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Grey theory study on the influence of karst-pile-soil coupling on the ultimate bearing capacity of pile foundation in karst development area

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Abstract. There are many factors influencing the ultimate bearing capacity of pile foundation in karst development area. The current design method usually adopts a large safety factor, which inevitably leads to an increase in engineering cost and a prolonged period. Grey theory is a kind of mathematical theory based on known information. It can describe the change law of unknown information by generating and developing some known information. Based on the above characteristics, combined with numerical simulation results, the effects of aspect ratio and karst scale on the ultimate bearing capacity of bridge piles are studied by using grey theory. The results show that the ultimate bearing capacity of piles increases with the increase of aspect ratio, and decreases with the increase of karst scale. The degree of influence of karst scale is larger than that of aspect ratio. Grey theory can better reflect the variation of ultimate bearing capacity of pile foundation under aspect ratio and karst scale, and the error between calculation result and numerical simulation result is about 9%. The grey theory can be applied to the estimation of the ultimate bearing capacity of bridge piles.

1. Introduction

When soluble rock (limestone, dolomite, gypsum, etc.) is subjected to long-term effects of groundwater and surface water, various karst phenomena are usually generated in the rock formation. On the one hand, karst caves with different morphological and physical properties are easily generated in the rock formation. On the other hand, due to the subsequent secondary weathering or the filling and covering of residuals, the karst caves are usually buried deep underground and become a potential hidden danger of the project. It is obviously unreasonable to construct the bridge structure directly in the karst development areas [1-3]. However, the factors affecting the ultimate bearing capacity of bridge piles in karst development areas are numerous. The size and location of the karst caves, surrounding rock characteristics, pile foundation parameters and construction technology all affect the bearing characteristics of the pile foundation and make the calculation of bearing capacity of bridge pile foundation in karst development area very complicated. The design method of ultimate bearing capacity



of bridge pile foundation in current karst development area usually takes a large safety factor, which masks potential safety hazards and increases engineering costs and cycles. [4-7].

Grey theory is an applied mathematics subject with uncertain systems as the research object. It can describe the change law of unknown information by generating and developing some known information. Because of the above characteristics, the current grey theory also has some applications in bridge pile foundation engineering [8, 9]. Guo [10] et al. used the GM(1,1) model in the grey theory to predict the load-settlement curve obtained from the incomplete static load test of the pile and calculated the ultimate bearing capacity of the single pile. Han [11] et al. studied the quasi-static loading method for bearing capacity measurement of bridge piles by using the GM(1,1) model of residual correction.

Combining numerical simulation and grey theory, this paper studies and analyzes the relationship between the aspect ratio and the karst scale of the pile foundation influencing the bearing capacity of bridge piles in karst development area, in order to provide reference and suggestion for the new design method of ultimate bearing capacity of bridge pile foundation in karst development area.

2. Numerical simulation

The establishment of the constitutive model and the selection of material parameters determine the correctness of the simulation results to some extent. In the study, the pile foundation structure of the bridge is made of concrete material, and the ideal elastic constitutive model is adopted in the analysis. The karst cave model is mainly rock-soil material. In the process of deformation of rock and soil, the relationship between stress and strain is nonlinear. In order to track the loading history, the total displacement, strain and stress are obtained. Therefore, the Mohr-Coulomb yield criterion is adopted.

2.1 Model establishment and parameter values

2.1.1 Model establishment. A variety of computing unit types are available in the Marc nonlinear finite element program. Taking into account the needs of the research, a twenty-node hexahedral element is used for cell meshing. When the entity is dispersed into a finite element unit, the pile foundation and the surrounding soil (rock) body unit are encrypted as much as possible, from near to far, from dense to sparse. This ensures computational accuracy and is easy to converge, saving computation time. The finite element analysis model is shown in Figure 1.

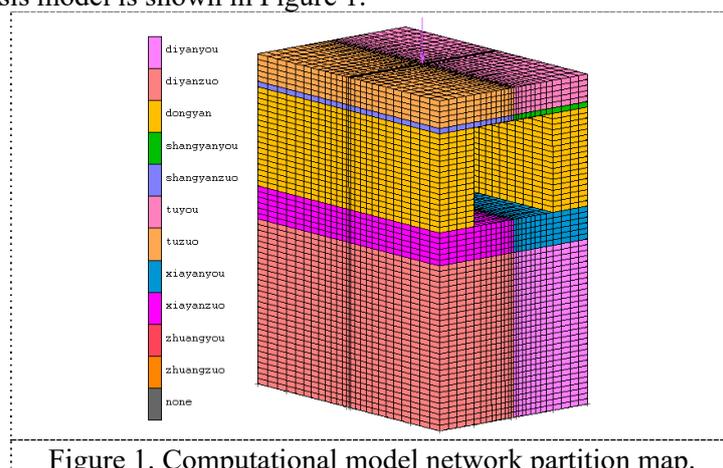


Figure 1. Computational model network partition map.

2.1.2 Selection of calculation parameters. In the numerical simulation analysis, by simulating the influence of the size change and the aspect ratio of the cave on the stress of the bridge pile foundation in the cave area, the bearing characteristics of the bridge pile foundation under the vertical load are studied. It provides an important theoretical basis for the design of bridge pile foundations in the cave area. The specific parameters are shown in Table 1.

Table 1. Finite element analysis model material parameters.

Material	Elastic Modulus (Pa)	Poisson's ratio	Cohesion (Pa)	Internal friction angle (°)	Volumetric weight (kN/m ³)
Pile	3×10^{10}	0.2	—	—	24
Clay	1.25×10^7	0.3	1.6×10^4	11	16
Middle weathered rock formation	2×10^{10}	0.23	1.7×10^6	25	23

2.2 Numerical simulation analysis scheme

The vertical loading of the pile top center is used to analyze the influence of different karst scales and aspect ratio on the bearing capacity of the bridge pile foundation. The calculation conditions are shown in Table 2.

Table 2. Calculation condition.

Karst scales (m ²)	Aspect ratio
6×6、10×10、14×14、18×18	15、20、25、30、35

2.3 Analysis of numerical simulation results

2.3.1 Analysis of the influence of the change of the aspect ratio on the bearing capacity of pile foundation.

The P-S curve of different aspect ratios when the karst scale is 6×6 m², 10×10 m², 14×14 m² and 18×18 m² are shown in Figure 2. The influence of the change of the aspect ratio on the ultimate bearing capacity of the pile foundation and its increasing is shown in Figure 3 and Figure 4.

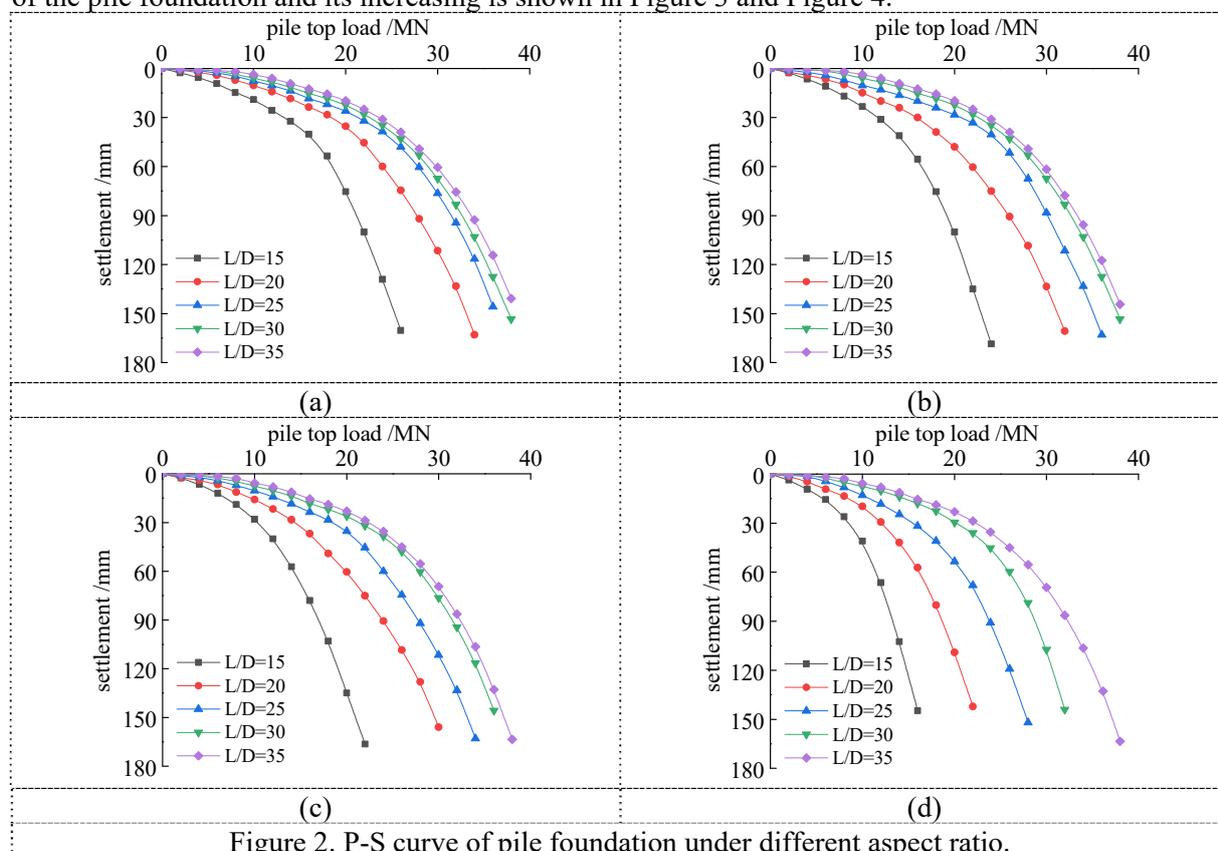


Figure 2. P-S curve of pile foundation under different aspect ratio.

It can be seen from Figure 2 to Figure 4 that the ultimate bearing capacity of the pile foundation increases with the increase of the aspect ratio when the karst scale is fixed. Taking the karst scale 10m

as an example, when the aspect ratio increases from 15 to 25, the ultimate bearing capacity increases from 14.3MN to 23.8MN, and the increase is 9.5MN with an increase of 66.2%, when the aspect ratio increases to 35, the ultimate bearing capacity increases to 26.1 MN, and the increase is 11.8 MN, an increase of 82.3%. It can be seen that when the karst scale is fixed, the larger the aspect ratio, the greater the ultimate bearing capacity of the pile foundation. At the same time, when the aspect ratio increases to a certain extent ($L/D=35$), the ultimate bearing capacity remains basically unchanged.

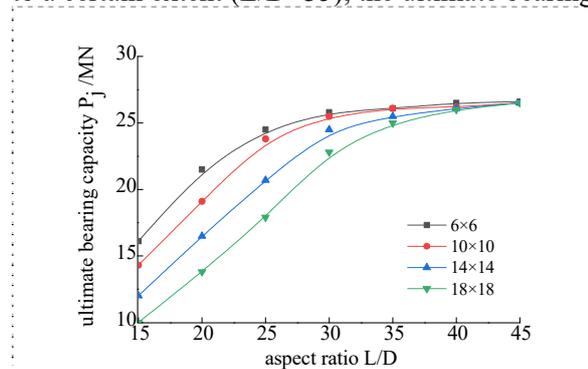


Figure 3. Effect of aspect ratio change on ultimate bearing capacity under different karst scales.

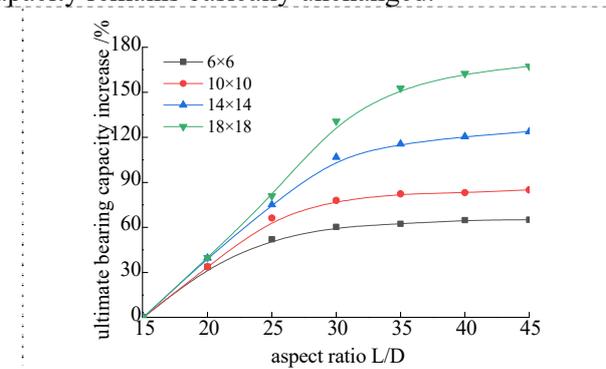
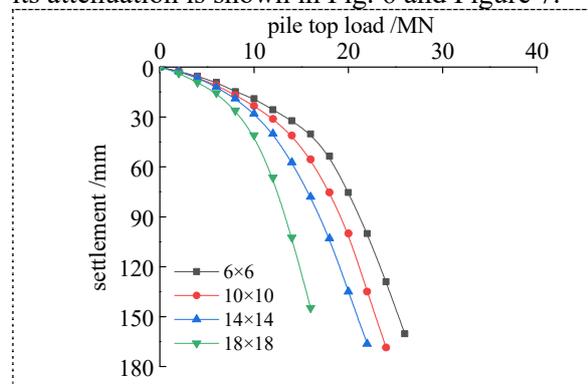
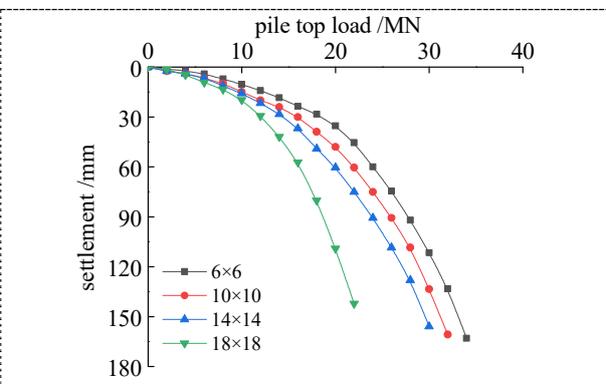


Figure 4. Effect of aspect ratio change on ultimate bearing capacity increase under different karst scales.

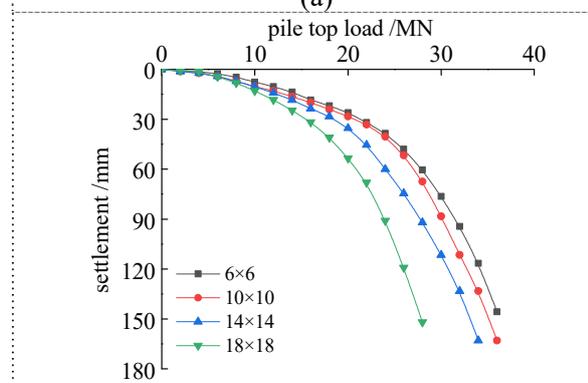
2.3.2 Analysis of Influence of Variation of Karst Scale on Bearing Capacity of Pile Foundation. The P-S curves of different karst scales when the aspect ratio is 15, 20, 25, 30 and 35 are shown in Figure 5. The influence of the variation of the karst scales on the ultimate bearing capacity of the pile foundation and its attenuation is shown in Fig. 6 and Figure 7.



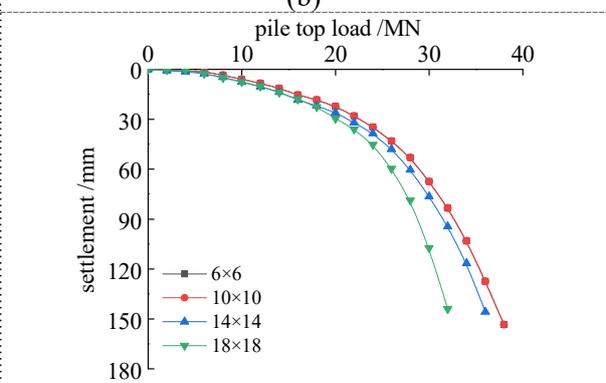
(a)



(b)



(c)



(d)

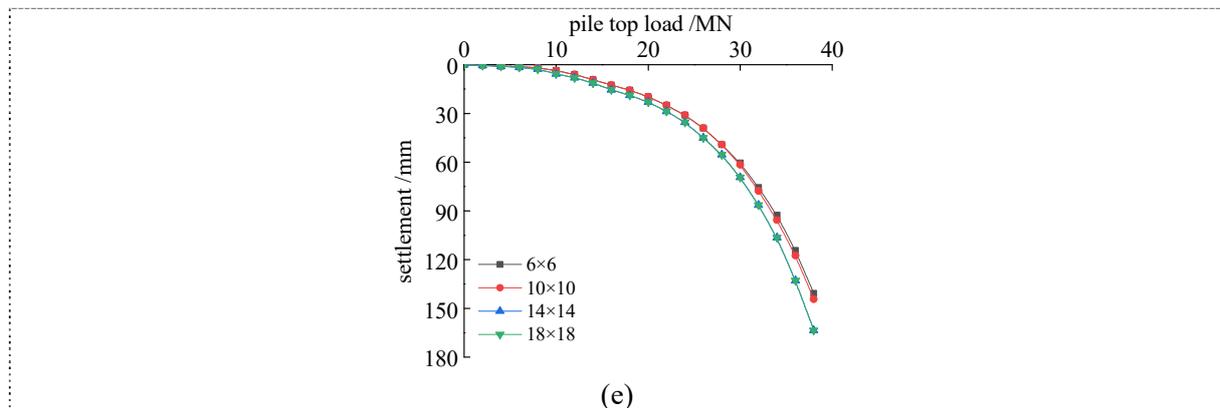


Figure 5. Curve of pile foundation under different Karst scales.

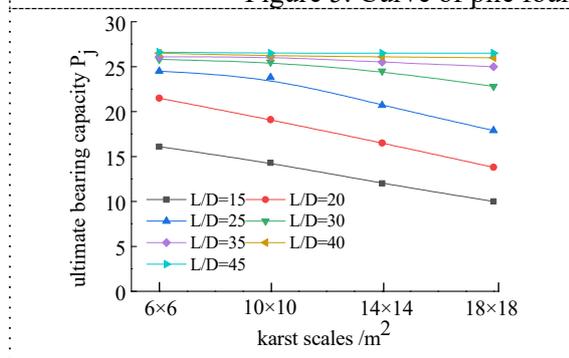


Figure 6. Influence of karst scales on ultimate bearing capacity.

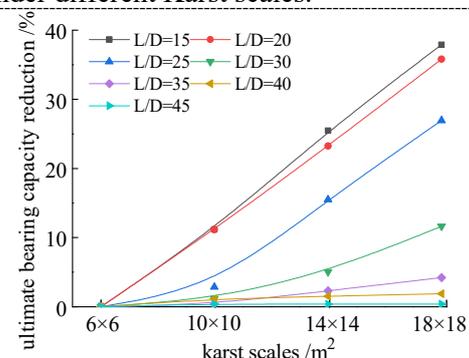


Figure 7. Influence of karst scales on the amplitude reduction of ultimate bearing capacity.

It can be seen from Figure 4 to Figure 7 that the ultimate bearing capacity of the pile decreases with the increase of the karst scales when the aspect ratio is constant. Taking the aspect ratio $L/D=25$ as an example, when the karst scale increases from 6m to 10m, the ultimate bearing capacity decreases from 24.5MN to 23.8MN, and the decrease is 0.7MN, which is 2.9%. When the karst scale is increased to 14m, the ultimate bearing capacity of the pile is reduced to 20.7MN. Compared with the pile foundation with the karst scale of 6m, the ultimate bearing capacity decreases by 3.8MN, with a decrease of 15.5%. When the karst scale increases to 18m, the ultimate bearing capacity of the pile is reduced to 17.9MN. Compared with the pile with a karst scale of 6m, the ultimate bearing capacity decreases by 6.6MN, which is 26.9%. The above analysis shows that when the aspect ratio is constant, the increase of the karst scale will significantly reduce the ultimate bearing capacity of the pile.

3. Calculating ultimate bearing capacity of pile foundation by using grey theory

3.1 Model establishment and calculation of grey system

The grey model is a model based on discrete series and fitted by differential equations. The grey model can quantitatively reflect the interrelationship of various factors within the transaction and the changing situation in the dynamic, so that the behavioral characteristics of things and systems can be described more accurately. The grey model is the basis of grey prediction. It has many types, such as GM (1, 1), GM (1, N), GM (2, 1), GM (n, h) models. The first number in the GM brackets indicates the order, and the second number indicates the number of variables. In this paper, the GM (1, N) model is used to calculate the ultimate bearing capacity of pile foundation.

3.2 GM(1,N) model

The GM(1,N) model is a grey model of first-order N variables. If there are n variables, there are n series

$$x_i^{(0)} = (x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)) \quad i = 1, 2, \dots, n \tag{1}$$

For the cumulative generation of $x_i^{(0)}$, it is usually an accumulation

$$x_i^{(1)}(k) = \sum_{m=1}^k x_i^{(0)}(m) \tag{2-a}$$

$$x_i^{(1)} = (x_i^{(1)}(1), x_i^{(1)}(2), \dots, x_i^{(1)}(n)) \quad i = 1, 2, \dots, n \tag{2-b}$$

Establishing a grey differential equation

$$\frac{dx_1^{(1)}}{dt} + ax_1^{(1)} = b_1x_2^{(1)} + b_2x_3^{(1)} + \dots + b_{n-1}x_n^{(1)} \tag{3}$$

Parameter column

$$\hat{a} = [a, b_1, b_2, \dots, b_{i-1}]^T \tag{4}$$

According to the least squares method, we can find \hat{a}

$$\hat{a} = (B^T B)^{-1} B^T y_n \tag{5}$$

Where

$$B = \begin{bmatrix} -\frac{1}{2}(x_1^{(1)}(1) + x_1^{(1)}(2)) & x_2^{(1)}(2) & \dots & x_n^{(1)}(2) \\ -\frac{1}{2}(x_1^{(1)}(2) + x_1^{(1)}(3)) & x_2^{(1)}(3) & \dots & x_n^{(1)}(3) \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2}(x_1^{(1)}(n-1) + x_1^{(1)}(n)) & x_2^{(1)}(n) & \dots & x_n^{(1)}(n) \end{bmatrix} \tag{6}$$

$$y_n = [x_1^{(0)}(2), x_1^{(0)}(3), \dots, x_1^{(0)}(n)]^T \tag{7}$$

This will get

$$x_1^{(1)}(k+1) = [x_1^{(0)}(1) - \frac{1}{a} \sum_{i=2}^n b_{i-1} x_i^{(0)}(k+1)] e^{-ak} + \frac{1}{a} \sum_{i=2}^n b_{i-1} x_i^{(1)}(k+1) e^{-ak} \tag{8}$$

3.3 Calculation and analysis by using GM(1,3) model

It is known that there are three variables in the analysis model, including the ultimate bearing capacity P_j , the aspect ratio N, and the karst scale S, corresponding to the original variables $x_1^{(0)}, x_2^{(0)}, x_3^{(0)}$ in the formula, there are 20 mappings for 3 variables. The variable curve is shown in Figure 8.

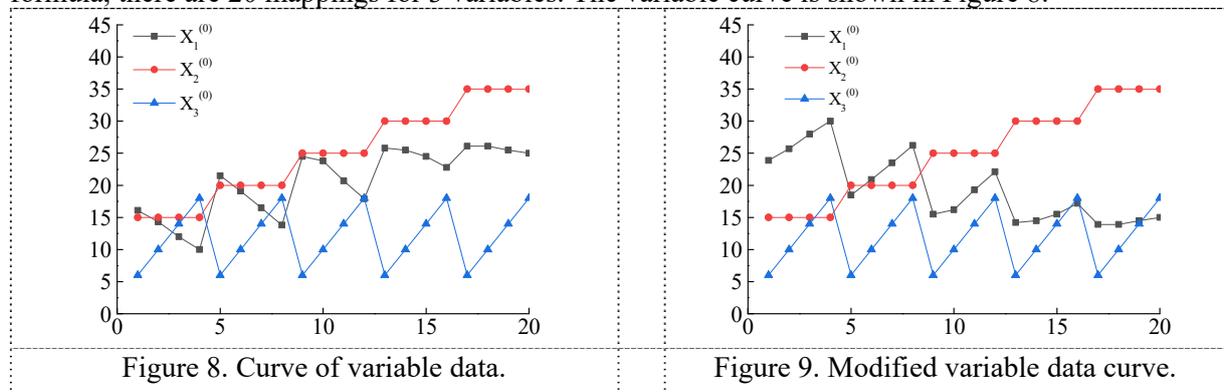


Figure 8. Curve of variable data.

Figure 9. Modified variable data curve.

In order to enhance the correlation of the data and reduce the error, the bearing capacity in the chart is generally rising, and the data of Series 1 are reduced by 40. The data curve is shown in Figure 9.

According to Formula (2-b), we can get $x_i^{(1)} = (x_i^{(1)}(1), x_i^{(1)}(2), \dots, x_i^{(1)}(20))$. The cumulative graph is shown in Figure 10.

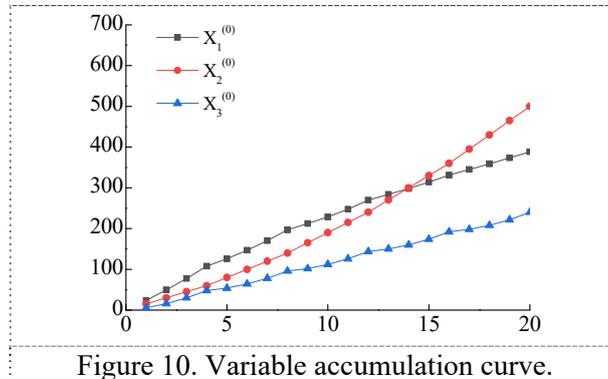


Figure 10. Variable accumulation curve.

According to Formula (6) and Formula (7), we can use the least squares method to find $\hat{a} = (B^T B)^{-1} B^T y_n$.

Where $\hat{a} = [a, b_1, b_2]^T$, we can solve a, b_1, b_2 as follows

$$a = -0.11, \quad b_1 = -0.08, \quad b_2 = 0.17$$

The extended mode of GM(1,N) is GM(1,N, $x^{(0)}$), and the extended model formula is

$$x_1^{(0)}(k) = \sum_{i=2}^N \beta x_i^{(0)}(k) + (1 - \alpha)x_1^{(0)}(k - 1)$$

Where $\alpha = \frac{a}{1 + 0.5a}, \beta = \frac{b}{1 + 0.5a}$,

We can solve $\alpha = -0.12, \beta_1 = -0.17, \beta_2 = 0.17, 1 - \alpha = 1.12$.

The calculation formula is

$$x_1^{(0)}(k) = -0.08x_2^{(0)}(k) + 0.17x_3^{(0)}(k) + 1.12x_1^{(0)}(k - 1)$$

Formula (11) is the formula for determining the bearing capacity derived from the grey theory. Finally, using 40 to subtract the calculated results, the formula for the influence of the aspect ratio and the karst scale on the ultimate bearing capacity of the pile under this condition is:

$$P_{(n)} = 40 + 0.18N_{(n)} - 0.17S_{(n)} - 1.12P_{(n-1)}$$

Bring the data into the calculation formula for bearing capacity test. The data change curve is shown in Figure 11.

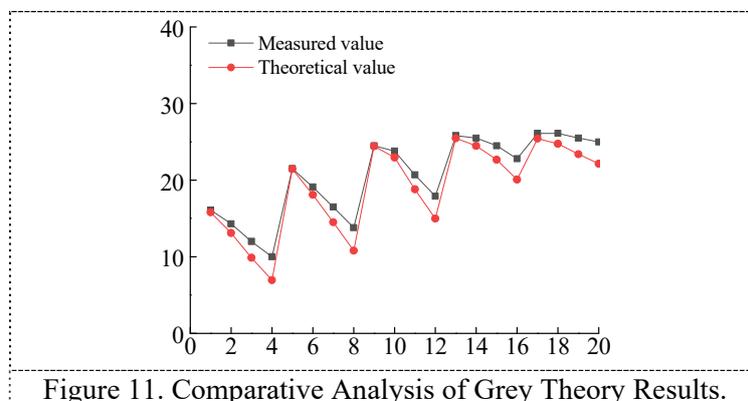


Figure 11. Comparative Analysis of Grey Theory Results.

It can be obtained from the analysis of Table 6 and Figure 11. The predicted value fluctuates greatly with respect to the actual value, but through the above calculation, the average error is 8.98%. According to the grey theory deviation analysis requirements, the above formula can be used as the calculation formula of the bearing capacity under the influence of the aspect ratio and the karst scale. It is also

confirmed once again that the combination of the aspect ratio and karst scale will affect the bearing capacity. Moreover, it can be concluded that $|b_2| > |b_1|$, that is, the influence of the karst scale on the bearing capacity is greater than the influence of the aspect ratio on the bearing capacity.

4. Conclusions

1. When the aspect ratio is constant, the larger the karst scale, the ultimate bearing capacity of the pile foundation will decrease significantly. When the karst scale is constant and the aspect ratio is increased, the ultimate bearing capacity of the pile foundation will increase significantly, but it is relatively small compared with the influence of the karst scale on the ultimate bearing capacity.

2. The grey theory pile bottom cave is based on the relationship between the existing aspect ratio, the karst scale and the ultimate bearing capacity of the pile foundation. The calculation formula of the relationship between the ultimate bearing capacity of the pile foundation and the aspect ratio and the karst scale is calculated. The average error between the calculation result and the numerical simulation result is about 9%, and the error is within the acceptable range. It indicates that the grey theory can better estimate the ultimate bearing capacity of the pile under the influence of the karst scale.

3. The application of grey theory in pile foundation engineering saves manpower and material resources and improves work efficiency. Because of its good practicability and accuracy, it will be more and more widely used in single pile static load test.

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