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The plane strain analysis for one-dimensional hexagonal piezoelectric quasicrystals strip in aperiodical plan

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Abstract. A new stress potential function is introduced, the non periodic plane problem in one-dimensional hexagonal piezoelectric quasicrystals is discussed and the physical equation of the stress-strain relationship in the non periodic plane is constructed. The exact solution of the straight crack in the periodic direction of the one-dimensional hexagonal piezoelectric quasicrystal is obtained. As an application, the problem of straight crack perpendicular to the direction of quasi-periodical in one-dimensional hexagonal piezoelectric quasicrystal with long and narrow body is solved. When the width of the long body becomes infinitely large, the Griffith crack solution is obtained. The results show that the stress at the crack tip remains singularity, which is basically consistent with the crack problem that penetrates along the quasi periodic direction. When the phonon field and the phase field get to zero, the above analytical solution degenerates into the fracture problem of isotropic piezoelectric materials, the results are in agreement with the existing results.

1. Introduction

A quasiperiodic crystal, or quasicrystal, is a structure that is ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but it lacks translational symmetry. While crystals, according to the classical crystallographic restriction theorem, can possess only two, three, four, and six-fold rotational symmetries, the Bragg diffraction pattern of quasicrystals shows sharp peaks with other symmetry orders, for instance five-fold.

Shechtman et al [1,2], firstly discovered the fivefold symmetry in the diffraction pattern of Al-Mn alloys and claimed that there is a new structure of solid state in nature. Levine and Steinhard named the new structure order as quasicrystals(QCs) and he was awarded the Nobel Prize in chemistry in 2011. As a new structure of solid matter, QCs have many desirable properties, such as high hardness, low friction coefficients, low surface energy, low heat-transfer, low adhesion, corrosion resistance and high wear resistance [3,4]. Recently, scientists have been considering to replace the traditional materials used to be employed in the aerospace industry with the quasicrystal materials, such as coating surface of spacecraft's wings and fuselage, as well as the thermal barrier coating.

Because of the particular structure of QCs, which is sensitive to electrical, thermal, magnetic and other physical and chemical properties, these properties are essentially different from ordinary crystals and have been investigated intensively [14–16]. In 2012, Altary and Dömeci [17], firstly gave the fundamental equations of piezoelectricity of QCs, which establish the theoretical foundation for the study of fracture mechanics of piezoelectricity of QCs. Li et al. [18] obtained the 3D fundamental solution for 1D hexagonal QCs with piezoelectric effect, and the propagation of cracks may lead to premature failure of these materials produced during their manufacturing process when QCs are subjected to mechanical and electrical loadings in service. Yang [19] reviewed the anti-plane shear problem of two symmetric cracks originating from an elliptical hole in ID hexagonal piezoelectric QCs. Yu and Guo [20,21] proposed the general solutions of plane problem and addressed complex variable method in ID hexagonal piezoelectric QCs. Jiang et al. [22] developed the interaction between a screw dislocation and a wedge-shaped crack in 1D hexagonal QCs with piezoelectric effect. However, to date, there has been relatively little research on the fracture problems of 1D hexagonal piezoelectric QCs in aperiodical plane.

2. Basic equations

We establish Cartesian coordinate system, let the coordinate axis x_3 is the quasi periodic directions of 1D hexagonal piezoelectric QCs with point group 6 mm and the plane perpendicular to the quasi periodic direction is the coordinate plane x_1-x_2, after Ref.[9], the generalized Hooke's law of elasticity problem in 1D hexagonal piezoelectric QC are given by

$$\left\{\begin{array}{l}
\sigma_{11}=C_{11}\varepsilon_{11}+C_{12}\varepsilon_{12}+C_{13}\varepsilon_{13}+R_1\omega_3-e_{31}^1E_3\\
\sigma_{22}=C_{12}\varepsilon_{11}+C_{22}\varepsilon_{22}+C_{13}\varepsilon_{33}+R_1\omega_3-e_{31}^1E_3\\
\sigma_{33}=C_{13}\varepsilon_{11}+C_{13}\varepsilon_{22}+C_{33}\varepsilon_{33}+R_2\omega_3-e_{33}^1E_3\\
\sigma_{23}=\sigma_{32}=2C_{44}\varepsilon_{32}+R_3\omega_2-e_{15}^1E_2\\
\sigma_{13}=\sigma_{31}=2C_{44}\varepsilon_{31}+R_3\omega_1-e_{15}^1E_1\\
\sigma_{12}=\sigma_{21}=2C_{66}\varepsilon_{12}\\
H_{31}=2R_3\varepsilon_{31}+\kappa_2\omega_1-e_{15}^2E_2\\
H_{32}=2R_3\varepsilon_{32}+\kappa_2\omega_2-e_{15}^2E_2\\
H_{33}=R_1(\varepsilon_{11}+\varepsilon_{22})+R_2\varepsilon_{33}+\kappa_1\omega_3-e_{31}^2E_3\\
D_1=2e_{15}^1\varepsilon_{31}+\varepsilon_{15}^2\omega_1+\in_{11}E_1\\
D_2=2e_{15}^1\varepsilon_{32}+e_{15}^2\omega_2+\in_{11}E_2\\
D_3=e_{31}^1(\varepsilon_{11}+\varepsilon_{22})+e_{33}^1\varepsilon_{33}+e_{33}^2\omega_3+\in_{33}E_3
\end{array}\right. \tag{1}$$

where ε_{ij}, ω_j, σ_{ij} and H_{ij} $(i,j=1,2,3)$ are the phonon strains, phason strain, the phonon stress and phason stress, respectively; E_j, D_j stand for the electric field and the clectric displacement, respectively; C_{ij}, k_i, R_i stand for the phonon elastic, phason elastic and phonon-phason coupling modulus, e_{ij}^1 and e_{ij}^2 denote piezoelastic constants of the phonon and phason fields, respectively; $\in_{11}$ and $\in_{33}$ denote the dielectric permittivity.Besides, the geometry equations of 1D hexagonal piezoelectric QCs are given by

$$\left\{\begin{array}{l}
\varepsilon_{ij}=\dfrac{1}{2}(\partial_j u_i+\partial_i u_j),\omega_j=\partial_j\omega\\
E_j=-\partial_j\phi \qquad (i,j=1,2,3)
\end{array}\right. \tag{2}$$

where u_i, ω, ϕ denote the displacement of phonon field and phason field and the electric potential, respectively.Equilibrium equations in the absence of body forces are given by

$$\begin{cases} \partial_1\sigma_{11}+\partial_2\sigma_{12}+\partial_3\sigma_{13}=0 \\ \partial_1\sigma_{21}+\partial_2\sigma_{22}+\partial_3\sigma_{23}=0 \\ \partial_1\sigma_{31}+\partial_2\sigma_{32}+\partial_3\sigma_{33}=0 \\ \partial_1 H_{11}+\partial_2 H_{22}+\partial_3 H_{33}=0 \\ \partial_1 D_1+\partial_2 D_2+\partial_3 D_3=0 \end{cases} \tag{3}$$

3. Plane elasticity in aperiodical plane

When defects such as cracks and hole, etc parallel to the periodic axis of 1D hexagonal piezoelectric QCs , the geometric properties of the material will not change in the quasi-periodic direction. If we take quasi-periodic axis of 1D hexagonal piezoelectric QCs for x_1 axis, then

$$\partial_1 u_i=0, \partial_1 w=0, \partial_1\sigma_{ij}=0, \partial_1 D_{ij}=0, \partial_1 D_j=0 \qquad (i,j=1,2,3) \tag{4}$$

Substituting Eqs.(4)into Eqs.(1)-(3), we get the physical equation in x_2-x_3 plane as follows:

$$\varepsilon_{22}=a_1\sigma_{22}+a_2\sigma_{33}+b_1H_{33}+c_1D_3 \tag{5}$$

$$\varepsilon_{33}=a_3\sigma_{22}+a_4\sigma_{33}+b_2H_{33}+c_2D_3 \tag{6}$$

$$\omega_2=a_{10}\sigma_{23}+d_2H_{32}+e_2D_2 \tag{8}$$

$$E_2=a_{11}\sigma_{23}+d_3H_{32}+e_3D_2 \tag{9}$$

$$\omega_3=a_5\sigma_{22}+a_6\sigma_{33}+b_3H_{33}+c_3D_3 \tag{10}$$

$$E_3=a_7\sigma_{22}+a_8\sigma_{33}+b_4H_{33}+c_4D_3 \tag{11}$$

where

$$a_1=\frac{\kappa_1c_{33}\in_{33}+c_{33}e_{31}^2e_{33}^2-R_2^2\in_{33}-R_2e_{33}^1e_{31}^2-R_2e_{33}^1e_{33}^2+\kappa_1(e_{33}^1)^2}{\Delta_1}$$

$$a_2=\frac{R_1R_2\in_{33}+R_1e_{33}^1e_{31}^2+R_2e_{31}^1e_{33}^2-\kappa_1(e_{33}^1)^2-\kappa_1c_{13}\in_{33}-c_{13}e_{31}^2e_{33}^2}{\Delta_1}$$

$$a_3=\frac{R_1R_2\in_{33}+R_2e_{31}^1e_{31}^2+R_1e_{33}^1e_{33}^2-\kappa_1e_{33}^1e_{31}^1-\kappa_1c_{13}\in_{33}-c_{13}e_{31}^2e_{33}^2}{\Delta_1}$$

$$a_4=\frac{\kappa_1c_{22}\in_{33}+c_{22}e_{31}^2e_{33}^2-R_1^2\in_{33}-R_1e_{31}^1e_{31}^2-R_1e_{31}^1e_{33}^2+\kappa_1(e_{31}^1)^2}{\Delta_1}$$

$$a_5=\frac{c_{13}R_2\in_{33}+c_{13}e_{33}^1e_{31}^2-c_{33}R_1\in_{33}-c_{33}e_{31}^1e_{31}^2-R_1(e_{33}^1)^2+R_2e_{31}^1e_{33}^1}{\Delta_1}$$

$$a_6=\frac{c_{33}R_2\in_{33}+c_{33}e_{33}^1e_{31}^2-c_{13}R_1\in_{33}-c_{13}e_{31}^1e_{31}^2-R_1e_{31}^1e_{33}^1+R_2(e_{31}^1)^2}{\Delta_1}$$

$$a_7=\frac{c_{13}\kappa_1e_{33}^1-c_{13}R_2e_{33}^2+c_{33}R_1e_{33}^2-c_{33}\kappa_1e_{31}^1-R_1R_2e_{33}^1+R_2^2e_{31}^1}{\Delta_1}$$

$$a_8=\frac{c_{22}R_2e_{33}^2-c_{22}\kappa_1e_{33}^1-c_{13}R_1e_{33}^2+c_{13}\kappa_1e_{31}^1+R_1^2e_{33}^1-R_1R_2e_{31}^1e_{33}^1}{\Delta_1}$$

$$a_9=\kappa_2\in_{11}+(e_{15}^2)^2,\ d_1=-(R_3\in_{11}+e_{15}^1e_{15}^2),\ e_1=-R_3e_{15}^2+\kappa_2e_{15}^1$$

$$a_{10}=-2(R_3\in_{11}+e_{15}^1e_{15}^2),\ d_2=2(c_{44}\in_{11}+(e_{15}^1)^2),\ e_2=2(c_{44}e_{15}^2-e_{15}^1R_3)$$
$$a_{11}=2(R_3e_{15}^2-\kappa_2e_{15}^1),\ d_3=-2(c_{44}e_{15}^2-R_3e_{15}^1),\ e_3=2(c_{44}-R_3^2)$$
$$b_2=\frac{R_1c_{13}\in_{33}+R_1e_{31}^1e_{33}^1+c_{13}e_{31}^1e_{33}^2-R_2(e_{31}^1)^2-R_2c_{22}c_{33}-c_{22}e_{33}^1e_{33}^2}{\Delta_1}$$
$$b_3=\frac{c_{22}c_{33}\in_{33}+c_{22}(e_{33}^1)^2-(c_{13})^2\in_{33}-2c_{13}e_{31}^1e_{33}^2+c_{33}(e_{33}^1)^2}{\Delta_1}$$
$$b_4=\frac{R_2c_{22}e_{31}^1+(c_{13})^2e_{33}^2+R_1c_{33}e_{31}^1-c_{22}c_{33}e_{33}^2-R_2c_{33}e_{31}^1-R_1c_{13}e_{31}^1}{\Delta_1}$$
$$c_1=\frac{c_{13}R_2e_{31}^2-c_{13}\kappa_1e_{33}^1-R_1c_{33}e_{31}^2+R_1R_2e_{33}^1+c_{33}\kappa_1e_{31}^1-R_2^2e_{31}^1}{\Delta_1}$$
$$c_2=\frac{\kappa_1c_{22}e_{33}^1+R_1c_{13}e_{31}^2+R_1R_2e_{31}^2-R_2c_{22}e_{31}^2-R_1^2e_{31}^1-\kappa_1c_{33}e_{33}^2}{\Delta_1}$$
$$c_3=\frac{c_{22}c_{33}e_{33}^2-R_2c_{22}e_{33}^1-(c_{13})^2e_{31}^2+R_1c_{13}e_{33}^1+R_2c_{13}e_{31}^1-R_1c_{33}e_{31}^1}{\Delta_1}$$
$$c_4=\frac{c_{22}c_{33}\kappa_1-c_{22}R_2^2-(c_{13})^2\kappa_1+c_{13}R_1R_2-R_1R_2c_{13}+R_1^2c_{33}}{\Delta_1}$$
$$d_1=-\frac{R_3\in_{11}+(e_{15}^1)^2}{\Delta_2},\quad e_2=\frac{\kappa_2e_{15}^1-R_3e_{15}^2}{\Delta_2},\ e_1=-R_3e_{15}^2+\kappa_2e_{15}^1$$
$$\begin{aligned}\Delta_1&=c_{22}c_{33}\kappa_1\in_{33}+c_{22}c_{33}e_{31}^1e_{33}^2-c_{22}R_2^2\in_{33}-c_{22}R_2e_{33}^1e_{31}^1-c_{22}R_2e_{33}^1e_{33}^2+c_{22}\kappa_1(e_{33}^1)^2\\&-C_{13}^2\kappa_1\in_{33}1-C_{13}^2e_{31}^1e_{33}^2+C_{13}R_1R_2\in_{33}+C_{13}R_2(e_{31}^1)^2+C_{13}R_1e_{33}^1e_{33}^2-C_{13}\kappa_1e_{31}^1e_{33}^1\\&+C_{13}R_1R_2\in_{33}-C_{13}R_1e_{31}^1e_{33}^1-C_{33}R_1^2\in_{33}-C_{33}R_1(e_{31}^1)^2-R_1^2(e_{33}^1)^2+R_1R_2e_{31}^1e_{33}^1\\&+C_{13}R_2e_{31}^1e_{33}^2-C_{13}\kappa_1e_{31}^1e_{33}^1-C_{33}R_1e_{31}^1e_{33}^2+C_{33}\kappa_1(e_{31}^1)^2+R_1R_2e_{31}^1e_{33}^1-R_2^2(e_{31}^1)^2\end{aligned}$$
$$\Delta_2=2[c_{44}\kappa_2\in_{11}+c_{44}(e_{15}^2)^2-(R_3)^2\in_{11}-R_3e_{15}^1e_{15}^2-2R_3(e_{15}^1)^2+\kappa_2(e_{15}^1)^2]$$

The corresponding equilibrium equations in plane x_2-x_3 are

$$\partial_2\sigma_{22}+\partial_3\sigma_{23}=0,\partial_2\sigma_{32}+\partial_3\sigma_{33}=0,\partial_2H_{32}+\partial_3H_{33}=0,\partial_2D_2+\partial_3D_3=0 \tag{12}$$

The distortion equation of compatibility are

$$\partial_3^2\varepsilon_{22}+\partial_2^2\varepsilon_{33}-2\partial_2\partial_3\varepsilon_{23}=0,\ \partial_2E_3-\partial_3E_2=0,\ \partial_2\omega_3-\partial_3\omega_2=0 \tag{13}$$

Now three new stress potential functions are introduced as followings

$$\sigma_{22}=\partial_3^2U,\ \sigma_{33}=\partial_2^2U,\ \sigma_{23}=-\partial_2\partial_3U,\ H_{32}=\partial_3V,\ H_{33}=-\partial_2V,\ D_{32}=\partial_3W,\ D_{33}=-\partial_2W \tag{14}$$

Where $U(x_2,x_3)$, $V(x_2,x_3)$, $W(x_2,x_3)$ are three new stress potential functions introduced. Equation (14) satisfies the equation (12). Substituting Eq. (14) into Eq. (5)-(9), then substituting the result into Eq. (13), by simple calculation, we have

$$L_1U-L_2V-L_3W=0,L_4U-L_5V-L_6W=0,L_7U-L_8V-L_9W=0 \tag{15}$$

where differential operator $L_i(i=1,2,3,4,5,6,7,8,9)$ such that

$$L_1=a_1\partial_3^4+(a_2+a_3+2a_9)\partial_2^2\partial_3^2+a_4\partial_2^4,L_2=(b_1+2d_1)\partial_2\partial_3^2+b_2\partial_2^3,$$
$$L_4=(a_5+a_{10})\partial_2\partial_3^2+a_6\partial_2^3,L_3=(c_1+2e_1)\partial_2\partial_3^2+c_2\partial_2^3,$$

$$L_5 = b_3\partial_2^2 + d_2\partial_3^2\,,\, L_6 = c_3\partial_2^2 + e_2\partial_3^2$$
$$L_7 = a_7\partial_2\partial_3^2 + a_8\partial_2^3 - a_{11}\partial_3^3\,,\, L_8 = b_4\partial_2^2 + d_3\partial_3^2\,,\, L_9 = c_4\partial_2^2 + e_3\partial_3^2 \tag{16}$$

By Eq. (15), we eliminate V and W, then we get a partial differential equation as follows:

$$(L_1L_5L_9 - L_1L_6L_8 - L_2L_4L_9 + L_2L_6L_7 + L_3L_4L_8 - L_3L_5L_7)\; U = 0 \tag{17}$$

Which is a partial differential equations of order eight. By literature[15], using four generalized analytic functions $\Phi_k(z_k)\ (k = 1,2,3,4)$, the solution of Eq. (17) can be expressed as

$$U(x_2, x_3) = 2\,\mathrm{Re}\sum_{k=1}^{4}\Phi_k(z_k)\,,\, z_k = x_2 + \mu_k x_3 \tag{18}$$

where *Re* stand for the real part of the corresponding complex function, $\mu_k = \alpha_k + i\beta_k$ $(k = 1,2,3,4, i = \sqrt{-1})$ are the characteristic roots of differential equation(17), α_k, β_k are real constants, which depends on the piezoelectric quasicrystal elasticity only. If the eigenvalue is repeated root. From Eq.(15) we can obtain

$$V(x_2, x_3) = 2\,\mathrm{Re}\sum_{k=1}^{4}\eta_k\Phi'_k(z_k)\,,\; W(x_2, x_3) = 2\,\mathrm{Re}\sum_{k=1}^{4}\zeta_k\Phi'_k(z_k) \tag{19}$$

where

$$\eta_k = \frac{[a_1\mu_k^4 + (a_2 + a_3 + 2a_9)\mu_k^2](c_3 + e_2\mu_k^2) - [(a_5 + a_{10})\mu_k^2 + a_6][(c_1 + 2e_1)\mu_k^2 + c_2]}{\prod(\mu_k)}$$

$$\zeta_k = \frac{[(b_1 - 2d_1)\mu_k^2 + b_2][(a_5 + a_{10})\mu_k^2 + a_6] - (b_3 + d_2\mu_k^2)[a_1\mu_k^4 + (a_2 + a_3 + 2a_9)\mu_k^2]}{\prod(\mu_k)}$$

$$\prod(\mu_k) = [(b_1 - 2d_1)\mu_k^2 + b_2](c_3 + e_2\mu_k^2) - [(c_1 + 2e_2)\mu_k^2 + a_6][b_3 + d_2\mu_k^2] \tag{20}$$

Substituting Eq. (18)and(19)into Eq.(14) yields

$$\sigma_{22} = 2\,\mathrm{Re}\sum_{k=1}^{4}\mu_k^2\varphi'_k(z_k)\,,\; \sigma_{33} = 2\,\mathrm{Re}\sum_{k=1}^{4}\varphi'_k(z_k)\,,\; \sigma_{23} = -2\,\mathrm{Re}\sum_{k=1}^{4}\mu_k\varphi'_k(z_k)$$
$$H_{32} = 2\,\mathrm{Re}\sum_{k=1}^{4}\eta_k\mu_k\varphi'_k(z_k)\,,\; H_{33} = -2\,\mathrm{Re}\sum_{k=1}^{4}\varphi'_k(z_k)\,,\; D_{32} = 2\,\mathrm{Re}\sum_{k=1}^{4}\xi_k\mu_k\varphi'_k(z_k)\,,$$
$$D_{33} = -2\,\mathrm{Re}\sum_{k=1}^{4}\xi_k\varphi'_k(z_k) \tag{21}$$

where $\varphi_k(z_k) = \partial_{z_k}\Phi_k(z_k) = \Phi'_k(z_k)$

By Eq.(14), the complex representation of boundary conditions can be represented by the following formula

$$\partial_2 U = 2\,\mathrm{Re}\sum_{k=1}^{4}\varphi_k(z_k) = -\int_s T_3 ds \tag{22}$$

$$\partial_3 U = 2\,\mathrm{Re}\sum_{k=1}^{4}\mu_k\varphi_k(z_k) = \int_s T_2 ds \tag{23}$$

$$V = 2\,\mathrm{Re}\sum_{k=1}^{4}\eta_k\varphi_k(z_k) = -\int_s T_h ds \tag{24}$$

$$W = 2\,\mathrm{Re}\sum_{k=1}^{4}\xi_k\varphi_k(z_k) = \int_s T_\kappa ds \tag{25}$$

Where T_2 and T_3 are plane stress acting on the boundary, T_h and T_κ stand for generalized stress acting on the phason space and the plane stress acting on electric field separately.

4. The problem of straight cracks in a strip of 1D hexagonal piezoelectric quasicrystal body in the direction perpendicular to direction of quasi-period

We now discuss a penetrating straight crack, along the periodic direction x_1, in a strip of 1D hexagonal piezoelectric QCs with point 6 mm. Then the geometric properties of the materials will not change in the periodic direction x_1, the problem in the plane perpendicular to periodic direction is also plane elasticity problem

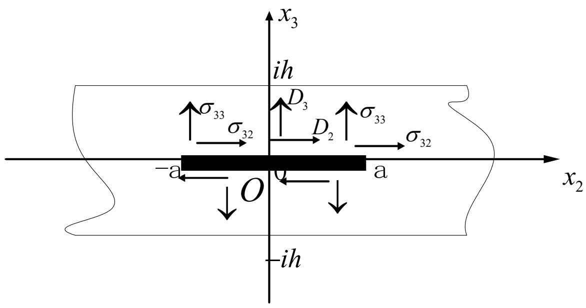


Fig.1 the Griffith crack in 1D hexagonal piezoelectric QCs strip

As shown in Fig.1, we have the follow boundary conditions：

$$\begin{cases}\sigma_{33}=P,\sigma_{32}=\tau_0,D_2=T_1,D_3=T_2 \\ H_{32}=\tau_1,H_{33}=\tau_2\end{cases},-a<x_2<a,x_3=0 \tag{26}$$

$$T_h=T_k=0 \qquad (|x_2|<a,x_3=0) \tag{27}$$

In order to obtain the complex potential function in the region in Ω, suppose that the Laurent expansion of function $\varphi_k(z_k)$ is:

$$\varphi_k(z_k)=C_k z_k+\sum_{j=2}^{\infty}C_k^{(j)}z_k^j+\varphi_k^0(z_k) \tag{28}$$

where

$$\varphi_k^{(0)}(\zeta_k)=a_{k_0}+\sum_{j=1}^{\infty}a_{k_j}z_k^{-j} \tag{29}$$

C_k, $C_k^{(j)}$ and a_{k_j} is the undetermined complex constant.

Substituting function $\varphi_k(z_k)$ into Eq.(21), by condition (27) we have

$$C_k^{(j)}=0,\ k=1,2,3,4;\ j=2,3,\ldots \tag{30}$$

$$2\operatorname{Re}\sum_{k=1}^{4}\mu_k^2C_k=0,\ 2\operatorname{Re}\sum_{k=1}^{4}C_k=p,\ 2\operatorname{Re}\sum_{k=1}^{4}\mu_kC_k=0$$

$$2\operatorname{Re}\sum_{k=1}^{4}\mu_k\eta_kC_k=0,\ 2\operatorname{Re}\sum_{k=1}^{4}\eta_kC_k=0 \tag{31}$$

There are 8 real constants in the above equations $\operatorname{Re}C_k$, $\operatorname{Im}C_k$ (k=1,2,3), but there are 7 independent equations only, therefore a constant can be chosen freely. Take $\operatorname{Re}C_3=0$

Substituting function $\varphi_k(z_k)$ into Eq.(22)-(25)again, by condition(27)we get

$$\sum_{k=1}^{4}[\varphi_k^0(z_k)+\overline{\varphi_k^0(z_k)}=-\sum_{k=1}^{4}[\mu_kz_k+\overline{\mu_kz_k}] \tag{32}$$

$$\sum_{k=1}^{4}[\mu_k\varphi_k^0(z_k)+\overline{\mu_k\varphi_k^0(z_k)}=-\sum_{k=1}^{4}[c_k\mu_k z_k+\overline{c_k\mu_k z_k}] \tag{33}$$

$$\sum_{k=1}^{4}[\eta_k\varphi_k^0(z_k)+\overline{\eta_k\varphi_k^0(z_k)}=-\sum_{k=1}^{4}[c_k\eta_k z_k+\overline{c_k\eta_k z_k}] \tag{34}$$

$$\sum_{k=1}^{4}[\zeta_k\varphi_k^0(z_k)+\overline{\zeta_k\varphi_k^0(z_k)}=-\sum_{k=1}^{4}[c_k\zeta_k z_k+\overline{c_k\zeta_k z_k}] \tag{35}$$

where we take the boundary value on the crack surface for z_k, and $\overline{()}$ stand for complex conjugate.

We introduce a generalized conformal mapping function [16]

$$z_k=\omega_k(\zeta_k)=\frac{\beta_k}{\pi}\log[\frac{\alpha}{2}(\zeta_k+\frac{1}{\zeta_k})+\beta] \tag{36}$$

where

$$\beta_k=\frac{\mu_k-\overline{\mu_k}}{2i},\quad \alpha=\frac{e^{\pi a/\beta_k H}-e^{-\pi a/\beta_k H}}{2},\quad \beta=\frac{e^{\pi a/\beta_k H}+e^{-\pi a/\beta_k H}}{2},\quad k=1,2,3$$

Which maps the interior area Ω_k of z_k-plane into the exterior of a unit circle in the ζ_k-plane, get $\zeta_k=\sigma=e^{i\theta}$, $0\le\theta\le 2\pi$, then there are three points on Ω_k can be transformed into the same point σ on the unit circle, and $\Phi_k^{(0)}(z_k)$ represented a function which is mapped by function $\varphi_k^0(z_k)$, then the condition(32)-(35)can be rewritten as：

$$\sum_{k=1}^{4}[\Phi_k^{(0)}(\sigma)+\overline{\Phi_k^{(0)}(\sigma)}]=\overline{l_1}\sigma+l_1\overline{\sigma} \tag{37}$$

$$\sum_{k=1}^{4}[\mu_k\Phi_k^{(0)}(\sigma)+\overline{\mu_k\Phi_k^{(0)}(\sigma)}]=\overline{l_2}\sigma+l_2\overline{\sigma} \tag{38}$$

$$\sum_{k=1}^{4}[\eta_k\Phi_k^{(0)}(\sigma)+\overline{\eta_k\Phi_k^{(0)}(\sigma)}]=\overline{l_3}\sigma+l_3\overline{\sigma} \tag{39}$$

$$\sum_{k=1}^{4}[\zeta_k\Phi_k^{(0)}(\sigma)+\overline{\zeta_k\Phi_k^{(0)}(\sigma)}]=\overline{l_4}\sigma+l_4\overline{\sigma} \tag{40}$$

where

$$l_1=\sum_{k=1}^{4}\frac{a(C_k+\overline{C_k})+ib(C_k\mu_k+\overline{C_k\mu_k})}{2}$$

$$l_2=\sum_{k=1}^{4}\frac{a(C_k\mu_k+\overline{C_k\mu_k})+ib(C_k\mu_k^2+\overline{C_k\mu_k^2})}{2}$$

$$l_3=\sum_{k=1}^{4}\frac{a(C_k\eta_k+\overline{C_k\eta_k})+ib(C_k\mu_k\eta_k+\overline{C_k\mu_k\eta_k})}{2}$$

$$l_4=\sum_{k=1}^{4}\frac{a(c_k\zeta_k+\overline{c_k\zeta_k})+ib(c_k\mu_k\zeta_k+\overline{c_k\mu_k\zeta_k})}{2}$$

Multiplying equations (37)-(40) by $\frac{d\sigma}{\sigma-\zeta}$, where ζ is point outside of the unit circle, according to Cauchy's integral formula for infinite region, we get

$$\int_\gamma\frac{\Phi_k^{(0)}(\sigma)d\sigma}{\sigma-\zeta}=-2\pi\Phi_k^{(0)}(\zeta_k),\int_\gamma\frac{\Phi_k^{(0)}(\overline{\sigma})d\sigma}{\sigma-\zeta}=0,\int_\gamma\frac{\sigma d\sigma}{\sigma-\zeta}=0,\int_\gamma\frac{d\sigma}{\sigma(\sigma-\zeta)}=-2\pi i\frac{1}{\zeta} \tag{41}$$

Which yields

$$\sum_{k=1}^{4}\Phi_k^0=\frac{l_1}{\zeta},\ \sum_{k=1}^{4}\mu_k\Phi_k^0=\frac{l_2}{\zeta},\quad \sum_{k=1}^{4}\eta_k\Phi_k^0=\frac{l_3}{\zeta},\sum_{k=1}^{4}\zeta_k\Phi_k^0=\frac{l_4}{\zeta} \tag{42}$$

When k takes 1,2,3,4, ζ correspond to ζ_k, The above solutions can be further represented as

$$\Phi_{\mathrm{k}}^0=\frac{l_1}{\zeta_k}\sum_{j=1}^{4}A_{kj}l_j,k=1,2,3,4 \tag{43}$$

where $$\left|A_{\mathrm{kj}}\right|=\frac{1}{\Delta}\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{pmatrix}$$

$$\Delta=\mu_1\eta_2(\zeta_4-\zeta_3)+\mu_1\eta_3(\zeta_2-\zeta_4)+\mu_1\eta_4(\zeta_3-\zeta_2)+\mu_2\eta_1(\zeta_3-\zeta_4)+\mu_2\eta_3(\zeta_4-\zeta_1)+\mu_2\eta_4(\zeta_1-\zeta_3)$$
$$+\mu_3\eta_1(\zeta_4-\zeta_2)+\mu_3\eta_2(\zeta_1-\zeta_4)+\mu_3\eta_4(\zeta_2-\zeta_1)+\mu_4\eta_1(\zeta_2-\zeta_3)+\mu_4\eta_2(\zeta_3-\zeta_1)+\mu_4\eta_3(\zeta_1-\zeta_2)$$

$$a_{11}=\mu_2\eta_3\zeta_4-\mu_2\eta_4\zeta_3-\mu_3\eta_2\zeta_4+\mu_3\eta_4\zeta_2+\mu_4\eta_2\zeta_3-\mu_4\eta_3\zeta_2,\ a_{12}=\eta_2\zeta_4+\eta_3\zeta_2+\eta_4\zeta_3-\eta_3\zeta_4-\eta_4\zeta_2-\eta_2\zeta_3$$
$$a_{13}=\mu_2\zeta_3+\mu_3\zeta_4+\mu_4\zeta_2-\mu_2\zeta_4-\mu_3\zeta_2-\mu_4\zeta_3,\ a_{14}=\mu_2\eta_4+\mu_3\eta_2+\mu_4\eta_3-\mu_2\eta_3-\mu_3\eta_4-\mu_4\eta_2$$
$$a_{21}=\mu_1\eta_4\zeta_3+\mu_3\eta_1\zeta_4+\mu_4\eta_3\zeta_1-\mu_1\eta_3\zeta_4-\mu_3\eta_4\zeta_1-\mu_4\eta_1\zeta_3\ \ a_{22}=\eta_1\zeta_3+\eta_3\zeta_4+\eta_4\zeta_1-\eta_1\zeta_4-\eta_3\zeta_1-\eta_4\zeta_3,$$
$$a_{23}=\mu_1\zeta_4+\mu_3\zeta_1+\mu_4\zeta_1-\mu_1\zeta_3-\mu_3\zeta_4-\mu_4\zeta_1,\ \ a_{24}=\mu_1\eta_3+\mu_3\eta_4+\mu_4\eta_2-\mu_1\eta_4-\mu_3\eta_1-\mu_4\eta_3$$
$$a_{31}=\mu_1\eta_2\zeta_4+\mu_2\eta_4\zeta_1+\mu_4\eta_1\zeta_2-\mu_1\eta_4\zeta_2-\mu_2\eta_1\zeta_4-\mu_4\eta_2\zeta_1,\ a_{32}=\eta_1\zeta_4+\eta_2\zeta_1+\eta_4\zeta_2-\eta_1\zeta_2-\eta_2\zeta_4-\eta_4\zeta_1,$$
$$a_{33}=\mu_1\zeta_2+\mu_2\zeta_4+\mu_4\zeta_1-\mu_1\zeta_4-\mu_2\zeta_1-\mu_4\zeta_2,\ \ a_{34}=\mu_1\eta_4+\mu_2\eta_1+\mu_4\eta_2-\mu_1\eta_2-\mu_2\eta_4-\mu_4\eta_1$$
$$a_{41}=\mu_1\eta_3\zeta_2+\mu_2\eta_1\zeta_3+\mu_3\eta_2\zeta_1-\mu_1\eta_2\zeta_3-\mu_2\eta_3\zeta_1-\mu_3\eta_1\zeta_2,\ \ a_{42}=\eta_1\zeta_2+\eta_2\zeta_3+\eta_3\zeta_1-\eta_1\zeta_3-\eta_2\zeta_1-\eta_3\zeta_2,$$
$$a_{43}=\mu_1\zeta_3+\mu_2\zeta_1+\mu_3\zeta_1-\mu_1\zeta_2-\mu_2\zeta_3-\mu_3\zeta_1,\ \ a_{44}=\mu_1\eta_2+\mu_2\eta_3+\mu_3\eta_1-\mu_1\eta_3-\mu_2\eta_1-\mu_3\eta_2$$

From Eqs. (28) and (36), we have

$$\varphi_k(z_k)=c_kz_k-\frac{\exp(\frac{\pi}{\beta_kH}z_k)-\beta-\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}}{\alpha}\sum_{j=1}^{4}A_{kj}l_j\quad(\ k=1,2,3,4\ ) \tag{44}$$

$$\varphi_k^{'}(z_k)=c_k-\frac{\frac{\pi}{\beta_kH}\exp(\frac{\pi}{\beta_kH}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}+\beta-\exp(\frac{\pi}{\beta_kH}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\quad(\ k=1,2,3,4\ ) \tag{45}$$

Substituting Eq.(45) into the Eqs. (21), all stress components of the elastic field of piezoelectric quasicrystals are obtained as follows:

$$\sigma_{22}=2\operatorname{Re}\sum_{k=1}^{4}\mu_k^2\{c_k-\frac{\frac{\pi}{\beta_kH}\exp(\frac{\pi}{\beta_kH}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}+\beta-\exp(\frac{\pi}{\beta_kH}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$\sigma_{33}=2\operatorname{Re}\sum_{k=1}^{4}\{c_k-\frac{\frac{\pi}{\beta_kH}\exp(\frac{\pi}{\beta_kH}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}+\beta-\exp(\frac{\pi}{\beta_kH}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$\sigma_{23}=-2\operatorname{Re}\sum_{k=1}^{4}\mu_k\{c_k-\frac{\frac{\pi}{\beta_kH}\exp(\frac{\pi}{\beta_kH}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}+\beta-\exp(\frac{\pi}{\beta_kH}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_kH}z_k)+\exp(\frac{2\pi}{\beta_kH}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$H_{32} = -2\operatorname{Re}\sum_{k=1}^{4}\eta_k\mu_k\{c_k - \frac{\frac{\pi}{\beta_k H}\exp(\frac{\pi}{\beta_k H}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}+\beta-\exp(\frac{\pi}{\beta_k H}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$H_{33} = -2\operatorname{Re}\sum_{k=1}^{4}\eta_k\{c_k - \frac{\frac{\pi}{\beta_k H}\exp(\frac{\pi}{\beta_k H}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}+\beta-\exp(\frac{\pi}{\beta_k H}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$D_{32} = 2\operatorname{Re}\sum_{k=1}^{4}\zeta_k\mu_k\{c_k - \frac{\frac{\pi}{\beta_k H}\exp(\frac{\pi}{\beta_k H}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}+\beta-\exp(\frac{\pi}{\beta_k H}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\}$$

$$D_{33} = -2\operatorname{Re}\sum_{k=1}^{4}\zeta_k\{c_k - \frac{\frac{\pi}{\beta_k H}\exp(\frac{\pi}{\beta_k H}z_k)[\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}+\beta-\exp(\frac{\pi}{\beta_k H}z_k)]}{\alpha\sqrt{1-2\beta\exp(\frac{\pi}{\beta_k H}z_k)+\exp(\frac{2\pi}{\beta_k H}z_k)}}\sum_{j=1}^{4}A_{kj}l_j\} \quad (46)$$

It is not difficult to find that there is singularity of $-\frac{1}{2}$ order at the crack tip($z=\pm a$). By document[14], the stress intensity factors of mode III crack of phonon field near the crack tip $z=a$ can be defined as follow

$$K=\begin{bmatrix} K_{\sigma_{32}}\text{-i}K_{\sigma_{33}} \\ K_{H_{32}}\text{-i}K_{H_{33}} \\ K_{D_2}\text{-i}K_{D_3} \end{bmatrix} = \lim_{x\to a^+}\sqrt{2\pi(x-a)}\begin{bmatrix} \sigma_{32}-i\sigma_{33} \\ H_{32}-iH_{33} \\ D_2-iD_3 \end{bmatrix} \quad (47)$$

Substituting Eq. (46) into the Eqs. (47), we can get

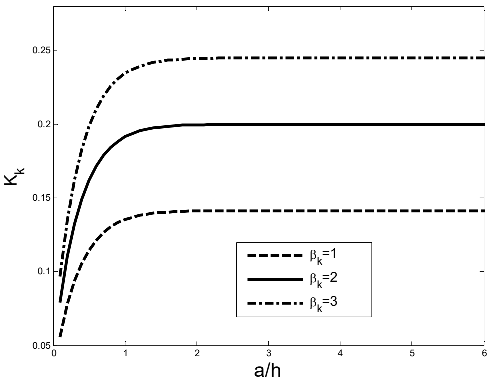
$$\begin{cases} K_{\sigma_{33}} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j, K_{\sigma_{32}} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\mu_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j \\ K_{H_{33}} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\eta_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j, K_{H_{32}} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\eta_k\mu_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j \\ K_{D_3} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\zeta_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j, K_{D_2} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\zeta_k\mu_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j \end{cases} \quad (48)$$

When the phonon field stress, phase field stress and their coefficients get to zero, Eq. (48) can be rewritten as

$$K_{D_3} = \sqrt{H}\operatorname{Re}\sum_{k=1}^{4}\zeta_k\sqrt{\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}}\sum_{k=1}^{4}A_{kj}l_j$$

where $l_3=0$, this is the plane elastic problem of a straight crack in an isotropic piezoelectric narrow body, which is in accordance with the results [23]. The stress intensity factor are given by

$$K_k = \sqrt{H\beta_k}\frac{\alpha+\beta-1}{\sqrt{\alpha(\alpha+\beta)}} \quad (49)$$



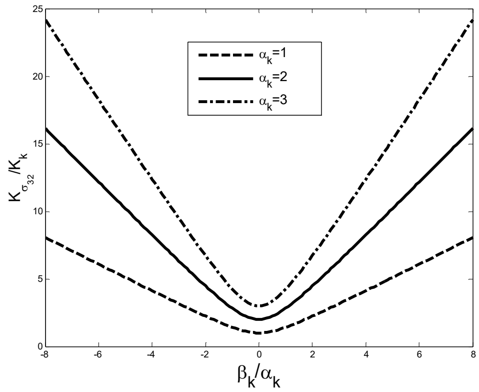


Fig.2 K_{k} versus a/h with different β_k Fig.3 $K_{\sigma_{32}}/K_{\mathrm{k}}$ versus β_k/α_k with different α_k

Fig. 2 reveals the law of K_{k} with crack length. It is shown that the magnitude of stress intensity factor always increases with the increase of crack length and decreases with the increase of strip width. Fig.3 indicate β_k/α_k has a strong influence on $K_{\sigma_{32}}/K_{\mathrm{k}}$.

As $h \to \infty$, Eqs. (45) can be reduced to

$$\varphi_k^{'}(z_k) = c_k - \frac{1}{a}[1 - \frac{z_k}{\sqrt{z_k^2 - a^2}}]\sum_{j=1}^{4} A_{ij} l_j \tag{50}$$

Substituting Eq. (50) into (21), the solution of the aperiodic plane problem for piezoelectric quasicrystals is obtained as follows

$$\begin{aligned}
\sigma_{22} &= 2\,\mathrm{Re}\sum_{k=1}^{4}\mu_k^2[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
\sigma_{33} &= 2\,\mathrm{Re}\sum_{k=1}^{4}[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
\sigma_{23} &= -2\,\mathrm{Re}\sum_{k=1}^{4}\mu_k[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
H_{32} &= -2\,\mathrm{Re}\sum_{k=1}^{4}\eta_k\mu_k[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
H_{33} &= -2\,\mathrm{Re}\sum_{k=1}^{4}\eta_k[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
\mathrm{D}_2 &= 2\,\mathrm{Re}\sum_{k=1}^{4}\zeta_k\eta_k[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j],\\
\mathrm{D}_3 &= 2\,\mathrm{Re}\sum_{k=1}^{4}\zeta_k[c_k - \frac{1}{a}(1 - \frac{z_k}{\sqrt{z_k^2 - a^2}})\sum_{j=1}^{4} A_{kj} l_j]
\end{aligned} \tag{51}$$

The analytical solution of the stress intensity factor at the tip of a Griffith crack in an infinite one-dimensional hexagonal piezoelectric quasicrystal can be obtained by equation (47)

$$\begin{cases}
K_{\sigma_{33}} = p\sqrt{\pi a}, \quad K_{\sigma_{32}} = \tau_0\mu_k\sqrt{\pi a}\\
K_{H_{33}} = \tau_2\eta_k\sqrt{\pi a}, \quad K_{H_{32}} = \tau_1\eta_k\mu_k\sqrt{\pi a}\\
K_{D_3} = T_2\zeta_k\sqrt{\pi a}, \quad K_{D_2} = \zeta_k\mu_k\sqrt{\pi a}
\end{cases} \tag{52}$$

When the piezoelectric constants, permittivity and quasicrystal elastic constants change into zero, the Eqs. (52) can be rewritten as

$$K_{\sigma_{33}} = p\sqrt{\pi a}$$

This is an analytical solution of the stress intensity factor at the tip of a Griffith crack in an infinitely isotropic material, which is consistent with the classical results.

5. Conclusion and Discussion

The theory of defects in the aperiodic plane of one dimensional six dimensional piezoelectric quasicrystals is established, and the governing equations and fundamental solutions of the elastic problems are given. As an application, the Griffith crack in an infinitely long and narrow body is studied by means of generalized conformal transformation in complex functions, the analytical solution of the elastic field is given. In the limit state, the solution of the crack problem is given. The result shows that the stress still has $-\frac{1}{2}$ order singularity at crack tip $z = \pm a$, this is basically the same as cracks passing through periodic plane in one-dimensional hexagonal quasicrystal. The stress field is related to the elastic constants of the phason field, which is different from that of quasi crystal.

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