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The Analytical Solution for Consolidation of Vertical Drain with Vacuum Preloading Based on Equivalent Annular Drain

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The Analytical Solution for Consolidation of Vertical Drain with Vacuum Preloading Based on Equivalent Annular Drain

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Abstract. Based on the equivalent annular band-shaped drain, and considering the vacuum preloading and the variation of radial permeability coefficient of the smeared soil zone, the mathematical model is established. An analytic method is used to derive the general solution for the consolidation of vertical drain based on equivalent annular and arbitrary variation of load with time. Based on the general solutions, the detailed closed-form solutions are further derived for the cases of instantaneously loading and linear loading. Based on the closed solution, the calculation program is compiled and the consolidation curves of the vacuum preloading shaft foundation based on the annular equivalent are drawn. The results show that the equivalent annular consolidation rate is less than that given by the equivalent circumference. With the increase of the area reduction coefficient, the consolidation rate of the equivalent annular approaches to the consolidation rate of the equivalent circumference. With the decrease of the permeability coefficient of the vertical drain, the influence of the permeability coefficient on the consolidation rate is weakened. The dissipation rate of pore water under instantaneous load is greater than that of linear pore water dissipation.

1. Introduction

Vacuum preloading is an effective drainage consolidation method, and has broad prospects for development [1]. Therefore, a large number of scholars have conducted theoretical research on the consolidation of vertical drain under vacuum preloading. Tran et al. have established the equivalent plane strain model of vertical drain pipe on soft foundation, and given the analytical solution of consolidation based on the combined vacuum preloading of embankment [2]. Zhang Yiping and others established the space axial symmetry deformation model of vacuum preloading. By using displacement method and Hansbo's sand well consolidation theory, the analytical solution of vacuum preloading consolidation of soft soil foundation was deduced [3]. Considering the influence of the original stress state of the soil, using the volumetric strain formula, the consolidation theory of the shaft foundation under vacuum preloading is analyzed by Fang Dechun and others[4], and the analytic solution of consolidation is obtained. In recent years, the vacuum preloading method has been widely applied in soft soil foundation treatment [5-10]. As a substitute for sand drains, plastic drainage boards have been greatly developed in accelerating consolidation of soft soil foundations [11-13]. Huang Chaoxuan and others assumed that the vertical drainage body was an elliptic cylinder, and the analytical solution of vertical drain consolidation was obtained [14] based on the elliptic cylinder coordinate system theory. Considering the clogging behavior of drainage plate, Chen Junsheng and others deduced the correction solution of sand well foundation consolidation under vacuum preloading,

and verified the correctness of consolidation theory by comparing with model test [15]. The above methods are all equivalent according to the circumference of drainage plate [16-18]. Lu Menmeng et al. put forward a new ring equivalent method. The circumference and cross-sectional area of the equivalent circular drainage are the same as those of the drainage plate, and the analytical solution of the consolidation of the drainage plate foundation under the ring equivalent is obtained [19]. Analytical solution of foundation consolidation based on annular equivalent preloading drainage plate has been studied by scholars, but the annular equivalent drainage plate foundation consolidation under vacuum preloading has not been studied.

Based on annular equivalent, the consolidation of plastic drainage slab foundation under vacuum preloading is studied analytically. And the analytical solution of the consolidation of plastic drainage slab foundation under vacuum preloading is deduced considering three modes of the variation of horizontal permeability coefficient of soil in the smeared area. Finally, the effect of annular equivalent is compared and analyzed.

2. Mathematical model

2.1 annular equivalent
The annular equivalence of plastic drainage boards is shown in Figure 1 [19].

（a）plastic drainage boards   （b）Equivalent annular drainage well
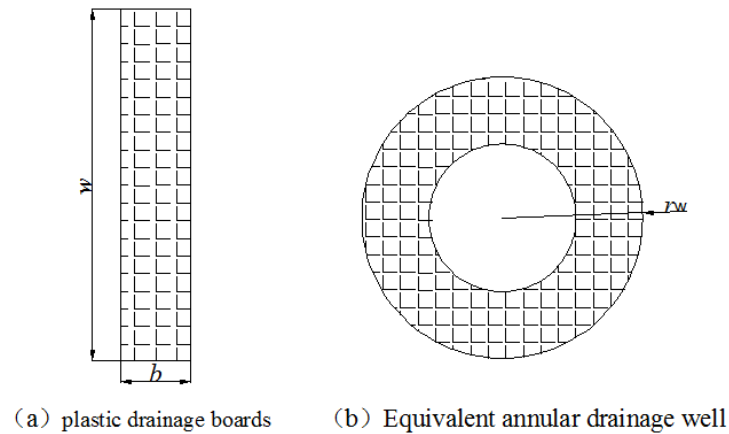
Fig.1   Band-shaped drain and the equivalent annular drain

The drainage board is transformed into circular drainage body according to the principle of equal circumference, i.e.

$$r_{\mathrm{w}} = \frac{w+b}{\pi} \tag{1}$$

where r_{w} is the external radius of circular drainage; w and b are the width and thickness of drainage boards respectively.

At the same time, it is required that the cross-sectional area of the annular drain body is equal to the cross section area of the drainage plate. The area reduction coefficient ζ is introduced here, and the area of the annular drainage body is equal to the product of the area of the outer circle and the area reduction coefficient, i.e.

$$wb = \zeta \pi r_{\mathrm{w}}^{2} \tag{2}$$

The formula of area reduction coefficient can be obtained by substituting Eq. (1) into Eq. (2).

$$\zeta = \frac{\pi \eta}{\left(1+\eta\right)^{2}} \tag{3}$$

where η is the ratio of thickness to width of drain plate, $\eta = b/w$.

2.2 Computing model
Fig. 2 is a diagram for calculating the vertical drain of radial consolidation. The top surface of the shaft foundation is completely permeable, and the bottom surface is completely impervious. Among them: the upper part is sand cushion; H is the thickness of soft soil foundation. r and z are radial and vertical coordinates respectively; r_w is the radius of shaft; r_s is the radius of smearing area; r_e is the radius of the shaft area; k_w is the permeability coefficient of shaft; $k_r(r)$ is the permeability coefficient of the soil in the smearing area; k_h is the horizontal permeability coefficient of undisturbed soil; m_v is the volume compression coefficient; u_w is the pore pressure at any point in the shaft; u_r is the pore pressure at any point in the soil; γ_w is the severity of water; $-P(t)$ is vacuum negative pressure.
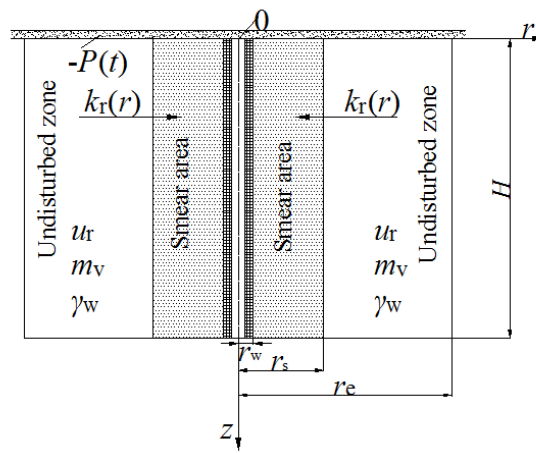

Fig.2 Circular equivalent calculation simple diagram

2.3 Basic assumption
The basic assumptions are as follows: (1) The soil mass is completely saturated; (2) Soil particles and water are incompressible, and the deformation of the soil is entirely caused by the discharge of pore water; (3) The use of plastic drainage board is considered radial seepage, and the seepage of water in the soil obeys Darcy law; (4) The condition of equal strain is established; (5) By introducing the area reduction factor, the continuous assumption of well circumferential flow is revised.

$$\left[2\pi r \frac{k_r(r)}{\gamma_w} \frac{\partial u_r}{\partial r} \right]_{r=r_w} = -\zeta \pi r_w^2 \frac{k_w}{\gamma_w} \frac{\partial^2 u_w}{\partial z^2} \tag{4}$$

(6)Vacuum degree remains unchanged along the depth. (7)Radial permeability coefficients of the smeared area exhibit three variation patterns, i.e. constant (pattern I), linear (pattern II) and parabolic (pattern III).

3.Governing equations and their boundary conditions
Only radial seepage is considered, the basic equation of radial seepage consolidation can be obtained.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{k_r(r)}{\gamma_w} r \frac{\partial u_r}{\partial r} \right) = -\frac{\partial \varepsilon_v}{\partial t} \tag{5}$$

where $k_r(r) = k_h f(r)$ is the radial permeability coefficient of the soil, $f(r)$ is a function of the radial distance away from the shaft, depending on the variation pattern of the horizontal permeability coefficient of the surrounding soil.

Based on equal strain conditions

$$\frac{\partial \varepsilon_v}{\partial t} = -m_v \frac{\partial \overline{u}_r}{\partial t} \qquad (6)$$

Average pore pressure

$$\overline{u}_r = \frac{1}{\pi\left(r_e^2 - r_w^2\right)} \int_{r_w}^{r_e} 2\pi r u_r \,\mathrm{d}r \qquad (7)$$

where ε_v is the volume strain at any point in the affected zone (equal to the vertical strain); $\overline{u}_r$ is the average pore pressure at any depth in the affected zone.

Radial boundary condition

$$r = r_e, \quad \frac{\partial u_r}{\partial r} = 0; \qquad (8)$$

$$r = r_w, \quad u_r = u_w. \qquad (9)$$

Using boundary conditions (8), we can integrate r on both sides of Eq. (5)

$$\frac{\partial u_r}{\partial r} = \frac{\gamma_w}{2k_h} \frac{\partial \varepsilon_v}{\partial t} \left(\frac{r_e^2}{rf(r)} - \frac{r}{f(r)} \right) \qquad (10)$$

The r on both sides of the Eq. (10) is again integrated by the boundary condition (9)

$$u_r = \frac{\gamma_w}{2k_h} \frac{\partial \varepsilon_v}{\partial t} \left(r_e^2 B_0 - C_0 \right) + u_w \qquad (11)$$

where

$$B_0 = \int_{r_w}^{r} \frac{1}{\xi f(\xi)} \mathrm{d}\xi, \quad C_0 = \int_{r_w}^{r} \frac{\xi}{f(\xi)} \mathrm{d}\xi \qquad (12)$$

The Eq. (11) is substituted by Eq. (7)

$$\overline{u}_r = \frac{\gamma_w r_e^2 F}{2k_h} \frac{\partial \varepsilon_v}{\partial t} + u_w \qquad (13)$$

where

$$F = \frac{2(r_e^2 B_1 - C_1)}{r_e^2(r_e^2 - r_w^2)} \qquad (14)$$

$$B_1 = \int_{r_w}^{r_e} r B_0 \,\mathrm{d}r, \quad C_1 = \int_{r_w}^{r_e} r C_0 \,\mathrm{d}r \qquad (15)$$

F corresponds to the three models of the horizontal permeability coefficient of soil in the smearing area. The specific expression is [20].

It can be obtained by Eq. (8) and $r = r_w$

$$\left. \frac{\partial u_r}{\partial r} \right|_{r=r_w} = \frac{\gamma_w r_w}{2k_s} (n^2 - 1) \frac{\partial \varepsilon_v}{\partial t} \qquad (16)$$

where $n = r_e / r_w$.

The Eq. (16) is substituted by Eq. (4)

$$\frac{\partial^2 u_{\text{w}}}{\partial z^2} = -\frac{\gamma_{\text{w}}}{\zeta k_{\text{w}}}\left(n^2 - 1\right)\frac{\partial \varepsilon_{\text{v}}}{\partial t} \tag{17}$$

By Eq. (6), E. (13) and Eq. (17) are obtained.

$$\frac{\partial \overline{u}_{\text{r}}}{\partial t} = -\frac{2k_{\text{h}}E_{\text{s}}}{\gamma_{\text{w}}Fr_{\text{e}}^2}\left(\overline{u}_{\text{r}} - u_{\text{w}}\right) \tag{18}$$

$$\frac{\partial^2 u_{\text{w}}}{\partial z^2} = -\frac{8k_{\text{h}}\left(n^2 - 1\right)}{\zeta k_{\text{w}}d_{\text{e}}^2 F}\left(\overline{u}_{\text{r}} - u_{\text{w}}\right) \tag{19}$$

where E_{s} is the modulus of compressibility of soil.

The governing equations of u_{w} are obtained by means of Eq. (18) and Eq. (19)

$$\frac{\partial^3 u_{\text{w}}}{\partial z^2 \partial t} + \frac{2k_{\text{h}}E_{\text{s}}}{\gamma_{\text{w}}}\frac{\partial^2 u_{\text{w}}}{\partial z^2} - \frac{8k_{\text{h}}\left(n^2 - 1\right)}{\zeta k_{\text{w}}d_{\text{e}}^2 F}\frac{\partial u_{\text{w}}}{\partial t} = 0 \tag{20}$$

Vertical boundary condition

$$z = 0 , \quad u_{\text{w}} = \overline{u}_{\text{r}} = -P(t) \tag{21}$$

$$z = H , \quad \frac{\partial u_{\text{w}}}{\partial z} = \frac{\partial \overline{u}_{\text{r}}}{\partial z} = 0 \tag{22}$$

At this point, the basic equations and boundary conditions for equal strain radial consolidation have been given.

4. Equation solution
Set the solution of equation (20) to be

$$u_{\text{w}} = -P(t) + \sum_{m=0}^{\infty} T(t)\sin\left(\frac{M}{H}z\right) \tag{23}$$

where $M = \dfrac{(2m+1)\pi}{2}$, $m = 0,1,2,3,\cdots$.

The Eq. (23) has already satisfied the boundary condition Eq. (21) and Eq. (22), and the average pore pressure is

$$\overline{u}_{\text{r}} = -P(t) + \sum_{m=0}^{\infty}\left(1 + \frac{\zeta k_{\text{w}}d_{\text{e}}^2 F}{8k_{\text{h}}(n^2 - 1)}\frac{M^2}{H^2}\right)T(t)\sin\left(\frac{M}{H}z\right) \tag{24}$$

The initial condition is

$$t = 0 , \quad \overline{u}_{\text{r}} = 0 。 \tag{25}$$

By Eq. (24) and Eq. (25), there are

$$-P(0) + \sum_{m=0}^{\infty}\left(1 + \frac{\zeta k_{\text{w}}d_{\text{e}}^2 F}{8k_{\text{h}}(n^2 - 1)}\frac{M^2}{H^2}\right)T(0)\sin\left(\frac{M}{H}z\right) = 0 \tag{26}$$

The two sides of Eq. (26) are multiplied by $\sin\left(\dfrac{M}{H}z\right)$ and integral to $[0, H]$ on z, and the orthogonality of trigonometric series can be obtained

$$T_m(0) = \frac{2}{M} P(0) \frac{1}{1 + \frac{\zeta k_w d_e^2 F}{8 k_h (n^2 - 1)} \frac{M^2}{H^2}} \tag{27}$$

The Eq. (23) is substituted by Eq. (20), and it can be obtained after arranging

$$\frac{\zeta k_w d_e^2 F}{8 k_h (n^2 - 1)} \frac{M^2}{H^2} \sum_{m=0}^{\infty} T'(t) \sin\left(\frac{M}{H} z\right) + \frac{\zeta k_w E_s}{\gamma_w (n^2 - 1)} \frac{M^2}{H^2} \sum_{m=0}^{\infty} T(t) \sin\left(\frac{M}{H} z\right) - \sum_{m=0}^{\infty} T'(t) \sin\left(\frac{M}{H} z\right) = -\frac{\partial P(t)}{\partial t}$$

$$（28）$$

Similarly, both sides of Eq. (28) are multiplied by $\sin\left(\frac{M}{H} z\right)$ and integral to $[0, H]$ on z, and the orthogonality of trigonometric functions series can be obtained

$$T'_m(t) + \beta T_m(t) = \frac{\frac{2}{M} \frac{\partial P(t)}{\partial t}}{1 + \frac{\zeta k_w d_e^2 F}{8 k_h (n^2 - 1)} \frac{M^2}{H^2}} \tag{29}$$

where

$$\beta = \frac{\frac{8 c_h}{F d_e^2}}{1 + \frac{8 k_h (n^2 - 1)}{\zeta k_w d_e^2 F} \frac{H^2}{M^2}} \tag{30}$$

The Eq. (28) is the first order linear differential equation of $T_m(t)$, and the Eq. (27) is its definite solution condition. The general solution is

$$T_m(t) = e^{-\beta t} \frac{\frac{2}{M}}{1 + \frac{\zeta k_w d_e^2 F}{8 k_h (n^2 - 1)} \frac{M^2}{H^2}} \left[P(0) + \int_0^t \frac{\partial P(\tau)}{\partial \tau} e^{\beta \tau} d\tau \right] \tag{31}$$

The average pore pressure of soil can be obtained by substituting Eq. (31) into Eq. (24)

$$\bar{u}_r = -P(t) + \sum_{m=0}^{\infty} \frac{2}{M} \left[P(0) + \int_0^t \frac{\partial P(\tau)}{\partial \tau} e^{\beta \tau} d\tau \right] \sin\left(\frac{M}{H} z\right) \tag{32}$$

The total average consolidation degree of the foundation is

$$U = \frac{\frac{1}{H} \int_0^H \bar{u}_r dz}{\frac{1}{H} \int_0^H -P_u dz} = \frac{P(t)}{P_u} - \frac{\sum_{m=0}^{\infty} \frac{2}{M^2} \left[P(0) + \int_0^t \frac{\partial P(\tau)}{\partial \tau} e^{-\beta \tau} d\tau \right] e^{-\beta t}}{P_u} \tag{33}$$

where P_u is the maximum vacuum degree.

5. Special solution
Figure 3 is the two common vacuum preloading distribution map. The above general solution is used to give a detailed solution.
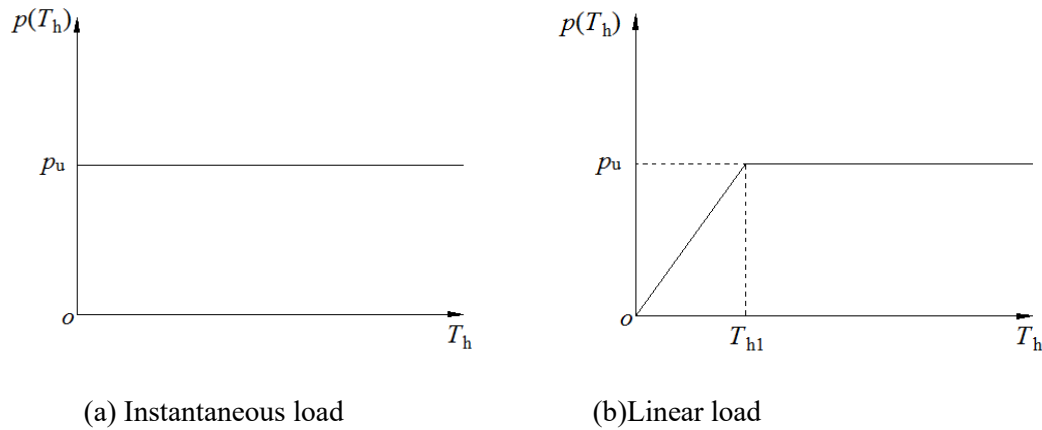
(a) Instantaneous load   (b)Linear load

Fig.3 Two special loads

(1) Instantaneous load
The degree of vacuum can be expressed as instantaneous load.

$$P(t) = P_u \tag{34}$$

By substituting Eq. (34) into Eq. (32) and Eq. (33), the solutions of average pore pressure and total average consolidation degree of soil under instantaneous load can be obtained as follows.

$$\bar{u}_r = -P_u + P_u \sum_{m=0}^{\infty} \frac{2e^{-\beta t}}{M} \sin\left(\frac{M}{H}z\right) \tag{35}$$

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} e^{-\beta t} \tag{36}$$

(2) Linear load
The degree of vacuum can be expressed as linear load.

$$P(t) = \begin{cases} P_u t/t_1 & (t \le t_1) \\ P_u & (t > t_1) \end{cases} \tag{37}$$

By substituting Eq. (37) into Eq. (32) and Eq. (33), the solutions of average pore pressure and average consolidation degree of soil under linear load can be obtained as follows.

$$\bar{u}_r = \begin{cases} -P(t) + P_u \sum_{m=0}^{\infty} \frac{2\left(1 - e^{-\beta t}\right)}{M\beta t_1} \sin\left(\frac{M}{H}z\right) & (t \le t_1) \\ -P(t) + P_u \sum_{m=0}^{\infty} \frac{2}{M\beta t_1} \left[e^{-\beta(t-t_1)} - e^{-\beta t}\right] & (t > t_1) \end{cases} \tag{38}$$

$$U = \begin{cases} \dfrac{t}{t_1} - \sum_{m=0}^{\infty} \dfrac{2}{M^2 \beta t_1}\left(1 - e^{-\beta t}\right) & (t \le t_1) \\ 1 - \sum_{m=0}^{\infty} \dfrac{2}{M^2 \beta t_1}\left[e^{-\beta(t-t_1)} - e^{-\beta t}\right] & (t > t_1) \end{cases} \tag{39}$$

6. Analysis of consolidation behavior

Based on the solution given, the influencing factors of foundation consolidation are calculated and analyzed for a kind of plastic drainage board with a width of 100 mm and a thickness of 4 mm, and the results are compared with the perimeter equivalence.

Fig. 4is three models next week looks and so on, but the length and width of the different drainage boards using perimeter equivalent and annular equivalent of the linear vacuum load consolidation degree comparison. The equivalent radius is calculated according to Eq. (1), but the girth is equivalent to the area reduction. The annular equivalent will reduce the area according to the thickness-width ratio. If the aspect ratio is smaller, the reduction factor is smaller. According to Fig. 4, it can be seen that the degree of consolidation considering area reduction is less than that without area reduction in the three modes. With the increase of area reduction coefficient, the annular equivalent degree of consolidation approaches the circumference equivalent degree of consolidation.
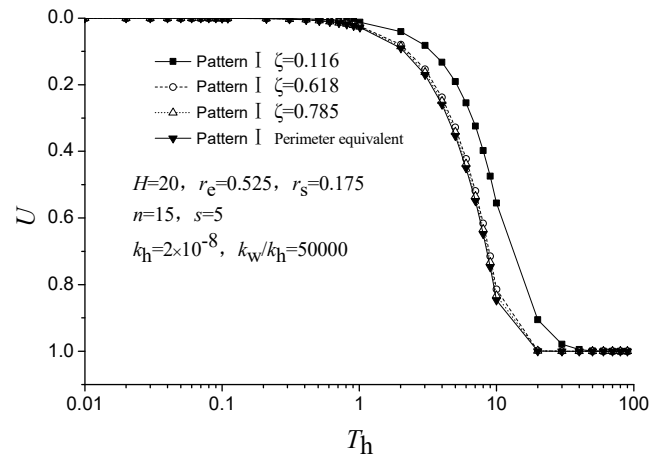
(a)
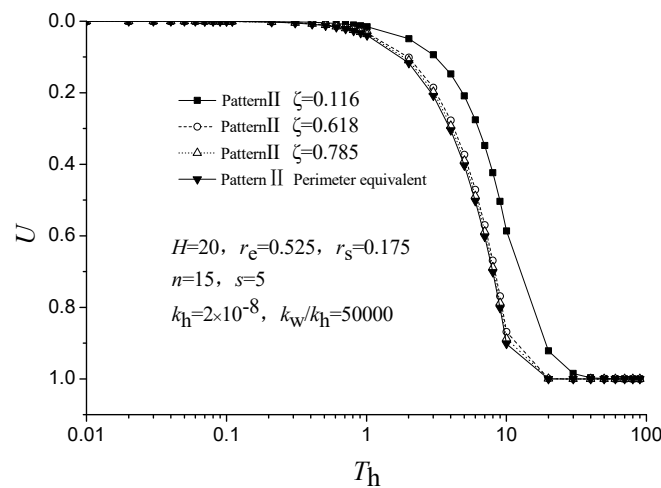
(b)
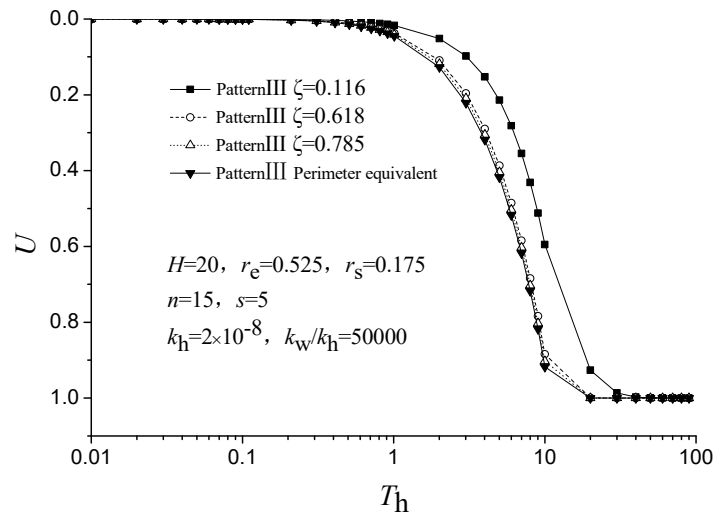
(c)

Fig.4 Influence of area reduction factor on consolidation degree curve (linear load)

Figure 5 is a comparison of the consolidation degree of the soil in the three smear areas when the drainage plate is subjected to different permeability coefficients under transient vacuum loading. It can be seen from the graph that the consolidation rate is the largest when the horizontal permeability coefficient of soil changes parabola, the second is linear change, and the smallest when the constant remains unchanged. With the decrease of permeability coefficient of drainage plate, the difference of consolidation rates calculated by different models is decreased.
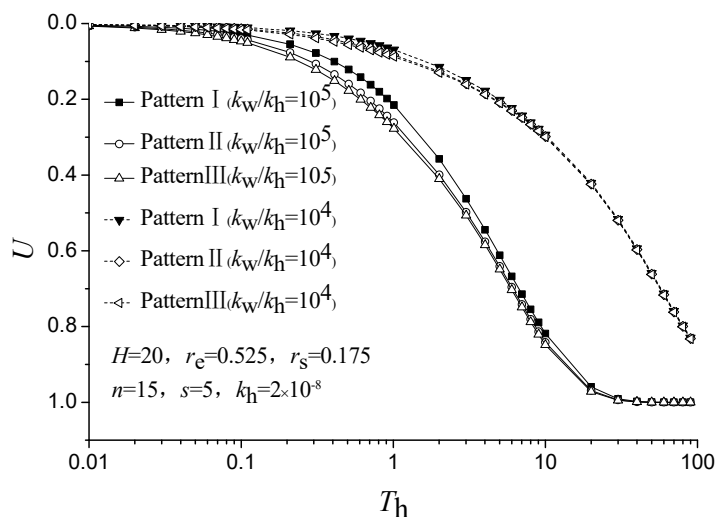

Fig.5 Comparison between the results of soil consolidation with three variation patterns of horizontal permeability of smeared soil（linear load）

Figure 6 is a comparison of the consolidation degree of the drainage plate's permeability coefficient when the instantaneous vacuum load changes the horizontal permeability coefficient of the soil in the smear area. It can be seen that with the smaller and smaller permeability coefficient of the drainage plate, the consolidation rate of the foundation becomes slower and slower.
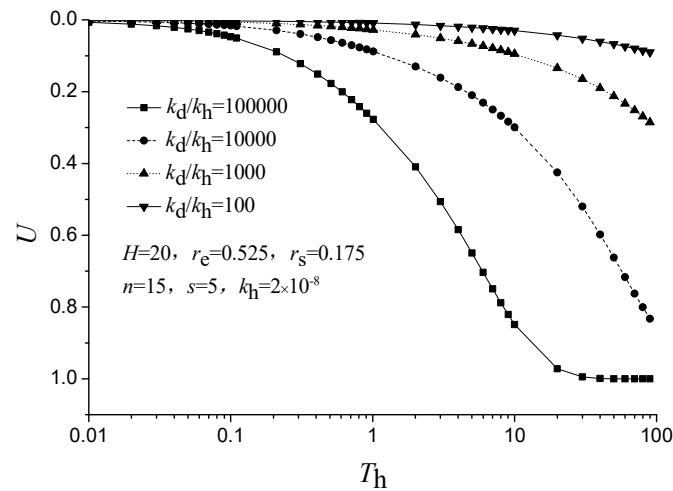
Fig.6 Influence of permeability coefficient of band-shaped drain on degree of consolidation（pattern Ⅲ，instantaneously loading）

Fig.7 shows the time-dependent dissipation rate of pore water under instantaneous and linear vacuum loads with different annular equivalent area reduction coefficients. It can be seen that the dissipation rate of pore water under the linear load is slower than that under instantaneous load. With the increase of reduction coefficient of annular equivalent area, the dissipation rate of pore water in foundation becomes larger and larger.
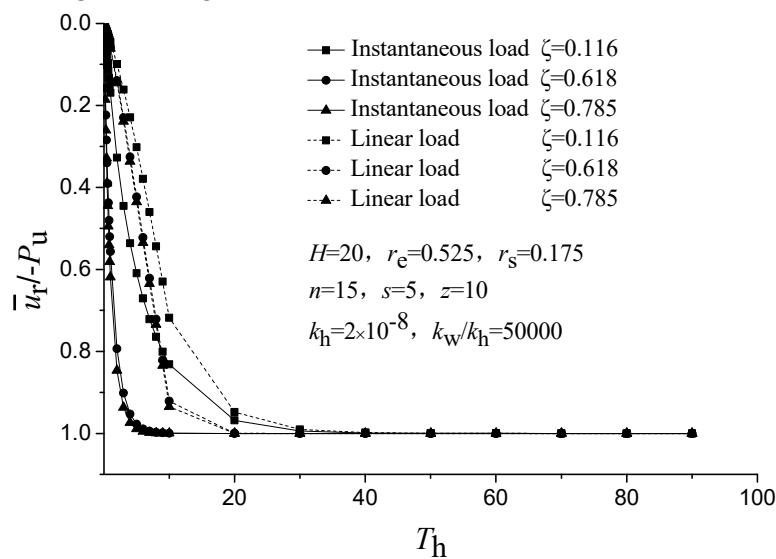

Fig.7 Effect of area reduction coefficient on the dissipation of pore water

7. Conclusion

Based on the annular equivalent method of drainage plate, the analytical solution of consolidation of vertical drain is derived, and the results are compared and analyzed. The following conclusions are drawn:

（1）The degree of consolidation obtained by considering circumference equivalence is greater than that obtained by ring equivalent. With the increase of the area reduction factor, the equivalent consolidation rate of the ring approaches the equivalent consolidation rate of girth.

（2）It is assumed that the consolidation rate of the shaft foundation is the highest when the horizontal permeability coefficient of the soil in the smeared area varies in a parabolic way, followed

by a linear one, and is the smallest when the constant remains unchanged. With the decrease of permeability coefficient of drainage plate, the difference of consolidation rates calculated by the three models also decreases. If the permeability coefficient of drainage plate is smaller, the consolidation rate of shaft foundation is slower.

（3）The dissipation rate of pore water under linear load is slower than that under instantaneous load. With the increase of reduction coefficient of annular equivalent area, the dissipation rate of pore water in foundation becomes larger and larger.

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