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# Correlation Study of Rainfall and Runoff in Xiangxi River Based on Archimedean Copula Function

Aili Yang<sup>1,\*</sup>, Yuli Si<sup>2</sup>, Yurui Fan<sup>3</sup>, Xiangming Kong<sup>4</sup> and Xiajing Lin<sup>1</sup>

<sup>1</sup>Key Laboratory of Environmental Biotechnology, Xiamen University of Technology, Xiamen, China, 361024

<sup>2</sup>People's Government of Datang Town in Zhuji City, Shaoxing, China, 311801

<sup>3</sup>Institute for Energy, Environment and Sustainable Communities, University of Regina, Regina, Saskatchewan, Canada, S4S 0A2

<sup>4</sup>College of Fundamental Science, Beijing Polytechnic, Beijing, China, 100176

\*E-mail: yangaili@xmut.edu.cn, Tel: 05926291620

**Abstract.** In the bivariate hydrologic correlation analysis framework, the correlation analysis between rainfall and runoff in Xiangxi River is constructed using Archimedean Copula method. Results show that: (1) the monthly rainfall and runoff in Xiangxi River water shed has a relatively strong positive correlation based on Kendall and Spearman's rank correlation coefficients; (2) the Frank Copula can be able to model the joint distributions of monthly rainfall and runoff according to goodness-of-fit statistic test. The obtained joint distribution can be used to simulate streamflows of Xiangxi River watershed according to the historical and predicted rainfall data, which can provide a basis for watershed water resources development.

## 1. Introduction

Univariate Hydrological frequency analysis procedures are efficient and widely applied for analysing the change rules of rainfall and runoff [1-2]. However, any variable in the hydrological events are not independent, hydrological processes often involve multidimensional characteristics [3]. Univariate hydrological frequency analysis often overlooked the combined effects of the variables, which could not reflect the correct variation of hydrological variables objectively. Consequently, statistical theories of joint probability analysis are progressively applied to the field.

In recent years, Copula theory have been applied to multivariate hydrological frequency analysis, such as relationship analysis of streamflow, peak value and duration [4-5]; storm or rainfall dependence analysis [6]; the relationship analysis of the extent and duration of drought [7-8]; simulation of monthly streamflow [3]; hydrologic risk analysis [9-10]. Copula function consists of marginal distributions and correlation analysis, and the calculation of the two parts is relatively independent, which makes the joint analysis of multivariate methods become popular [11]. The objective of this article is to develop an Archimedean Copula method for simulation the correlation of rainfall and runoff in Xiangxi River (e.g. the largest tributary of Yangtze River in the Hubei part of the Three Gorges Reservoir area).



## 2. Methodology

### 2.1. Copula Function Theory

Copula theory was proposed by Sklar (1959) [12]. Copula function has many types including normality of Copula, t-Copula functions, and Archimedean Copula function family [13]. However, Archimedean Copula function family plays an important role in hydrologic frequency analysis with a simplified structure, diversification and practicability and other relevant characteristics [6]. The main Archimedean Copula function family includes: Gumbel-Hougaard Copula function, Clayton Copula function and Frank Copula functions. In this paper, we choose Frank Copula function to calculate and represent the correlation of rainfall and runoff in Xiangxi River.

Let  $C(u_1, u_2, \dots, u_t)$  is Copula function,  $u_i$  is marginal of variable,  $t$  is number of variable,  $\theta$  is the parameter of copula function, the cumulative probability distribution of the Frank Copula function can be expressed as follows:

$$C(u_1, u_2, \dots, u_t) = -\frac{1}{\theta} \ln \left\{ 1 + \frac{[\exp(-\theta u_1) - 1][\exp(-\theta u_2) - 1] \cdots [\exp(-\theta u_t) - 1]}{[\exp(-\theta) - 1]^{t-1}} \right\}; \theta \in R \quad (1)$$

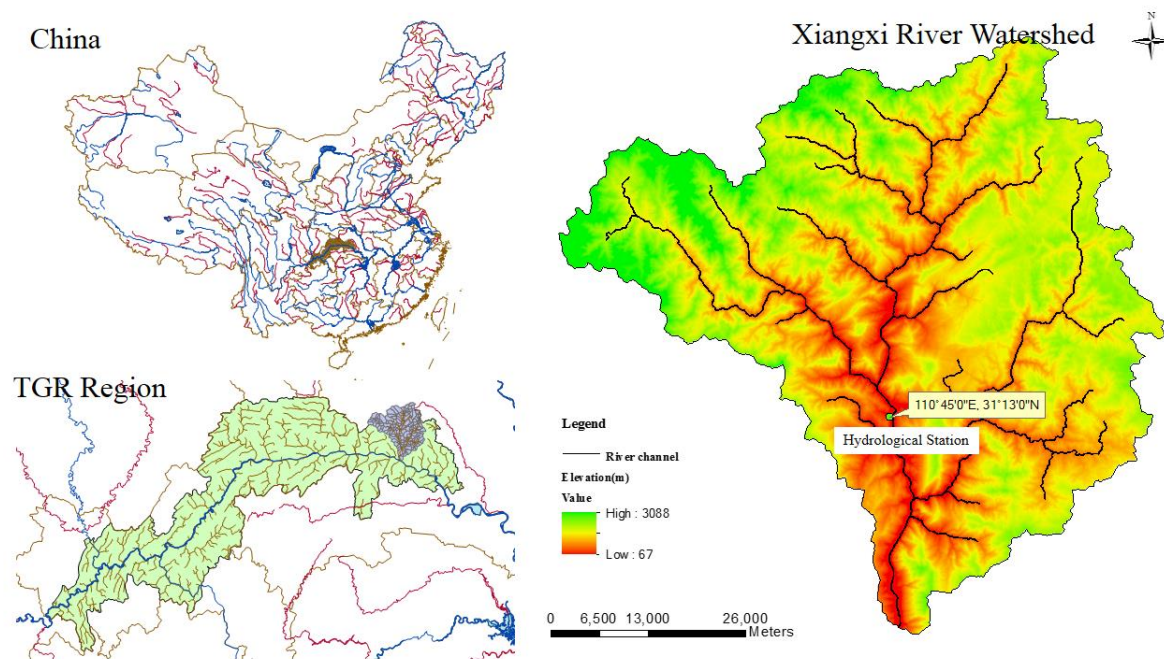
Expression of generator:

$$\varphi = -\ln \frac{\exp(-\theta t) - 1}{\exp(-\theta) - 1} \quad (2)$$

The successful key to apply Copula function lies in efficient estimation of marginal distributions. In this paper, the marginal distributions are estimated by the Gamma distribution, the Generalized extreme value distribution and Lognormal distribution, respectively. The root mean square error (RMSE) and Akaike Information Criterion (AIC) values are calculated to identify the appropriate marginal distributions. And then, the value of parameter  $\theta$  in the Frank Copula function is estimated by the maximum likelihood method. Finally, the joint probability distribution is estimated, and goodness-of-fit statistic test is performed (for more calculation details, readers can refer to Kong et al. (2015) [3]).

### 2.2. Case Study

The Xiangxi River basin located between  $30.96 \sim 31.67^\circ \text{N}$  and  $110.47 \sim 111.13^\circ \text{E}$  in Hubei part of China Three Gorges Reservoir region (see Figure 1) was selected to validate the applicability of the developed method in correlation study of rainfall and runoff. The Xiangxi River originates in the Shennongjia Nature Reserve, with the mainstream length of 94 km and a catchment area of  $3,099 \text{ km}^2$  [14]. The Xiangxi River experiences a northern subtropics climate, and the main rainfall season is from May to September with the annual precipitation of 1100 mm [15]. In this study, forty seven year's daily streamflow data (1991~2008) from Xingshan Hydrometric Station provided by the Hydrologic Bureau of Xingshan County would be used for analysis.



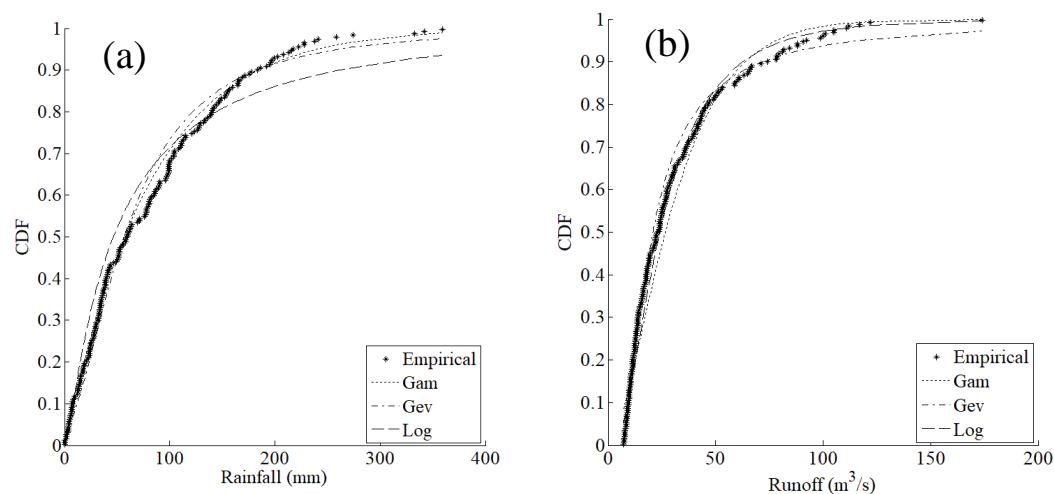
**Figure 1.** The studied area [9]

### 3. Results Analysis

A plot of empirical cumulative distribution functions (i.e. CDFs) and generated marginal cumulative distribution functions for monthly rainfall and runoff by the Gamma distribution, the Generalized extreme value distribution and Lognormal distribution are shown in Figure 2. From Table 1 and Figure 2, we can see that the Gamma-based marginal distribution can appropriately represent the observed rainfall data, while the Generalized-extreme-value-based marginal distribution can appropriately represent the observed runoff data.

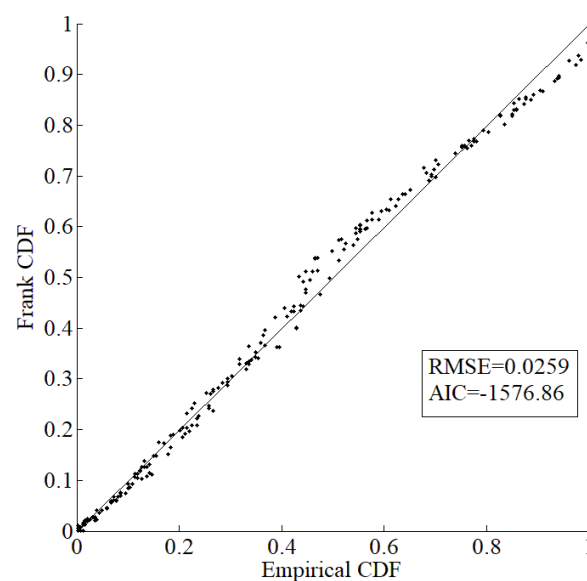
**Table 1.** The RMSE and AIC analysis for marginal distributions

Probability distribution	RMSE		AIC	
	Rainfall	Runoff	Rainfall	Runoff
Gam	0.024	0.056	-1624.15	-1242.93
Gev	0.038	0.0319	-1404.66	-1481.28
Log	0.059	0.0349	-1222.06	-1445.28



**Figure 2.** Comparison of the generated and empirical marginal CDFs for monthly rainfall (a) and runoff (b) in Xiangxi River watershed.

The monthly rainfall and runoff joint probability distribution at Xingshan Hydrometric station in Xiangxi River watershed is estimated by the Frank Copula function. The Kendall's rank correlation coefficient is 0.5666 and the Spearman rank correlation coefficient is 0.7676. It is indicated that a relatively strong positive correlation existing between monthly rainfall and runoff.

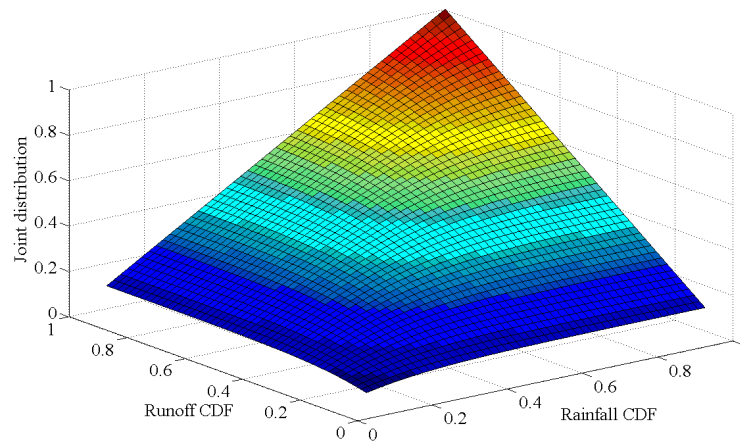


**Figure 3.** Comparison of the generated and empirical joint CDFs for rainfall and runoff in Xiangxi River.

According to Figure 3, Frank-Copula-based joint distribution can be able to appropriately represent the relevant characteristics of monthly rainfall and runoff of Xiangxi River. So, in this study, Frank Copula function is used to calculate the joint distribution of monthly rainfall and runoff. Figure 4 is the joint CDF of rainfall and runoff.

In this study, Anderson-Darling test (i.e. AD test) has been chosen to investigate whether or not the joint distributions are suitable for the description of the dependencies for different rainfall-runoff pairs

in Xiangxi River based on the historical records. Results are given in Table 2. It can be seen that the method performed well for modelling variable pairs. Statistics are less than threshold values.



**Figure 4.** The joint CDF of rainfall and runoff.

**Table 2.** Goodness-of-fit test

Fit test	Test statistics	$\alpha$ -the critical value of sub-sites				
		0.20	0.15	0.10	0.05	0.01
AD	1.20	1.41	1.46	1.58	2.02	3.15

#### 4. Conclusions

In this study, an Archimedean Copula method has been proposed for monthly rainfall and runoff correlation analysis in Xiangxi River, China. The different distributions were used to obtain a suitable marginal distribution. The correlation of monthly rainfall and runoff was estimated through the Frank Copula. Kendall and Spearman's rank correlation coefficients, RMSE and AIC values were adopted to clarify Copula function. The joint distribution of monthly rainfall and runoff was then derived based on the selected Copula. The obtained joint distribution could be used to simulate streamflows of Xiangxi River according to the historical and predicted rainfall data, which could provide a basis for water resources development.

#### Acknowledgments

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