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Geometric nonlinear formulation of plate buckling structure

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Abstract. The analytical or exact mathematical formulation of stresses and displacements for plate buckling structure become impossible to develop if the plate geometry is so complicated. Numerical technique is one another approach to solve this problem and it is chosen in this study for plate structure under in-plane and out-plane load. The formulation of elastic stiffness matrix (k_e) and geometric nonlinear stiffness matrix (k_g) of the plate structure due to buckling is presented and based on virtual displacement principle. The geometric nonlinear stiffness matrix (k_g) is found function of internal stresses. The direct iteration technique is applied to find nodal displacements. Under this technique, the Gauss points stresses are initialized as zeros, then the k_g matrix is updated, and then a new nodal displacement vector is found for the next approximation of internal stresses. Iterative process is done until convergence of displacement is satisfied. The rectangular plate with one fixed edge supported is used to test the proposed nonlinear formulation and procedure. The compressive in-plane load and moment is considered and applied for the tested plate. The plate is discretized with appropriate number of triangular finite element mesh. It is found that, the convergence of displacement is satisfied by using direct iteration technique. The load - deflection curve shows nonlinear relationship and approach to critical load. This finding shows that the direct iteration method can be accepted for the analyzing of plate buckling by considering geometric nonlinear assumption.

1. Introduction

Plate is a two-dimensional structure which its thickness is very small compared to other dimensions. It can present either flat or curve surface. Plate can serve either as complete structure like slab or as structural component. One typical of structural components in engineering structures can be classified as plates is that the web of I beam [1].

While a plate is subjected to in plane load, initial deformation occurs in-plane directions which called in plane displacement. But, increasing the in-plane load leads to changing the behaviour of plate from flat state to curved state (transverse deformation). This load must be considered as load buckle which make the plate unstable [2].

The first mathematical approach of plate problem was done by Euler [1]. He found solution for vibration analysis of plate problems. Another impetus to plate vibration research was done by the German physicist Chladni who discover various modes of free vibrations. Sophie Germain developed a plate differential equation that lacked the warping term. Cauchy and Poisson were first to formulate the problem of plate bending on the basis of general equations of theory of elasticity [3]. Also Navier



consider the plate thickness in the general plate equation as a function of rigidity, D . He introduces governing differential equation of plates subjected to transverse load. He also introduced an exact solution for plate problem by using Fourier trigonometric series.

The first complete and extended theory of thin plate bending was developed by Kirchhoff [1]. His very important contribution to plate theory was the introduction of supplementary boundary forces. He also contributes to the theory of plate that consider large deflection. On the other hand, Reissner and Mindlin introduce theory of plate bending for moderately thick plate. The plastic analysis of plate, a well known as yield line theory application was introduced by Johansen, but the material is assumed perfectly plastic.

The numerical methods normally used to analyse plate structure is finite difference method and finite element method. Finite difference method is a straightforward to solve governing differential equation proposed by Navier. Its limitation is just for rectangular shape of plate and limited to linear elastic analysis. The finite element method is an advanced numerical technique to analyse plate problems. In this technique, the plate is discretized into quadrilateral or triangular elements and can be used to analyse the elastic plates, plastic plates or geometric nonlinear plates.

Generally, there are not many studies carried out about the behaviour of plate under compression load to satisfy a good level of safety because any changing of the plate or web (I steel beam) configurations lead to the failure. Most of previous studies were presented rely on the plate subjected to transverse load [4,5]. Furthermore, if the compression load is present on plate, most of studies focus on in-plane displacement due to compression load and ignoring the buckling or geometric nonlinear effects [6,7].

Newton-Raphson method is the technique normally used in finite element method to perform structural analysis for plastic or nonlinear materials. In this technique, the member forces or stresses are initializing as zeros or any initial values, then a new member forces and stresses are updated using iteration process. The convergence of member forces and stresses are successfully satisfied for one-dimensional, beam and two-dimensional elasto-plastic structures [8].

In the present work, a formulation for nonlinear geometric analysis of plate structure is developed and presented. The formulation is derived based on virtual displacement principle. Based on sample of plate structure, the analysis is conducted by applying direct iteration technique. The plate structure is modelled by finite element mesh, stresses at Gauss points are initialize to zeros, the direct iteration technique is applied until convergence of stresses is satisfied. The convergence, by using direct iteration technique is studied for a sample plate problem.

2. Geometric nonlinear formulation

2.1 Bending strain formulation

Figure 1 shows a triangular plate element with displacements u_1, v_1, w_1 at node 1, displacements u_2, v_2, w_2 at node 2 and displacements u_3, v_3, w_3 at node 3.

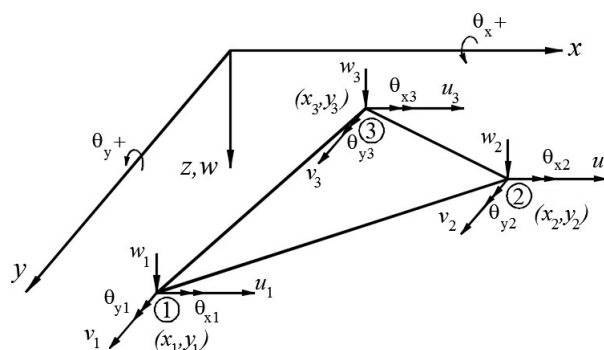


Figure. 1. Triangular plate element with deformation variables.

The deflection function, $w(x,y)$, is assumed a polynomial of the form

$$w(x,y) = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8(x^2y + xy^2) + a_9y^3 = \mathbf{X}\mathbf{a} \quad (1)$$

where

$$\mathbf{X} = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & x^3 & x^2y + xy^2 & y^3 \end{bmatrix}$$

$$\mathbf{a} = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \quad a_9]^T$$

The slopes, which are dependent variables to $w(x,y)$ are given as follow

$$\theta_x = \frac{\partial w}{\partial y} = 0a_1 + 0a_2x + a_3 + 0a_4x^2 + a_5x + 2a_6y + 0a_7x^3 + a_8(x^2 + 2xy) + 3a_9y^2$$

$$\theta_y = -\frac{\partial w}{\partial x} = -0a_1 - a_2 - 0a_3y - 2a_4x - a_5y - 0a_6y^2 - 3a_7x^2 - a_8(2xy + y^2) - 0a_9y^3$$

Apply boundary conditions, the constants a_1, a_2, \dots, a_9 are found and given as follow

$$\mathbf{a} = \overline{\mathbf{X}}^{-1} \mathbf{q} \quad (2)$$

where

$$\overline{\mathbf{X}} = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & x_1y_1 & y_1^2 & x_1^3 & x_1^2y_1 + x_1y_1^2 & y_1^3 \\ 0 & 0 & 1 & 0 & x_1 & 2y_1 & 0 & x_1^2 + 2x_1y_1 & 3y_1^2 \\ 0 & -1 & 0 & -2x_1 & -y_1 & 0 & -3x_1^2 & -(2x_1y_1 + y_1^2) & 0 \\ 1 & x_2 & y_2 & x_2^2 & x_2y_2 & y_2^2 & x_2^3 & x_2^2y_2 + x_2y_2^2 & y_2^3 \\ 0 & 0 & 1 & 0 & x_2 & 2y_2 & 0 & x_2^2 + 2x_2y_2 & 3y_2^2 \\ 0 & -1 & 0 & -2x_2 & -y_2 & 0 & -3x_2^2 & -(2x_2y_2 + y_2^2) & 0 \\ 1 & x_3 & y_3 & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & x_3^2y_3 + x_3y_3^2 & y_3^3 \\ 0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & x_3^2 + 2x_3y_3 & 3y_3^2 \\ 0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3^2 & -(2x_3y_3 + y_3^2) & 0 \end{bmatrix} \text{ and}$$

$$\mathbf{q} = [w_1 \quad \theta_{x1} \quad \theta_{y1} \quad w_2 \quad \theta_{x2} \quad \theta_{y2} \quad w_3 \quad \theta_{x3} \quad \theta_{y3}]^T$$

Substituting Equation (2) into Equation (1), gives

$$w(x,y) = \mathbf{X}\overline{\mathbf{X}}^{-1} \mathbf{q} \quad (3)$$

From Kirchhoff's plate theory assumption [1], the bending and shear strains can be related to deflection as follow

$$\boldsymbol{\epsilon}^b = \begin{Bmatrix} \epsilon_x^b \\ \epsilon_y^b \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z \frac{\partial^2 w}{\partial x^2} \\ -z \frac{\partial^2 w}{\partial y^2} \\ -2z \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (4)$$

Substituting Equation (3) into Equation (4), and re-arrange, it is found that

$$\boldsymbol{\varepsilon}^b = \mathbf{B}\mathbf{q} \quad (5)$$

where

$$\mathbf{B} = -z \mathbf{L} \bar{\mathbf{X}}^{-1}$$

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2x & 6y \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4(x+y) & 0 \end{bmatrix}$$

Constitutive equation of classical plate material is given by [1]

$$\boldsymbol{\sigma}^b = \begin{Bmatrix} \sigma_x^b \\ \sigma_y^b \\ \tau_{xy} \end{Bmatrix} = \mathbf{D} \boldsymbol{\varepsilon}^b \quad (6)$$

where

$$\mathbf{D} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

and E is modulus of elasticity, and ν is a Poisson ratio of the plate.

2.2 Axial strain formulation

The axial strain formulation is derived based on second order effect. The in-plane displacements, $u(x,y)$ and $v(x,y)$, are assumed linear functions and given follow:

$$u(x,y) = N_1 u_1 + N_2 u_2 + N_3 u_3 = \mathbf{N} \mathbf{q}_u \quad (7)$$

$$v(x,y) = N_1 v_1 + N_2 v_2 + N_3 v_3 = \mathbf{N} \mathbf{q}_v \quad (8)$$

where

$$\mathbf{N} = [N_1 \quad N_2 \quad N_3]$$

$$\mathbf{q}_u = [u_1 \quad u_2 \quad u_3]^T$$

$$\mathbf{q}_v = [v_1 \quad v_2 \quad v_3]^T$$

$$N_1 = \frac{1}{2A}(\alpha_1 + \beta_1 x + \gamma_1 y); \quad N_2 = \frac{1}{2A}(\alpha_2 + \beta_2 x + \gamma_2 y); \quad N_3 = \frac{1}{2A}(\alpha_3 + \beta_3 x + \gamma_3 y)$$

$$\alpha_1 = (x_2 y_3 - x_3 y_2) \quad \alpha_2 = (x_3 y_1 - x_1 y_3) \quad \alpha_3 = (x_1 y_2 - x_2 y_1)$$

$$\beta_1 = (y_2 - y_3) \quad \beta_2 = (y_3 - y_1) \quad \beta_3 = (y_1 - y_2)$$

$$\gamma_1 = (x_3 - x_2) \quad \gamma_2 = (x_1 - x_3) \quad \gamma_3 = (x_2 - x_1)$$

A = Area of triangular element

The axial strains, ε_x^a and ε_y^a , are formulated based on second order effect, as follow

$$\varepsilon_x^a = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad (9)$$

$$\varepsilon_y^a = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \quad (10)$$

The axial stresses, σ_x^a and σ_y^a , are derived by substituting Equation (9) and (10) into equations $\sigma_x^a = E\varepsilon_x^a$ and $\sigma_y^a = E\varepsilon_y^a$, and then considering Equation (3), (7), (8) for w , u and v , respectively. The detail derivation is given as follow:

$$\begin{aligned} \sigma_x^a &= E\varepsilon_x^a = E \left(\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) = E \left\{ \frac{\partial}{\partial x} (\mathbf{N}\mathbf{q}_u) + \frac{1}{2} \frac{\partial}{\partial x} (\mathbf{X}\bar{\mathbf{X}}^{-1}\mathbf{q}) \frac{\partial}{\partial x} (\mathbf{X}\bar{\mathbf{X}}^{-1}\mathbf{q}) \right\} \\ &= E \left\{ \frac{\partial \mathbf{N}}{\partial x} \mathbf{q}_u + \frac{1}{2} \frac{\partial \mathbf{X}}{\partial x} \bar{\mathbf{X}}^{-1} \mathbf{q} \frac{\partial \mathbf{X}}{\partial x} \bar{\mathbf{X}}^{-1} \mathbf{q} \right\} \end{aligned} \quad (11)$$

$$\begin{aligned} \sigma_y^a &= E\varepsilon_y^a = E \left(\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right) = E \left\{ \frac{\partial}{\partial y} (\mathbf{N}\mathbf{q}_v) + \frac{1}{2} \frac{\partial}{\partial y} (\mathbf{X}\bar{\mathbf{X}}^{-1}\mathbf{q}) \frac{\partial}{\partial y} (\mathbf{X}\bar{\mathbf{X}}^{-1}\mathbf{q}) \right\} \\ &= E \left\{ \frac{\partial \mathbf{N}}{\partial y} \mathbf{q}_v + \frac{1}{2} \frac{\partial \mathbf{X}}{\partial y} \bar{\mathbf{X}}^{-1} \mathbf{q} \frac{\partial \mathbf{X}}{\partial y} \bar{\mathbf{X}}^{-1} \mathbf{q} \right\} \end{aligned} \quad (12)$$

2.3 Formulation of Stiffness Matrix

Figure. 2 shows a triangular plate element under real forces. Each node has five internal forces, i.e. out-plane load P , bending moments M_x and M_y , and in-plane loads N_x and N_y . The load P and moments M_x and M_y are the forces normally considered in the normal plate bending without buckling.

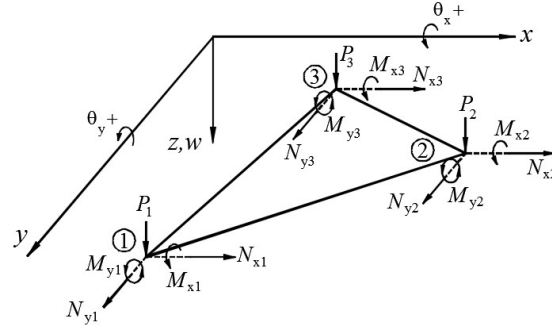


Figure 2. System of real forces.

The stiffness matrix is found by using principle of virtual work for deformable bodies. Under this principle, the deformable structure is in equilibrium state under a system of forces if the virtual external work done by the real external forces and moments acting through the virtual external displacements and rotations is equal to the virtual strain energy stored in the structure. Thus,

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \underbrace{\int_V (\delta \boldsymbol{\varepsilon}^b)^T \boldsymbol{\sigma}^b dV}_{\text{bending}} + \underbrace{\int_V \delta \varepsilon_x^a \sigma_x^a dV + \int_V \delta \varepsilon_y^a \sigma_y^a dV}_{\text{axial stability}} \quad (13)$$

where

$$\begin{aligned} \mathbf{F} &= [P_1 \quad M_{x1} \quad M_{y1} \quad P_2 \quad M_{x2} \quad M_{y2} \quad P_3 \quad M_{x3} \quad M_{y3}]^T \\ \mathbf{N}_x &= \begin{Bmatrix} N_{x1} \\ N_{x2} \\ N_{x3} \end{Bmatrix}; \quad \mathbf{q}_u = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}; \quad \mathbf{N}_y = \begin{Bmatrix} N_{y1} \\ N_{y2} \\ N_{y3} \end{Bmatrix}; \quad \mathbf{q}_v = \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \end{aligned}$$

and δ denote virtual term.

Substituting Equation (5), (6), (9) and (10) into Equation (13), gives

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \int_V [\mathbf{B} \delta \mathbf{q}]^T \mathbf{D} \boldsymbol{\varepsilon}^b dV + \int_V \delta \left(\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) \sigma_x^a dV + \int_V \delta \left(\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right) \sigma_y^a dV \quad (14)$$

Substituting Equation (5) into Equation (14) and re-arrange, we found that

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \delta \mathbf{q}^T \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV \mathbf{q} + \int_V \left(\frac{\partial \delta u}{\partial x} \right) \sigma_x^a dV + \int_V \left(\frac{\partial \delta v}{\partial y} \right) \sigma_y^a dV + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial x} \right)^2 \right) \sigma_x^a dV + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial y} \right)^2 \right) \sigma_y^a dV \quad (15)$$

The first, second and third terms at right-hand side of Equation (15) represent linear terms which come from first order bending and axial effects. The last two terms at right-hand side of this equation represent nonlinear term which come from second order effect of axial stresses.

Substituting $\sigma_x^a = E \partial u / \partial x$ and $\sigma_y^a = E \partial v / \partial y$ on linear terms of Equation (15), gives

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \delta \mathbf{q}^T \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV \mathbf{q} + \int_V \left(\frac{\partial \delta u}{\partial x} \right) E \frac{\partial u}{\partial x} dV + \int_V \left(\frac{\partial \delta v}{\partial y} \right) E \frac{\partial v}{\partial y} dV + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial x} \right)^2 \right) \sigma_x^a dV + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial y} \right)^2 \right) \sigma_y^a dV \quad (16)$$

Substituting Equation (7) and (8) into Equation (16) and re-arrange the equation, gives

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \delta \mathbf{q}^T \left(\int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV \right) \mathbf{q} + \delta \mathbf{q}_u^T \left(E \int_V \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} dV \right) \mathbf{q}_u + \delta \mathbf{q}_v^T \left(E \int_V \frac{\partial \mathbf{N}^T}{\partial y} \frac{\partial \mathbf{N}}{\partial y} dV \right) \mathbf{q}_v + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial x} \right)^2 \right) \sigma_x^a dV + \int_V \left(\frac{1}{2} \delta \left(\frac{\partial w}{\partial y} \right)^2 \right) \sigma_y^a dV \quad (17)$$

Application of variational calculus to the last two terms in Equation (17), leads

$$\delta \mathbf{q}^T \mathbf{F} + \delta \mathbf{q}_u^T \mathbf{N}_x + \delta \mathbf{q}_v^T \mathbf{N}_y = \delta \mathbf{q}^T \left(\int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV \right) \mathbf{q} + \delta \mathbf{q}_u^T \left(E \int_V \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} dV \right) \mathbf{q}_u + \delta \mathbf{q}_v^T \left(E \int_V \frac{\partial \mathbf{N}^T}{\partial y} \frac{\partial \mathbf{N}}{\partial y} dV \right) \mathbf{q}_v + \int_V \left(\frac{\partial \delta w}{\partial x} \right) \frac{\partial w}{\partial x} \sigma_x^a dV + \int_V \left(\frac{\partial \delta w}{\partial y} \right) \frac{\partial w}{\partial y} \sigma_y^a dV \quad (18)$$

Substituting Equation (3) into Equation (18) and cancelled out the virtual displacement vectors at left and right hand side of the equation, the expanded form yields

$$\Rightarrow \begin{Bmatrix} \mathbf{F} \\ \mathbf{N}_x \\ \mathbf{N}_y \end{Bmatrix} = \underbrace{\begin{bmatrix} \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & E \int_V \frac{\partial \mathbf{N}^T}{\partial x} \frac{\partial \mathbf{N}}{\partial x} dV & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & E \int_V \frac{\partial \mathbf{N}^T}{\partial y} \frac{\partial \mathbf{N}}{\partial y} dV \end{bmatrix}}_{\mathbf{k}_e} \begin{Bmatrix} \mathbf{q} \\ \mathbf{q}_u \\ \mathbf{q}_v \end{Bmatrix} + \underbrace{\begin{bmatrix} \left(\bar{\mathbf{X}}^{-1} \right)^T \left[\int_V \left(\frac{\partial \mathbf{X}}{\partial x} \right)^T \left(\frac{\partial \mathbf{X}}{\partial x} \right) \sigma_x^a dV + \int_V \left(\frac{\partial \mathbf{X}}{\partial y} \right)^T \left(\frac{\partial \mathbf{X}}{\partial y} \right) \sigma_y^a dV \right] \bar{\mathbf{X}}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathbf{k}_g} \begin{Bmatrix} \mathbf{q} \\ \mathbf{q}_u \\ \mathbf{q}_v \end{Bmatrix} \quad (19)$$

Equation (19) is a proposed equilibrium equation for isotropic plate under buckling. The in-plane displacement vectors, \mathbf{q}_u and \mathbf{q}_v , are additional variables to normal plate without buckling. The first matrix at right-hand side of Equation (19) represent the linear stiffness matrix which is under normal plate without buckling. The second matrix at right-hand side of this equation represent the proposed nonlinear stiffness matrix of plate under buckling state.

2.4 Analysis procedure using direct iteration technique

Once formulations has been setup, the analysis procedure of plate structure considering geometric nonlinear analysis is suggested as follow:

1. Discretize the plate structure into a number of triangular elements.
2. Calculate \mathbf{k}_e matrix (Equation (19)) for all elements
3. Initialize the stresses, σ_x^a and σ_y^a , at every Gauss points equal to zeros.
4. Calculate \mathbf{k}_g matrix (Equation (19)) for all elements
5. Assemble all elements \mathbf{k}_e and \mathbf{k}_g matrices to form \mathbf{K}_e and \mathbf{K}_g matrices, respectively. Then find the structure stiffness matrix \mathbf{K} .
6. Solve for the nodal displacement vector, \mathbf{Q} , using $\mathbf{Q} = \mathbf{K}^{-1}\mathbf{R}$, where \mathbf{R} = Nodal external force vector
7. Extract the element displacement vector \mathbf{q} , \mathbf{q}_u and \mathbf{q}_v , from \mathbf{Q} for each and every element in the structure
8. For each element, calculate the new stresses, σ_x^a and σ_y^a , at every Gauss points by using Equation (11) and (12).
9. Check the convergence of σ_x^a and σ_y^a . If not converge, repeat steps 4 through 8 as many times as possible until convergence.

A schematic representation of the above procedure is shown in Figure. 3 for a one degree of freedom structure. In performing the above procedure, the complete load-deflection response of the plate can be traced, and the stability limit point is obtained as the peak point of this load-deflection curve.

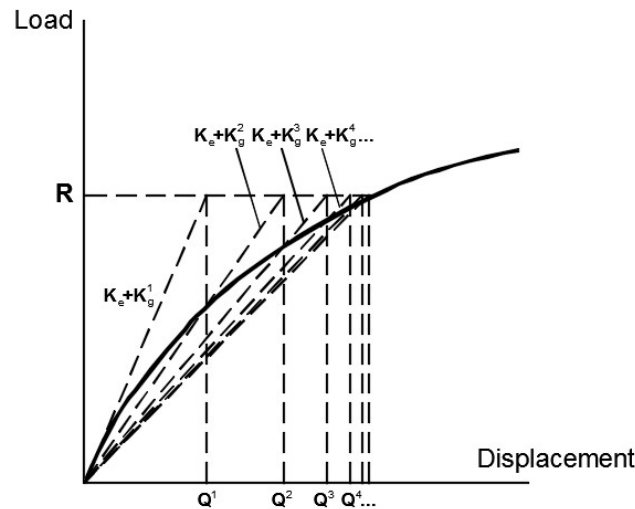


Figure 3. Direct iteration technique for instability plate analysis.

3. Implementation of proposed formulation to plate sample

3.1 Analysis of plate sample

The proposed formulation given in Equation (19) and the convergence of direct iteration technique applied to plate problem are tested to sample plate structure shown in Figure. 4. The plate is meshing by 36 triangular elements with 28 nodes. The bottom edge of the plate is fixed and the left and right edges are restrained in x direction. The load variable, P , is applied to the plate and the constant moment, M_x , is applied at top edge of the plate to initiate the instability. The modulus of elasticity, E , Poisson's ratio, ν , and thickness of the plate are 200 kN/mm², 0.2 and 8 mm, respectively.

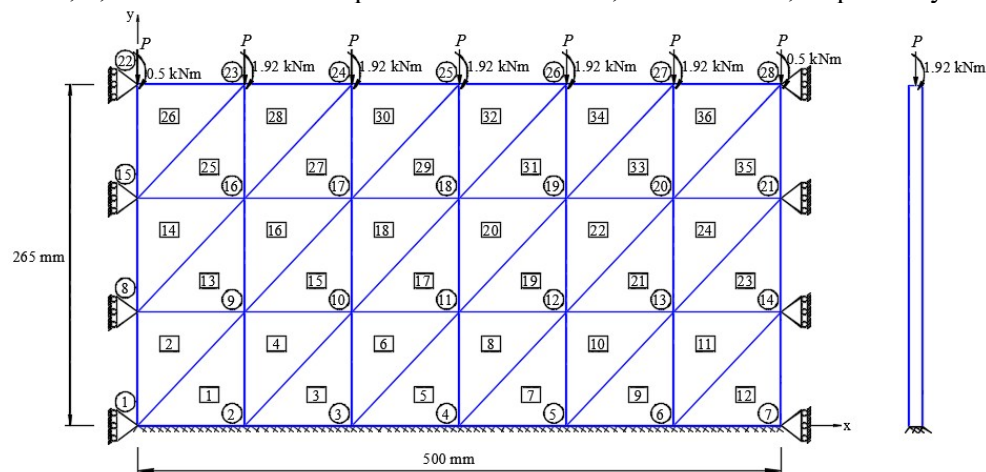


Figure 4. Plate structure with triangular element meshing.

The analysis is done iteratively for eight load levels, i.e. $P = 1.92$ kN, 3.84 kN, 7.68 kN, 15.36 kN, 17 kN, 20 kN, 22 kN and 23 kN using the procedure given in Section 2.4 and with the aid of MATLAB software [9]. The first step is to introduce four Gauss points coordinates in each triangular element. Then, for each element, find \mathbf{k}_e matrix by using the first part formula at right-hand side of Equation (19). We use Gaussian quadrature formula with four Gauss points to evaluate all integrations in \mathbf{k}_e matrix given in Equation (19) [9]. The \mathbf{k}_g matrix in Equation (19) is evaluated by initiating the stresses, σ_x^a and σ_y^a , at every Gauss points equal to zero, then evaluated numerically for all

integrations by using four points Gaussian quadrature formula. Once all \mathbf{k}_e and \mathbf{k}_g were found for all elements, standard assembly process of \mathbf{k}_e and \mathbf{k}_g matrices is done. Then, the nodal displacements are found. By using Equation (11) and (12), the values of σ_x^a and σ_y^a at every Gauss points are evaluated. The new value of \mathbf{k}_g matrix is evaluated and updated again for each element and the process of analysis (previously explained) is repeated until all σ_x^a and σ_y^a converge to a certain value.

3.2 Results and discussion

Table 1 shows the deflections at node 25 for several load levels and a few iteration steps. It is found that, when the load levels less or equal to 7.68 kN, the convergence of deflection is achieved at second iteration. For load levels between 15.36 kN and 20 kN, the convergence of deflections is achieved at fourth iterations. For load levels 22 kN and 23 kN, the convergence is achieved at iteration 6 and 10, respectively. This finding shows that the direct iteration technique with the proposed nonlinear formulation given in Equation (19) can be used to analyse the plate instability structure.

Table 1. Deflections at node 25 for different load levels and iteration steps.

Load, P (kN)	Deflections (mm)				
	Iteration 2	Iteration 4	Iteration 6	Iteration 10	Iteration 12
1.92	0.0065	0.0065	0.0065	0.0065	0.0065
3.84	0.0592	0.0592	0.0592	0.0592	0.0592
7.68	0.0726	0.0726	0.0726	0.0726	0.0726
15.36	0.1313	0.1309	0.1309	0.1309	0.1309
17	0.1584	0.1576	0.1576	0.1576	0.1576
20	0.2536	0.2483	0.2483	0.2483	0.2483
22	0.4215	0.3887	0.3878	0.3878	0.3878
23	0.629	0.529	0.515	0.5128	0.5128

It is also found that, when the load level is greater than 23 kN, such as 24 kN, the convergence of deflection is not satisfactory. This result show that the 23 kN is an estimated critical load for the sample plate chosen and the convergence of deflection is cannot achieve when the load level is greater than the critical load.

Based on the results given in Table 1, the load, P versus deflection is plotted and shown in Figure. 5. From this Figure, the shape of load-deflection graph is nonlinear and the load converge to limit load or critical load, P_{cr} . The slope of the graph decreased and show that the deflection increment or instability of the plate is increased at higher load level. The deformed shape of the plate or geometric effect will play important role to reduce the stiffness of the plate. Based on these results, predictions using equilibrium equation given in Equation (19) and considering direct iteration technique in analysis of plate structure are found satisfactory.

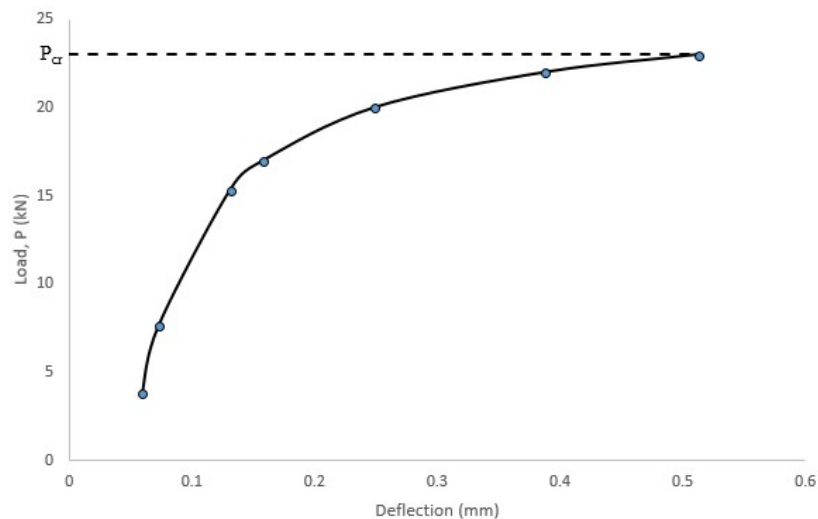


Figure 5. Nonlinear P -deflection graph.

4. Concluding remarks

To conclude this paper, the following remarks are noted.

1. The stiffness matrix for plate structure considering geometric nonlinearity or instability is successfully develop and based on the principle of virtual work for deformable bodies.
2. By using direct iteration technique, the convergence of iteration process at any load steps for plate structure considering geometric nonlinearity is successfully achieved.
3. The nonlinearity of load-deflection graph was found and the predicted loads converge to critical load, P_{cr} .

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