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## Analysis of The Joint Impact of Synchronous Discharges in Estimating the Flood Risk: Case Study on Hron River

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# Analysis of The Joint Impact of Synchronous Discharges in Estimating the Flood Risk: Case Study on Hron River

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**Abstract.** The paper presents the methodology of bivariate statistical analysis of the joint impact of synchronous discharges in estimating the flood risk on Hron River and its tributary Slatina in Central Slovakia. Basic statistical analysis approach gives satisfactory results in the case of simple systems, for example, where the main river does not capture major tributaries. These conventional approaches may not give satisfactory results for the evaluation of flood risk in situations where floods occur on two or more rivers and join together at the same time. As input data mean daily discharges were used. Some Archimedean copula functions as a mathematical tool for joint as well as conditional probability distribution calculation of the synchronous variables were used. This class of copulas is popular in empirical applications for flexibility, easy construction and includes a whole suite of closed-form copulas that covers a wide range of dependency structures, including comprehensive and non-comprehensive copulas, radial symmetry and asymmetry, and asymptotic tail dependence and independence. The first part of the paper presents the preparation of the input data and choice of appropriate marginal probability distributions. The next part, presents the testing and selection of the appropriate copula function for the bivariate joint statistical analysis of the synchronous discharges. Tested Archimedes copula functions have achieved relatively equal calculated distribution probabilities. Probabilities calculated using the Gumbel-Hougaard copula function achieved the least error of the estimation. This copula function has been selected as the most appropriate for illustrating the joint distribution probability and consequently to determine the joint probability of occurrence of the synchronic discharges. The results showed that the joint probability of maximum discharges is relatively low, but not unlikely. In the context of climatic extreme events, statistical techniques such as event coincidence analysis will be relevant for investigating the impacts of anthropogenic climate change on human societies and ecosystems worldwide. The results obtained by the bivariate analysis of the variables which characterize the hydrological regime can contribute to a more reliable assessment of the flood risks.

## 1. Introduction

Assessment of the statistical significance of floods in the complex hydrological conditions that exist at the confluence of the main stream and its tributaries, as well as the choice of hydrological design parameters for dimensioning of flood protection in these areas are one of the priority tasks in the current hydrology. Flood wave is realization of the discharge process, which under the natural conditions has stochastic character. Determining of the basic flood wave characteristics for design of the water management project is based on adequate mathematical technique such as statistical theory of probability. The basic mathematical technique is based on the evaluation of the select univariate variable, which characterizes some event. Basic statistical analysis approach gives satisfactory results



in the case of simple systems, for example, where the main river does not capture major tributaries. These conventional approaches may not give satisfactory results for the evaluation of flood risk in situations where floods occur on two or more rivers and join together at the same time. Natural water circulation is increasingly affected or disturbed by the artificial interventions in river basins. Regulation of rivers as well as adjustments of the river basins often brings changes in concentration of basins drainage as well as increasing of the speed of flood wave. As consequence of it can be coincidence of the flood waves of the main river and its tributary. The bivariate approach to statistical analysis of flood events should be further developed and defined at neighbouring profiles on mainstream and its tributaries. Besides that, bivariate analysis of the simultaneously occurred variables can significantly contribute to a more reliable assessment of flood danger. In Slovak territory the coincidence of multiple flood waves caused the flood with time return period of 100-year on Tisa and Bodrog river in year 2000. For example the flood occurred in August, 2002 in Czech Republic on the Vltava River and Dyje River showed an increase in return period of the discharges with an increase in area of the basin. It was caused by coincidence of the flood waves in profiles of the river network [1]. Espinoza [2] analysed formation of the floods on the Amazonia River and its tributary. They focused on the catastrophic flood occurred in 2012, when coincidence of two large flood waves has occurred. Prohaska [3] dealt with synchronously occurring flood waves on the Danube and its tributaries. Their analysis was based on the theory of bivariate variable statistics and results confirmed that flood wave genesis is very complex within the Danube basin. Matúš [4] was dealing with hydrological modelling of joint events on the river Morava using aggregation operators.

The paper presents the methodology of bivariate statistical analysis, choice of appropriate marginal probability distributions, testing of the Archimedean copula functions and joint as well as conditional probability distribution using copula functions. Then, the joint occurrence probability of the variables was calculated. In the context of climatic extreme events, statistical techniques such as event coincidence analysis will be relevant for investigating the impacts of anthropogenic climate change on human societies and ecosystems worldwide. The results obtained by the bivariate (as well as threevariate) analysis of the variables which characterize the hydrological regime can contribute to a more reliable assessment of the flood risks.

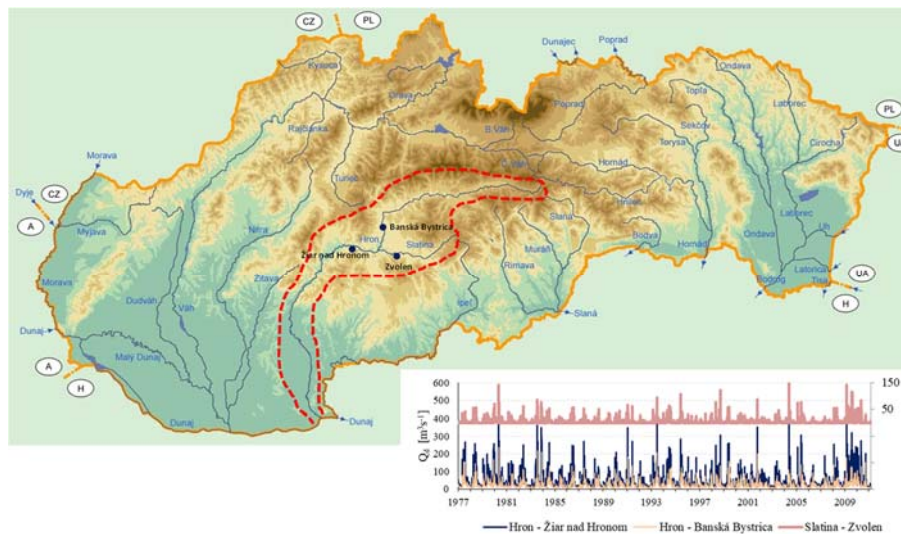
## 2. Location and data

### 2.1. Choice and preparing of the data

The Hron River is a 298-kilometre (185 mi) long left tributary of the Danube and the second-longest river in Slovakia. The Hron River originates between Low Tatras Mountains and Spišsko-Gemersky karst on level of 980 m a.s.l. and through central and southern Slovakia, flows into the Danube River near Štúrovo. The river's basin covers approximately 11 percent of Slovakia's territory.

The Slatina stream is the largest left tributary of the Hron River. The length of the river is 55.2 km. The Slatina River springs in Veporské hills on level of 930 m a.s.l. and discharges into the Hron River on level 272 m a.s.l. near town Zvolen (Central Slovakia).

The scheme of the main Slovak basins as well as the course of the mean daily discharges on Hron River (upstream and downstream profiles) and its tributary Slatina during the period 1977–2011 is presented in Figure 1 and selected water measured gauging station are listed in table 1.



**Figure 1.** The map of the main Slovak basins (border of Hron basin – red) and mean daily discharges on Hron River (upstream and downstream profiles-lower course) and its tributary Slatina (upper course) during the period 1977–2011.

**Table 1.** List of the selected gauging stations and analysed periods

Main river	Gauging station (UP)	Gauging station (DWN)	Tributary	Gauging station (TR)	Period
Hron	Banská Bystrica	Žiar nad Hronom	Slatina	Zvolen	1977–2011

The mean daily discharges of the Hron River and its tributary Slatina were used as input data for the bivariate coincidence analysis. The data for the given periods was provided by Slovak Hydrometeorological Institute. The choice of flow waves, which were included in the analysed basic statistical series, was based on the POT method. The threshold in our calculation was determined at the level of 40-50% of long-term maximum annual discharge. This ensures waves independence and includes all significant events in the year (table 2) to statistical series. From these waves, the maximum mean daily discharges and correspondence discharges were selected according to the following scheme of the combination of variables:  $Q_{maxup} - Q_{cor1dwn}$ ;  $Q_{maxup} - Q_{cor1tr}$ ;  $Q_{maxdwn} - Q_{cor2up}$ ;  $Q_{maxdwn} - Q_{cor2tr}$ ;  $Q_{maxtr} - Q_{cor3up}$ ;  $Q_{maxtr} - Q_{cor3dwn}$ .

**Table 2.** Values of the POT level at Hron and Slatina at current stations

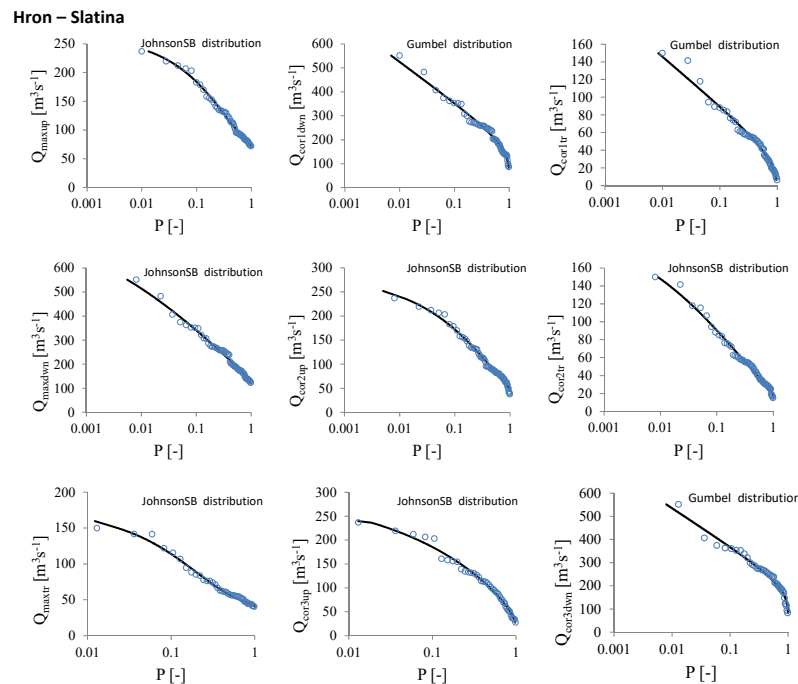
Main river	Gauging station (UP)	$\bar{Q}_{amax}$ [m³s⁻¹]	Gauging station (DWN)	$\bar{Q}_{amax}$ [m³s⁻¹]	Tributary	Gauging station (TR)	$\bar{Q}_{amax}$ [m³s⁻¹]
Hron	B. Bystrica	157	Žiar nad Hronom	308	Slatina	Zvolen	95

Where:  $Q_{maxup}$  - maximum daily discharge on the main river upstream from the tributary;  $Q_{cor1dwn}$ ,  $Q_{cor1tr}$  - corresponding discharges on the main river downstream from the tributary as well as tributary in the moment of occurrence of the maximum daily discharge on the main river upstream from the tributary;  $Q_{maxdwn}$  - maximum daily discharge on the main river downstream from the tributary;  $Q_{cor2up}$ ,  $Q_{cor2tr}$  - corresponding discharges on the main river upstream from the tributary as well as tributary in the moment of occurrence of the maximum daily discharge on the main river downstream from the tributary;  $Q_{maxtr}$  - maximum daily discharge on the tributary;  $Q_{cor3up}$ ,  $Q_{cor3dwn}$  - corresponding discharges on the main river upstream as well as downstream from the tributary in the moment of occurrence of the maximum daily discharge on the tributary.

### 3. Methodology

#### 3.1. Univariate Analysis of the synchronous hydrological variables - discharges

The first step of the bivariate analysis is to identify univariate (marginal) distributions for hydrological variables. The random variables may have different properties and thus need to be converted to variables having interval of [0, 1] by scaling the data. Knowing the marginal distribution, we are able to separate marginal behaviour and dependence structure. The dependence structure is fully described by the joint distribution of uniform variables obtained from marginal distribution. In order to determine univariate parametric distribution functions (marginal distributions), standard MLM (maximum likelihood method) method was used. According to the goodness-of-fit tests (Kolmogorov-Smirnov and  $\chi^2$ ) the marginal distributions were selected. The JohnsonSB and Gumbel parametric distributions the best fitted most variables derived by method POT. The fitted distributions and  $p$ -value of the goodness-of-fit tests showed, that we cannot reject the hypothesis that selected distributions fit well to the observed data at a 5% significance level (table 3). Subsequently, the empiric distribution evaluated with Cunnane [5] formula has been fitted with selected univariate parametric cumulative distribution functions (Figure 2)).



**Figure 2.** Comparison of the empirical distribution with fitted univariate parametric distribution functions

**Table 3.** The best fitted parametrical univariate distributions of the variables and  $p$ -values of the Kolmogorov-Smirnov and  $\chi^2$  tests

	$Q_{maxup}$	$Q_{cor1dwn}$	$Q_{cor1tr}$	$Q_{maxdwn}$	$Q_{cor2up}$	$Q_{cor2tr}$	$Q_{maxtr}$	$Q_{cor3up}$	$Q_{cor3dwn}$
Hron - Slatina									
Distrib.	Joh. SB	Gumbel	Gumbel	Joh. SB	Joh. SB	Joh. SB	Joh. SB	Joh. SB	Gumbel
K-S	0.893	0.618	0.927	0.696	0.679	0.889	0.669	0.966	0.609
$p$ -value									
$\chi^2$	0.85	0.753	0.684	0.421	0.411	0.745	0.481	0.895	0.984
$p$ -value									

### 3.2. Bivariate analysis of the synchronous discharges

Copula functions were used as mathematical tool for determining a joint cumulative distribution of two dependent variables. We used the Archimedean class of copula functions. Among existing types of copulas, the Archimedean one is the very popular class used in hydrological applications [6 - 9] ...etc.). This class of copulas is popular in empirical applications for flexibility, easy construction and includes a whole suite of closed-form copulas that covers a wide range of dependency structures, including comprehensive and non-comprehensive copulas, radial symmetry and asymmetry, and asymptotic tail dependence and independence. The Clayton, Gumbel-Hougaard and Frank copulas were selected for this study (table 4). The copula parameter  $\theta$  was estimated using a mathematical relationship between the Kendall's coefficient of rank correlation and the generating function  $\varphi(t)$  [10]. The values of the estimated parameters of selected Archimedean copula functions are listed in table 5.

**Table 4.** Probability functions, parameter space, generating function and relationship of non-parametric dependence measure with association parameter for the most frequently used Archimedean copulas in hydrology

Copula	C (u, v, $\theta$ )	Parameter space $\theta$	Kendall's $\tau$	generator $\varphi(t)$
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$[-1, \infty) \setminus \{0\}$	$\frac{\theta}{\theta + 2}$	$\frac{1}{\theta}(t^{-\theta} - 1)$
Gumbel-Hougaard	$\exp[-((-\ln u)^\theta + (-\ln v)^\theta)^{1/\theta}]$	$[1, \infty)$	$\frac{\theta - 1}{\theta}$	$(-\ln t)^\theta$
Frank	$-\frac{1}{\theta} \ln[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)}]$	$(-\infty, \infty) \setminus \{0\}$	$1 + \frac{4}{\theta}[D_1(\theta^*) - 1]$	$-\ln \frac{e^{-\theta} - 1}{e^{-\theta} - 1}$

**Table 5.** Copula parameters (C - Clayton, G-H - Gumbel-Hougaard, F – Frank), selected combinations of the variables

	$Q_{maxup}-Q_{cor1dwn}$	$Q_{maxup}-Q_{cor1tr}$	$Q_{maxdwn}-Q_{cor2up}$	$Q_{maxdwn}-Q_{cor2tr}$	$Q_{maxtr}-Q_{cor3up}$	$Q_{maxtr}-Q_{cor3dwn}$
Hron - Slatina						
C	3.75	1.86	2.74	3.15	1.07	2.81
G-H	2.87	1.93	2.37	2.58	1.54	2.41
F	9.65	5.41	7.35	8.05	3.54	7.3

### 3.3. Testing the copula function suitability for simulation of dependence between variables

In our work, firstly the Gringorten [11] plot-position formula (Equation 1) was used for the visual assessment of the copula function fitting. Comparison of empirical joint probability distribution for selected combinations of the synchronous discharges and the corresponding probability values derived by parametric copulas is presented in Figure 3. The empirical and parametric probabilities are plotted against ascending ranks of observation. The graphical comparison showed the range of the parametric copula area which corresponds to selected a series of observed data.

$$F(x, y) = P(X \leq x_i, Y \leq y_j) = \frac{\sum_{m=1}^i \sum_{l=1}^j n_{ml} - 0,44}{N + 0,12} \quad (1)$$

Where,  $N$  is the total number of the variables,  $j$  and  $i$  ascending ranks of  $x_i$  and  $y_i$ ,  $n_{ml}$  is the number of occurrence of the combinations of  $x_i$  and  $y_i$ .

For an evaluation of the difference between the parametric (Clayton, Gumbel Hougaard and Frank) and empirical copula functions, the following statistical indicators of the goodness-of-fit of the estimation were used:

Mean absolute error

$$MAE = \frac{1}{N} \sum_{t=1}^N |C_{\theta} - C_E|, \quad (2)$$

Square mean error

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (C_{\theta} - C_E)^2}, \quad (3)$$

Maximum absolute error

$$ME = \max |C_{\theta} - C_E|. \quad (4)$$

Where the  $C_{\theta}$  and  $C_E$  are empirical joint probabilities and parametric joint probabilities computed according the copula function.

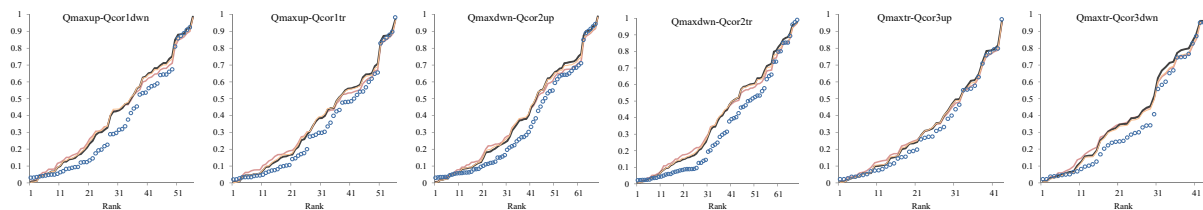
Results of the comparison of the joint empirical probabilities with fitted parametric copula showed that computed probabilities reached relatively comparable values for all three tested Archimedean copula functions. The best match between empirical and parametric copula function was achieved for combinations of variables with a lower rank Kendall correlation coefficient. The lowest values of the statistical indicators achieved the Gumbel-Hougaard copula function (Figure 4).

Subsequently, the Kolmogorov-Smirnov (KS test) goodness-of-fit test was used for an evaluation the copula function suitability. The two-sample KS test is one of the most useful and general nonparametric methods for comparing two samples, as it is sensitive to differences in both location and shape of the empirical cumulative distribution functions of the two samples. The test provides a result of the null hypothesis that two samples originate from the distributions with the same deviations, as opposed to the alternative that the differences are not the same in the fundamental distributions. At the significance level  $\alpha = 0.05$ , the critical value of the statistical variable  $D_k$  from the KS test table (e.g. [12]) was determined for a given range of variables. According to the Equation (5) [13], the value of the statistic  $D_v$  was calculated. If,  $D_k \geq D_v$  than we cannot reject the  $H_0$  hypothesis: the two one-dimensional random variables come from the same probability distribution. The results of the KS test are listed in table 6.

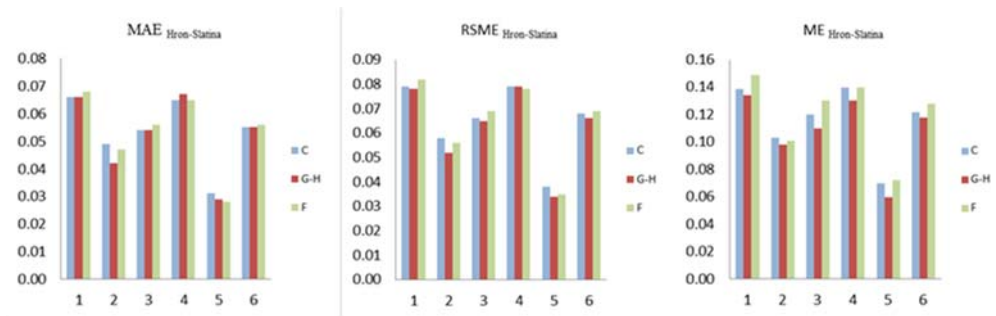
$$D_v = \max_{1 \leq k \leq n} \left\{ \left| C_k - \frac{m_k}{n} \right|, \left| C_k - \frac{m_{k-1}}{n} \right| \right\}, \quad (5)$$

where  $C_k$  is observed copula value of  $(x_k, y_k)$ ,  $m_k$  is number of pairs  $(x_k, y_k)$  suitable for  $x \leq x_k$  and  $y \leq y_k$ .

Hron-Slatina



**Figure 3.** Comparison of the empirical (points) with fitted parametric copula probabilities (Clayton, Gumbel-Hougaard – black line and Frank)



**Figure 4.** Statistical errors from comparison of the empirical copula JCDF and parametric copulas JCDF (1 –  $Q_{maxup}-Q_{cor1dwn}$ , 2 –  $Q_{maxup}-Q_{cor1tr}$ , 3 –  $Q_{maxdwn}-Q_{cor2up}$ , 4 –  $Q_{maxdwn}-Q_{cor2tr}$ , 5 –  $Q_{maxtr}-Q_{cor3up}$ , 6 –  $Q_{maxtr}-Q_{cor1dwn}$ ).

**Table 6.** Evaluation of Archimedean copula functions for computing the joint distribution function by Kolmogorov-Smirnov test

Hron - Slatina	N	Rank $\tau / \rho$	Clayton			Gumbel-Hougaard		Frank	
			$D_k$	$D_v$	p-value	$D_v$	p-value	$D_v$	p-value
$Q_{maxup} - Q_{cor1dwn}$	56	0.65 / 0.84	0.181	0.179	0.303	<b>0.161</b>	<b>0.43</b>	0.163	0.430
$Q_{maxup} - Q_{cor1tr}$	56	0.48 / 0.67	0.181	0.167	0.387	<b>0.132</b>	<b>0.693</b>	0.139	0.619
$Q_{maxdwn} - Q_{cor2up}$	69	0.58 / 0.78	0.163	0.150	0.317	<b>0.130</b>	<b>0.570</b>	0.128	0.568
$Q_{maxdwn} - Q_{cor2tr}$	69	0.61 / 0.82	0.163	0.217	0.066	<b>0.188</b>	<b>0.154</b>	0.203	0.102
$Q_{maxtr} - Q_{cor3up}$	43	0.35 / 0.48	0.203	0.140	0.865	<b>0.116</b>	<b>0.918</b>	0.117	0.917
$Q_{maxtr} - Q_{cor3dwn}$	43	0.58 / 0.78	0.203	0.209	0.269	<b>0.203</b>	<b>0.270</b>	0.209	0.269

According the KS test we reject the null hypothesis  $H_0$  for pairs of variables, which reached relatively high rank correlation:  $Q_{maxdwn} - Q_{cor2tr}$  (Hron – Slatina). This applies to all three tested copula functions. Because the value of the statistical variable  $D_k$  increases with decreasing N for the pair of variables  $Q_{maxup} - Q_{cor1dwn}$  (Hron) in this case  $H_0$  cannot be rejected (table 6). With respect to these results additional  $\chi^2$  test for the given pairs was used to confirm the rejection of the null hypothesis  $H_0$ . This test did not confirm rejection of  $H_0$  at the significance level  $\alpha = 0.05$  and achieved the highest values of p-value for the Gumbel-Hougaard copula. Based on the results of nonparametric tests the Gumbel-Hougaard copula was used to determine the joint probability distribution of the synchronous variables.

#### 4. Simulation of synchronous discharges using the copula function to determine the joint occurrence probability

Subsequently, the Gumbel-Hougaard copula was used for simulation of 3000 pairs for all combination of variables (Figure 5). Simulated pairs were performed to determine the joint probability distribution using the Gumbel-Hougaard copula (JDF-G-H) and consequently to determine the joint occurrence probability of the synchronic variables (table 7) including the following: a) the probability that at least one variable exceeds its threshold values (Eq.6); b) the probability that both variables exceeding certain threshold value (Eq. 7); c) the probability that  $Q_{maxup}$ ,  $Q_{maxdwn}$ ,  $Q_{maxtr}$  exceeds a specific threshold when corresponding variable is also high (Eq. 8).

$$P_a = P \cup (x, y) = 1 - F_{(max, cor)}; \quad (6)$$

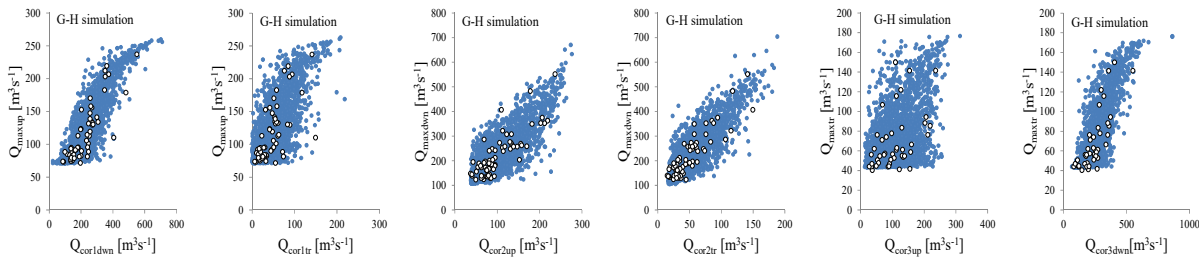
$$P_b = P \cap (x, y) = 1 - F_{(cor)} - F_{(max)} + F_{(max, cor)}; \quad (7)$$



$$P_c = \frac{P \cap (x, y)}{P(X > x)} = \frac{1 - F_{(cor)} - F_{(max)} + F_{(max, cor)}}{1 - F_{(cor)}}. \quad (8)$$

Where  $F_{(max)}$ ,  $F_{(cor)}$  are marginal distribution function and  $F_{(max, cor)}$  is joint distribution estimated through the copula function.

### Hron-Slatina



**Figure 5.** Scatter plots of 3000 data pairs generated from Gumbel-Hougaard copula and measured data of the synchronous variables for selected combinations (cannot reject hypothesis  $H_0$ ).

**Table 7.** Values of the JDF-G-H for maximal synchronous discharges and joint exceedance probabilities  $P_a$ ,  $P_b$  and  $P_c$

Hron-Slatina							
$Q_{maxup}$ [m <sup>3</sup> s <sup>-1</sup> ]	236.7	$Q_{maxdwn}$ [m <sup>3</sup> s <sup>-1</sup> ]	551.1	$Q_{maxtr}$ [m <sup>3</sup> s <sup>-1</sup> ]	149.8	$Q_{maxtr}$ [m <sup>3</sup> s <sup>-1</sup> ]	149.8
$Q_{max1cortr}$ [m <sup>3</sup> s <sup>-1</sup> ]	149.8	$Q_{max2cortr}$ [m <sup>3</sup> s <sup>-1</sup> ]	149.8	$Q_{max3corup}$ [m <sup>3</sup> s <sup>-1</sup> ]	236.7	$Q_{max3cordwn}$ [m <sup>3</sup> s <sup>-1</sup> ]	551.1
$F_{maxup}$	0.988	$F_{maxdwn}$	0.994	$F_{maxtr}$	0.977	$F_{maxtr}$	0.977
$F_{max1cortr}$	0.991	$F_{max2cortr}$	0.991	$F_{max3corup}$	0.982	$F_{max3cordwn}$	0.992
Par. $\theta$	1.93	Par. $\theta$	2.58	Par. $\theta$	1.54	Par. $\theta$	2.41
JDF-G-H	0.985	JDF-G-H	0.990	JDF-G-H	0.968	JDF-G-H	0.976
$P_a$ [%]	<b>1.515</b>	$P_a$ [%]	<b>1.010</b>	$P_a$ [%]	<b>3.210</b>	$P_a$ [%]	<b>2.371</b>
$P_b$ [%]	<b>0.585</b>	$P_b$ [%]	<b>0.490</b>	$P_b$ [%]	<b>0.890</b>	$P_b$ [%]	<b>0.729</b>
$P_c$ [%]	<b>65.037</b>	$P_c$ [%]	<b>54.396</b>	$P_c$ [%]	<b>49.448</b>	$P_c$ [%]	<b>91.116</b>

## 5. Results and discussions

In the presented paper, we analysed the suitability of the most commonly used class of copula functions in hydrology - Archimedean copula functions (Clayton, Gumbel-Hougaard and Frank). All tested Archimedes copula function achieved relatively equal calculated values of the probabilities. Based on the statistical criteria and visual comparison the Gumbel-Hougaard copula function was selected as the most suitable to illustrate the joint probability distribution of the synchronous variables. The best match between empirical and parametric copula function was achieved for combinations of variables with a lower Kendall correlation coefficient. The non-parametric goodness-of-fit test Kolmogorov-Smirnov for evaluation of the selected copula was used. According to the KS test the null hypothesis  $H_0$  (variables come from same distribution) was rejected for all pairs with relative high values of the Kendall  $\tau$  a Spearman  $\rho$ . The best agreement between empirical and parametric copula function was achieved for combination of variables with a lower Kendall  $\tau$  coefficient and for Gumbel-Hougaard copula function. For confirmation of the rejection of the null hypothesis  $H_0$ , another nonparametric  $\chi^2$  test was used. This test did not confirm the rejection of  $H_0$  at the

significance level  $\alpha = 0.05$  and achieved the highest values of p-value for the Gumbel-Hougaard copula.

Testing the copula function suitability for simulation of dependence between variables and using the goodness-of-fit is still discussed as a topic in literature. For example, [14, 15] used some of their criteria for testing the goodness-of-fit of the copulas. The so called “pseudo-likelihood ratio test”, inspired by a semi-parametric adaptation of the criterion AIC is introduced in [16]. Next goodness-of-fit test based on the Cramér-von Mises statistic (measure of distance between parametric copula and empirical copula) used in their work e.g.: [17, 18].

Based on the achieved results the Gumbel-Hougaard copula was used for simulation of 3000 pairs for all combination of variables on selected rivers. Simulated pairs were performed to determine the joint probability distribution and consequently to determine the joint probability occurrence of exceedance the synchronic variables. Although the joint exceedance probability  $P_b$  (the probability that both variables exceeding certain threshold value) of analysed maximum synchronous discharges was relatively low we cannot exclude that in the future such situation occur. E.g. Tadić [19] analysed the joint occurrence probability of floods on the rivers Danube and Drava near Osijek. Their results showed that the probability of such situation is low (0.79%) but they reminded that such a situation occurred in 1966 and it was one of the biggest floods.

## 6. Conclusions

The aim of the paper was show methodology of bivariate statistical analysis of the joint impact of synchronous discharges in estimating the flood risk. The joint occurrence probability of the variables was calculated by using copula functions. Although the probability that both variables exceeding maximal threshold values of synchronous discharges was relatively low, but we cannot exclude that in the future such situation occur. In the context of climatic extreme events, statistical techniques such as event coincidence analysis will be relevant for investigating the impacts of anthropogenic climate change on human societies and ecosystems worldwide. The results obtained by the bivariate (as well as threevariate) analysis of the variables which characterize the hydrological regime can contribute to a more reliable assessment of the flood risks.

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