

PAPER • OPEN ACCESS

Stress-Dilatancy of Sands with Inherent Fabric Anisotropy for Direct Shear

To cite this article: Zenon Szypcio 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **221** 012012

View the [article online](#) for updates and enhancements.

Stress-Dilatancy of Sands with Inherent Fabric Anisotropy for Direct Shear

Zenon Szypcio ¹

¹Bialystok University of Technology, Faculty of Civil and Environmental Engineering, Wiejska 45E, 15-351 Bialystok, Poland.

z.szypcio@pb.edu.pl

Abstract. Granular materials deposited under gravity present inherent fabric anisotropy. The mechanical properties of such materials depend on the angle between shear plane and bedding plane perpendicular to gravity direction. From the literature data of direct shear of glass beads, Fujian and mica sands, the stress-dilatancy relationships are analyzed based on *Frictional State Theory*. The angle of critical frictional state, and parameters α and β may describe stress-dilatancy relationship for direct sheared granular materials with inherent anisotropy. It is shown that parameters α and β are different for pre-peak and post-peak stage of shearing, contrary to isotropic sand behaviour. It is also shown that Taylor's and Bolton's relationships are correct only for some experiment for which the angle between shear and bedding plane is 0° . The critical frictional angle Φ° is almost equal to the residual angle of granular material.

1. Introduction

Granular materials deposited under gravity possess inherent fabric anisotropy. Stress-dilatancy and other mechanical properties depend on the direction of loading with respect to the deposition direction. Quantifying influence of anisotropy on mechanical properties, especially strength, is of great engineering significance because anisotropy universally exists in nature [1, 2, 3]. During shear, the volumetric strain (dilatancy) can be observed, which is caused by the granular nature of soils. The influence of dilatancy on sand strength was investigated by Taylor [4] and Bolton [5]. Lately, the *Frictional State Theory* for isotropic soils has been worked out [6], and a new stress-dilatancy relationship for direct shear has been developed [7]. The linear stress-dilatancy relationships may be described by three parameters: Φ° - critical frictional state angle, α and β parameters. For granular soils it can be assumed that $\Phi^\circ \approx \Phi'_{cv} \approx \Phi_f$ [6, 7, 8]. The parameters α and β represent changes of fabric during shear. For direct shear, the deformation is located in a narrow shear band and parameters α and β for pre-failure and post-failure regions are equal [7]. The influence of inherent anisotropy on α and β values will be analyzed in this paper. Based on direct shear of mica sand, Fujian sand and glass beads with inherent anisotropy conducted by Tong et al. [1], the stress-dilatancy relationships will be found and parameters α and β for pre- and post-failure region will be calculated. Generally, it will be shown that Taylor's [4] and Bolton's [5] relationships are not correct for anisotropic granular materials.

The influence of anisotropy on stress-strain and strength of granular materials was experimentally [e.g. 9, 10, 11] and theoretically investigated [12].



2. Direct shear tests

The Fujian sand, mica sand and glass beads were directly sheared in a special shear box by Tong et al. [1]. The modified shear allows to fabricate samples by deposition of granular material from an arbitrary angle between 0° to 180° with respect to shear plane. Therefore, sheared samples had initial inherent anisotropy quantified by angle Ψ_b between the loading direction and the bedding plane schematically shown in Figure 1.

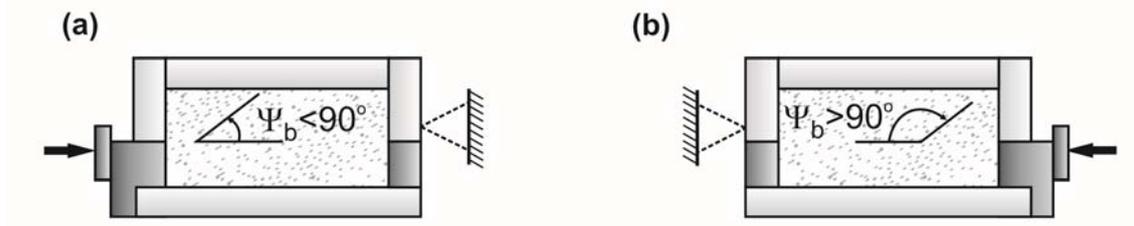


Figure 1. Scheme of direct shear: (a) $\Psi_b < 90^\circ$; (b) $\Psi_b > 90^\circ$.

The Fujian sand has predominantly quartz, relatively rounded and moderately elongated particles. Mica sand contains approximately 18% mica flakes by mass, and other quartz, flaky and angular particles. Mica sand can show strong fabric anisotropy. Glass beads have similar gradation to Fujian sand with rounded and smooth surface particles [1].

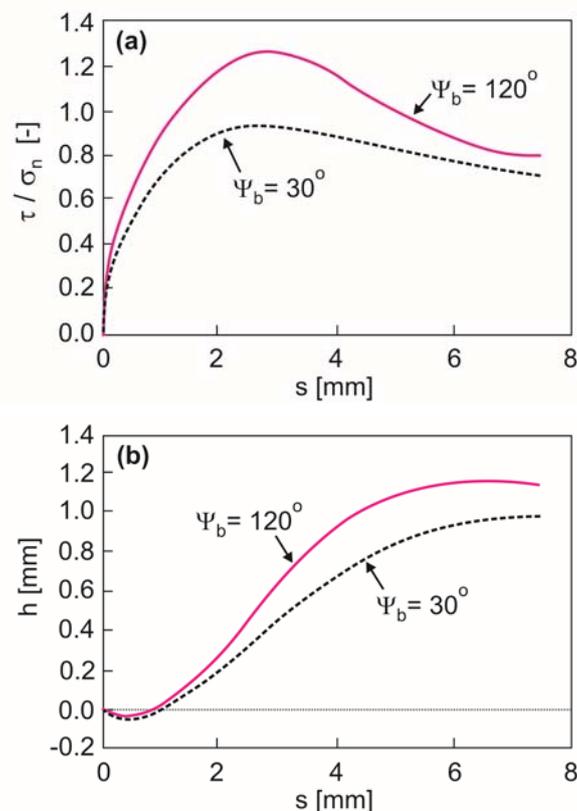


Figure 2. Characteristic behaviour of material during direct shear: (a) $(\tau/\sigma_n) - s$; (b) $h - s$.

These three granular materials were direct sheared at normal stresses range from 150 kPa to 300 kPa with inclination of bedding plane Ψ_b ($=0^\circ/180^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ, 105^\circ, 120^\circ, 135^\circ, 150^\circ$). The change of stress ratio (τ/σ_n) and sample height (h) as a function of shear displacement (s)

was measured and shown in the paper [1] for some experiments. The typical characteristic examples of experimental relationships $(\tau/\sigma_n) - s$ and $h - s$, are shown in Figure 2.

3. Stress-dilatancy for direct shear

Taylor's relationship for direct shear has a form

$$\tan \Phi' = \tan \Phi_r + \delta h / \delta s \quad (1)$$

where Φ_r is residual shear angle. Residual values of shear angles are 24.2° , 32.3° and 35° for glass beads, Fujian and mica sands respectively [1]. Bolton [5] proposed relation between peak value of shear angle (Φ_p') and maximum dilation angle (Ψ_d) in the form

$$\Phi_p' = \Phi_r + 0.8 \Psi_d^{\max} \quad (2)$$

where

$$\Psi_d = \arctan(\delta h / \delta s) \quad (3)$$

Based on *Frictional State Theory* we can write [6, 7]

$$\tan \Phi' = \frac{\tau}{\sigma_n} = \frac{\sqrt{3} \eta \cos \Phi^o \cos \theta}{3 + \eta \left\{ \sin \theta - \sqrt{3} \sin \Phi^o \sin \theta \right\}} \quad (4)$$

where

$$\eta = M_b^o - A_b^o (\alpha + \beta D) \quad (5)$$

$$M_b^o = \frac{3 \sin \Phi^o}{\sqrt{3} \cos \theta - \sin \Phi^o \sin \theta} \quad (6)$$

$$A_b^o = \frac{1}{\cos(\theta - \theta_\varepsilon)} \left\{ 1 - \frac{2}{3} M_b^o \sin\left(\theta + \frac{2}{3} \pi\right) \right\} \quad (7)$$

$$\tan \theta_\varepsilon = \frac{1}{\sqrt{3}} \frac{\delta h / \delta s}{\sqrt{1 + (\delta h / \delta s)^2}} \quad (8)$$

$$D = -\sqrt{3} \frac{\delta h / \delta s}{\sqrt{1 + \frac{4}{3} (\delta h / \delta s)^2}} = -\frac{3 \sin \Psi_d}{\sqrt{3 + \sin^2 \Psi_d}} \quad (9)$$

where Φ^o is the critical frictional state angle, α and β are *Frictional State Theory* parameters. For granular soils, it may be assumed that $\Phi^o = \Phi_r$ and for direct shear as a special plane strain condition $\theta = 15^\circ$ [7].

The relationships: $\tau/\sigma_n - s$ and $h - s$ for some representative experiments were approximated by high degree polynomials and stress-dilatancy $(\tau/\sigma_n - \delta h/\delta s)$ relationships were calculated. An example of stress-dilatancy relationship obtained from calculation is shown in Figure 3.

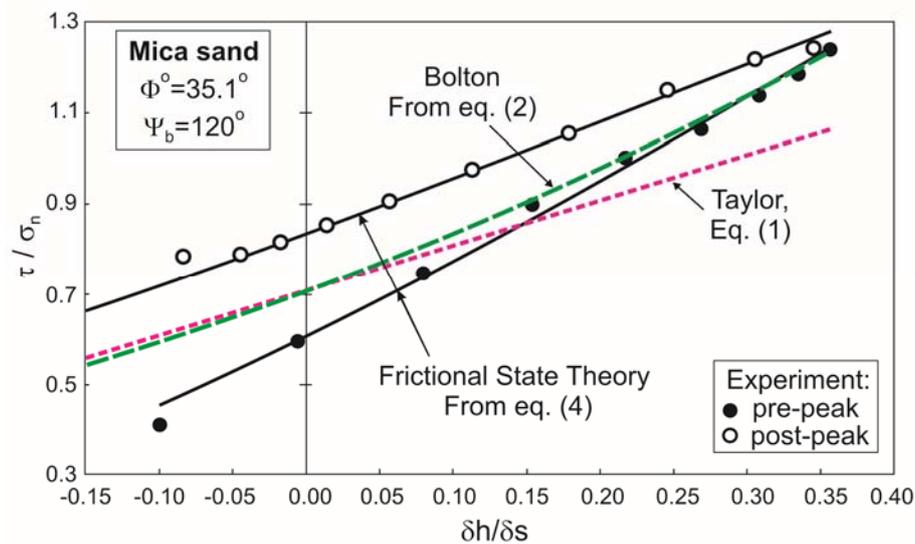


Figure 3. Stress-dilatancy relationship for direct shear.

It can be seen that stress-dilatancy relationships are linear in pre-peak and post-peak phases of shearing. The straight lines described by equation (4) are defined by Φ° , α and β . The $\Phi^\circ = \Phi_r$ and α , β parameters are different for pre-peak and post-peak phases of shearing. The values α and β for pre-peak and post-peak phases of shearing are marked as α_1 , β_1 and α_2 , β_2 , respectively shown in Table 1.

Table 1. Stress-dilatancy parameters.

Angle Ψ_b [°]	Tested materials															
	Mica sand				Fujian sand				Glass beads							
	Φ° [°]	α_1 [-]	β_1 [-]	α_2 [-]	β_2 [-]	Φ° [°]	α_1 [-]	β_1 [-]	α_2 [-]	β_2 [-]	Φ° [°]	α_1 [-]	β_1 [-]	α_2 [-]	β_2 [-]	
0		0.45	1.60	0.00	0.75		0.15	1.15	0.015	0.85		-	-	0.05	0.85	
30		0.25	1.15	0.00	0.75		0.45	1.60	0.075	0.80		-	-	0.065	0.80	
60		0.25	1.20	-0.05	0.75		0.40	1.65	0.025	0.90		-	-	0.015	0.90	
90	35.0	0.25	1.36	-0.025	1.00	32.3	0.45	1.90	-0.025	0.85	24.2	-	-	-0.035	0.85	
120		0.15	1.40	-0.17	0.90		0.35	1.50	0.025	0.85		-	-	0.015	0.85	
150		0.45	1.75	0.00	0.85		0.35	1.40	0.00	0.65		-	-	0.00	0.65	
180		0.45	1.60	0.00	0.75		0.15	1.15	0.015	0.85		-	-	0.05	0.85	

The methodology presented in this paper does not allow to correctly calculate parameters α_1 and β_1 for glass beads at the pre-peak phase of shearing.

It is seen that for granular materials with inherent fabric anisotropy, by contrast to isotropic material, values α and β are different for pre-peak and post-peak phases of shearing. For isotropic granular material in direct shear, as special case of plane strain conditions, are $\alpha=0$ and $\beta=1.4$ [7]. Therefore, the behaviour of granular material with inherent fabric anisotropy is very different from that of isotropic material. The strength of material is defined by a cross point of stress-dilatancy lines approximated to pre-peak and post-peak relationships ($\tau/\sigma_n - \delta h/\delta s$) obtained from *Frictional State Theory*.

Generally, Taylor's relation underestimates the shear strength. Bolton's relationship predicts more or less correctly only the shear strength of the materials under examination [1].

Frictional State Theory can describe correctly not only the shear strength but also the stress-dilatancy relationships at the pre-peak and post-peak phases of shearing. The $\alpha \neq 0$ and $\beta \neq 1$ for post-peak phase of shearing suggest that intrinsic fabric anisotropy is not fully erased during direct shear.

4. Conclusions

During direct shear, an inherent fabric anisotropy is not fully erased, and Taylor's and Bolton's relationships are not correct for these materials generally.

Frictional State Theory developed for isotropic soils can describe stress-dilatancy relationship in the pre-peak and post-peak phases of direct shear.

The influence of inherent fabric anisotropy on the strength of granular materials is observed not only for sands with angular grains but also for glass beads with rounded smooth surface grains.

References

- [1] Z. Tong, P. Fu, S. Zhou, and Y. F. Dafalias, "Experimental investigation of shear strength of sands with inherent fabric anisotropy", *Acta Geotechnica*, Vol. 9, No. 2, pp.257-275, 2014.
- [2] A. A. Al-Karni, and M. A. Al-Sharmani, "Study of the effect of soil anisotropy on slope stability wing method of slices." *Computers and Geotechnics*, Vol. 26, No. 2, pp. 83-103, 2000.
- [3] T. Kimura, O. Kusakabe, and K. Saitoh, "Geotechnical Model Tests of Bearing Capacity Problems in Centrifuge." *Géotechnique*, Vol. 35, No. 1, pp. 33-45, 1985.
- [4] D. W. Taylor, "Fundamentals of Soil Mechanics", John Wiley & Sons, New York, 1948.
- [5] M. D. Bolton, "The strength and dilatancy of sands", *Géotechnique*, Vol. 36, No. 1, pp. 65-78, 1986.
- [6] Z. Szypcio, "Stress-dilatancy for soils. Part I: The Frictional State Theory", *Studia Geotechnica et Mechanica*, Vol. 38, pp.51-57, 2016.
- [7] Z. Szypcio, "Stress-dilatancy for soils. Part IV: Experimental validation for simple shear condition." *Studia Geotechnica et Mechanica*, Vol. 39, pp. 81-88, 2017.
- [8] K. Dołyżk-Szypcio, "Stress-Dilatancy for Crushed Latite Basalt", *Studia Geotechnica et Mechanica*, Vol. 40, No. 1, pp. 6-10, 2018.
- [9] P.J. Guo, "Modified Direct Shear Test for Anisotropic Strength of Sand", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 134, No. 9, ASCE, pp. 1311-1318, 2008.
- [10] W. K. Lam, F. Tatsuoka, "Effects of critical anisotropic fabric and σ_z on strength and deformation characteristics of sand." *Soils and Foundations*, Vol. 28, No. 1, pp. 89-106, 1988.
- [11] Z. X. Yang, X. S. Li, and J. Yang, "Quantifying and modeling fabric anisotropy of granular soils", *Géotechnique*, Vol. 58, No. 4, pp. 237-248, 2008.
- [12] C. S. Chang, Z. Y. Yin, "Micromechanical modeling for inherent anisotropy in granular materials". *Journal of Engineering Mechanics*, Vol. 136, No. 7, pp. 830-839, 2010.