $Q_0 := Q_0 - Q_n$.

The 5th stage. The value of Q_0 should be analyzed:

1. If $Q_0 > 0$, then two cases are possible:

1.1. If the capacities of all the arcs (v_0, v_j) ($v_j \in \Gamma(v_0)$) are equal to 0 or $\sum_{(v_0, v_j) \in E} u_{0j} < Q_0$, then the

problem does not have the solution, and only Q_0 units of goods can be delivered from v_0 .

1.2. Otherwise, the capacities of the arcs $(v_0, v_{i_1}), (v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots, (v_{i_n}, v_n)$, which are not equal to 0, decrease by the value of Q_n , and their weights are equal to the transportation cost of the unit of goods. At the next iteration, c_{rij} is taken as 0 for these arcs.

We consider that $k=k+1$ and then we get back to the 2nd stage.

2. If $Q_0=0$, then the problem is resolved: the optimal routes and the quantities of goods being transported from v_0 to v_n are found.

Then, we calculate the minimal costs for the transportation of wood volume Q , construction (restoration) of roads, using the formula (1).

As a result of calculations using the above-mentioned algorithm, the route of minimal cost of wood transportation can be determined.

This algorithm has the following assumptions and special features:

- it includes both, costs which depend on the volume of wood being transported and costs which do not depend on this;
- construction (restoration) and transportation costs may vary depending on the volume of wood being transported;
- the capacity of the route sections is taken into account;
- natural and climatic conditions of the route sections that have the most significant impact on the cost of transportation are taken into account.

3. Conclusion

The developed algorithm allows solving the problem of finding the optimal route of harvested wood transportation from the cutting area to the consumer with minimal total cost. This algorithm takes into account the costs that vary depending on the cargo volume transported on this route. The obtained results can be used in solving similar problems in other conditions, for example, when transporting other types of cargo.

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