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To cite this article: Xiaotang Lv *et al* 2019 *IOP Conf. Ser.: Earth Environ. Sci.* **218** 012015

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Interaction between a semi-cylindrical hill and a shallow buried fixed rigid cylindrical inclusion under SH wave

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Abstract. This paper analyses the interaction between a semi-cylindrical hill and a shallow buried fixed rigid cylindrical inclusion under SH wave. The whole calculation model includes two regions. Firstly the displacement solutions that satisfy the boundary conditions in two regions are constructed by the complex variable method. Then a set of infinite algebraic equations for the problem can be obtained by the boundary condition of the fixed rigid inclusion and that of the public boundary of two regions. And to illustrate the interaction between the semi-cylindrical hill and the shallow buried fixed rigid cylindrical inclusion, the variation of both the surface displacements of the hill and the dynamic response of the inclusion with different parameters are provided. The numerical results show that the existence of fixed rigid inclusion significantly reduces the displacement amplitude of the semi-cylindrical hill.

1. Introduction

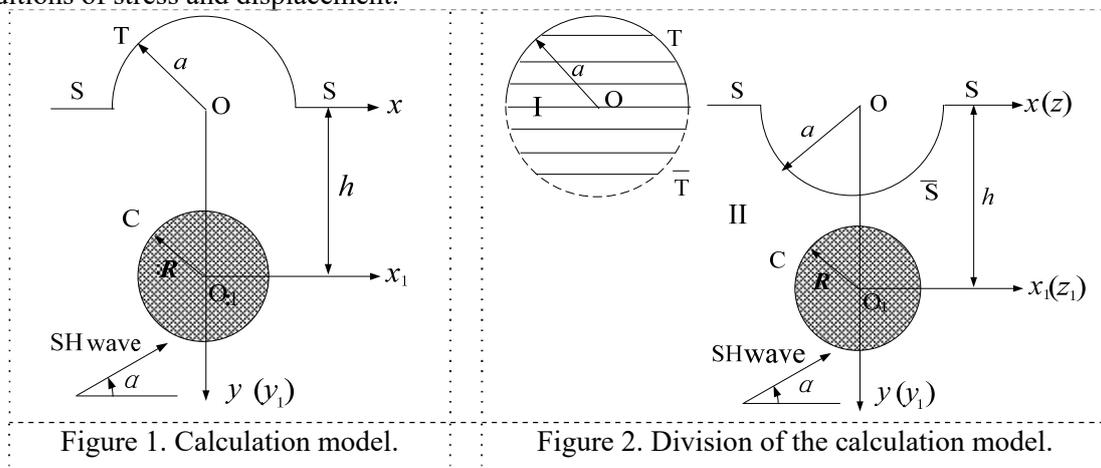
Study of the ground motion under the seismic waves has always been an important topic in earthquake engineering. Local topography and structures underground are the important factors affecting the ground motion. With the development of engineering technology, more underground structures have been built such as subway and underground tunnels. Many scholars devote themselves to studying the safety conditions of underground structures and the influence of these underground structures on the ground motion under the dynamic load, and a lot of meaningful results have been achieved[1]-[6]. Chen and Qi discussed the dynamic of circular cavity and inclusion in layer half-space through the large-arc assumption method [7]. Using wave function expansion and complex function method, Liu and Wang solved the problem that scattering of SH wave by a semi-cylindrical hill above a subsurface cavity, and the influence of the subsurface cavity on the surface displacement of semi-cylindrical hill and of horizontal boundary has been analysed [8]. In 2014, Lv investigated the dynamic stress concentration of a subsurface elastic cylindrical inclusion below a semi-cylindrical hill, and the variation of dynamic stress concentration factor on the edge of inclusion has been shown under the incidence of SH wave [9].

The aim of this paper is to consider a problem that a shallow buried fixed rigid cylindrical inclusion near a semi-cylindrical hill in half space, and analyse the interaction between them by the complex variable method. Under SH wave, the influences of the inclusion on the surface displacements of the hill and that of the hill on the dynamic concentration factors of the inclusion are discussed with different parameters such as the radius of fixed rigid inclusion and the embedded depth of it.

2. Description of the problem



In elastic half-space, the calculation model of a semi-cylindrical hill with a shallow buried fixed rigid cylindrical inclusion is shown in figure 1. For the semi-cylindrical hill, the boundary is T, the radius is a , the centre is O, and the horizontal boundary on both sides of the hill is S. For the rigid cylindrical inclusion, the boundary, the radius and the centre can be expressed by C, R and O_1 . Under SH wave, studying the interaction between a semi-cylindrical hill and a shallow buried fixed rigid cylindrical inclusion, which means to find the governing equations satisfying the following conditions: boundary conditions of horizontal boundary S, of the hill T and of the fixed rigid cylindrical inclusion C. For this purpose, the calculation model shown in figure 1 can be divided into two domains as shown in figure 2. Domain I is a circular area including the hill boundary T, domain II is composed of the horizontal boundary S, the semi-circular canyon boundary \bar{S} and the rigid cylindrical inclusion C. The boundaries \bar{T} and \bar{S} are the public boundaries of two domains, on which should satisfy the continuity conditions of stress and displacement.



3. Solving the problem

3.1 Governing equation

In homogeneous and isotropic media, the displacement function of steady-state SH wave should satisfy the following governing equation in complex plane (z, \bar{z}) :

$$\frac{\partial^2 W}{\partial z \partial \bar{z}} + \frac{1}{4} k^2 W = 0 \tag{1}$$

where W is the displacement function.

The stress expressions can be written as:

$$\tau_{rz} = \mu \left(\frac{\partial G}{\partial z} e^{i\theta} + \frac{\partial G}{\partial \bar{z}} e^{-i\theta} \right) \tag{2}$$

$$\tau_{\theta z} = i\mu \left(\frac{\partial G}{\partial z} e^{i\theta} - \frac{\partial G}{\partial \bar{z}} e^{-i\theta} \right) \tag{3}$$

3.2 Wave field in domain I

Under SH wave, the standing wave in domain I is:

$$W_{(z, \bar{z})}^{(st)} = W_0 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_m \frac{J_{m-1}(k_1 a) - J_{m+1}(k_1 a)}{J_{n-1}(k_1 a) - J_{n+1}(k_1 a)} \times a_{mn} J_n(k_1 |z|) \left[\frac{z}{|z|} \right]^n \tag{4}$$

which satisfies the conditions that stress free at T and arbitrary at \bar{T} [8], and the expression of a_{mn} can also be found in reference [8].

The stress expression from equation (4) is:

$$\tau_{rz}^{(st)} = \frac{\mu_1 k_1 W_0}{2} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} C_m \frac{J_{m-1}(k_1 a) - J_{m+1}(k_1 a)}{J_{n-1}(k_1 a) - J_{n+1}(k_1 a)} a_{nm} [J_{n-1}(k_1 |z|) - J_{n+1}(k_1 |z|)] \left[\frac{z}{|z|} \right]^n \quad (5)$$

3.3 Wave field in domain II

3.3.1 Incident wave and reflected wave. In complex plane, set α is incident angle, incident wave and reflected wave can be written as:

$$W_{(z,\bar{z})}^{(i)} = W_0 e^{\frac{ik_1}{2} [z e^{i\alpha} + \bar{z} e^{-i\alpha}]} \quad (6)$$

$$W_{(z,\bar{z})}^{(r)} = W_0 e^{\frac{ik_1}{2} [z e^{-i\alpha} + \bar{z} e^{i\alpha}]} \quad (7)$$

The stresses from equation (6) and (7) are:

$$\tau_{rz}^{(i)} = i\mu_1 k_1 W_0 \cos(\theta + \alpha) e^{ik_1 |z| \cos(\theta + \alpha)} \quad (8)$$

$$\tau_{rz}^{(r)} = i\mu_1 k_1 W_0 \cos(\theta - \alpha) e^{ik_1 |z| \cos(\theta - \alpha)} \quad (9)$$

In complex plane (z_1, \bar{z}_1) , equation (6) to equation (9) take the forms:

$$W_{(z_1, \bar{z}_1)}^{(i)} = W_0 e^{\frac{ik_1}{2} [(z_1 + d) e^{i\alpha} + (\bar{z}_1 + \bar{d}) e^{-i\alpha}]} \quad (10)$$

$$W_{(z_1, \bar{z}_1)}^{(r)} = W_0 e^{\frac{ik_1}{2} [(z_1 + d) e^{-i\alpha} + (\bar{z}_1 + \bar{d}) e^{i\alpha}]} \quad (11)$$

$$\tau_{rz_1}^{(i)} = i\mu_1 k_1 W_0 \cos(\theta_1 + \alpha) e^{\frac{ik_1}{2} [(z_1 + d) e^{i\alpha} + (\bar{z}_1 + \bar{d}) e^{-i\alpha}]} \quad (12)$$

$$\tau_{rz_1}^{(r)} = i\mu_1 k_1 W_0 \cos(\theta_1 - \alpha) e^{\frac{ik_1}{2} [(z_1 + d) e^{-i\alpha} + (\bar{z}_1 + \bar{d}) e^{i\alpha}]} \quad (13)$$

where d is the complex coordinates of the rigid inclusion center in the plane with origin at O, and \bar{d} is the conjugate of complex variable d .

3.3.2 Scattering waves. The scattered wave field in domain II should satisfy the boundary condition that stress free at horizontal surface. There are two kinds of scattering waves in domain II, one is from the semi-circular canyon \bar{S} and can be represented by $W_{\bar{S}}^{(s)}$, the other is from the fixed rigid inclusion C and can be represented by $W_C^{(s)}$. In complex plane (z, \bar{z}) , the scattering waves are constructed to satisfy the condition stated above, which take the forms:

$$W_{\bar{S},(z,\bar{z})}^{(s)} = W_0 \sum_{m=0}^{\infty} A_m H_m^{(1)}(k_1 |z|) \left\{ \left[\frac{z}{|z|} \right]^m + \left[\frac{\bar{z}}{|\bar{z}|} \right]^{-m} \right\} \quad (14)$$

$$W_{C,(z,\bar{z})}^{(s)} = W_0 \sum_{m=-\infty}^{\infty} B_m \left\{ H_m^{(1)}(k_1 |z-d|) \left[\frac{z-d}{|z-d|} \right]^m + H_m^{(1)}(k_1 |z-\bar{d}|) \left[\frac{z-\bar{d}}{|z-\bar{d}|} \right]^{-m} \right\} \quad (15)$$

The corresponding stresses are:

$$\tau_{rz,s}^{(s)} = \frac{\mu_1 k_1 W_0}{2} \sum_{m=0}^{\infty} A_m [H_{m-1}^{(1)}(k_1 |z|) - H_{m+1}^{(1)}(k_1 |z|)] \left\{ \left[\frac{z}{|z|} \right]^m + \left[\frac{\bar{z}}{|\bar{z}|} \right]^{-m} \right\} \quad (16)$$

$$\begin{aligned} \tau_{rz,C}^{(s)} = & \frac{\mu_1 k_1 W_0}{2} \sum_{m=-\infty}^{\infty} B_m \left\{ \left[H_{m-1}^{(1)}(k_1 |z-d|) \left[\frac{z-d}{|z-d|} \right]^{m-1} \right] - H_{m+1}^{(1)}(k_1 |z-d|) \left[\frac{z-d}{|z-d|} \right]^{-(m+1)} \right] e^{i\theta} \\ & + \left[-H_{m+1}^{(1)}(k_1 |z-d|) \left[\frac{z-d}{|z-d|} \right]^{m+1} + H_{m-1}^{(1)}(k_1 |z-d|) \left[\frac{z-d}{|z-d|} \right]^{-(m-1)} \right] e^{-i\theta} \right\} \end{aligned} \tag{17}$$

In complex plane (z_1, \bar{z}_1) , equation (14) to equation (17) take the forms:

$$W_{\bar{s}_1(z_1, \bar{z}_1)}^{(s)} = W_0 \sum_{m=0}^{\infty} A_m H_m^{(1)}(k_1 |z_1 + d|) \left\{ \left[\frac{z_1 + d}{|z_1 + d|} \right]^m + \left[\frac{z_1 + d}{|z_1 + d|} \right]^{-m} \right\} \tag{18}$$

$$W_{C_1(z_1, \bar{z}_1)}^{(s)} = W_0 \sum_{m=0}^{\infty} B_m \left\{ H_m^{(1)}(k_1 |z_1|) \left[\frac{z_1}{|z_1|} \right]^m + H_m^{(1)}(k_1 |z_1 - \bar{d} + d|) \left[\frac{z_1 - \bar{d} + d}{|z_1 - \bar{d} + d|} \right]^{-m} \right\} \tag{19}$$

$$\begin{aligned} \tau_{rz_1, \bar{s}}^{(s)} = & \frac{\mu_1 k_1 W_0}{2} \sum_{m=-\infty}^{\infty} A_m \left\{ \left[H_{m-1}^{(1)}(k_1 |z_1 + d|) \left[\frac{z_1 + d}{|z_1 + d|} \right]^{m-1} \right] - H_{m+1}^{(1)}(k_1 |z_1 + d|) \left[\frac{z_1 + d}{|z_1 + d|} \right]^{-(m+1)} \right] e^{i\theta_1} \\ & + \left[-H_{m+1}^{(1)}(k_1 |z_1 + d|) \left[\frac{z_1 + d}{|z_1 + d|} \right]^{m+1} + H_{m-1}^{(1)}(k_1 |z_1 + d|) \left[\frac{z_1 + d}{|z_1 + d|} \right]^{-(m-1)} \right] e^{-i\theta_1} \right\} \end{aligned} \tag{20}$$

$$\begin{aligned} \tau_{rz_1, C}^{(s)} = & \frac{\mu_1 k_1 W_0}{2} \sum_{m=-\infty}^{\infty} B_m \left\{ \left[H_{m-1}^{(1)}(k_1 |z_1|) \left[\frac{z_1}{|z_1|} \right]^{m-1} \right] - H_{m+1}^{(1)}(k_1 |z_1 + d - \bar{d}|) \left[\frac{z_1 + d - \bar{d}}{|z_1 + d - \bar{d}|} \right]^{-(m+1)} \right] e^{i\theta_1} \\ & + \left[-H_{m+1}^{(1)}(k_1 |z_1|) \left[\frac{z_1}{|z_1|} \right]^{m+1} + H_{m-1}^{(1)}(k_1 |z_1 + d - \bar{d}|) \left[\frac{z_1 + d - \bar{d}}{|z_1 + d - \bar{d}|} \right]^{-(m-1)} \right] e^{-i\theta_1} \right\} \end{aligned} \tag{21}$$

4. Equations for solving the problem

According to the boundary conditions of the public boundaries (\bar{T} and \bar{S}) and of the rigid inclusion, a series infinite algebraic equation solving the unknown coefficients A_m, B_m, C_m, D_m can be written as:

$$\begin{cases} W_{(z, \bar{z})}^{(st)} = W_{(z, \bar{z})}^{(i)} + W_{(z, \bar{z})}^{(r)} + W_{s, (z, \bar{z})}^{(s)} + W_{C, (z, \bar{z})}^{(s)} & \text{on } \bar{S} \\ \tau_{rz}^{(st)} = \tau_{rz}^{(i)} + \tau_{rz}^{(r)} + \tau_{rz, \bar{s}}^{(s)} + \tau_{rz, T}^{(s)} & \text{on } \bar{S} \\ 0 = W_{(z_1, \bar{z}_1)}^{(i)} + W_{(z_1, \bar{z}_1)}^{(r)} + W_{s, (z_1, \bar{z}_1)}^{(s)} + W_{C, (z_1, \bar{z}_1)}^{(s)} & \text{on } C \end{cases} \tag{22}$$

5. Numerical results

Take figure 1 as the calculating example. In the following discussion, a dimensionless frequency $\eta = 2a / \lambda$ is defined, where λ is the wavelength of shear wave in the half space; and α is incident angle.

Figure 3 shows the surface displacement of the semi-cylindrical hill and that of the horizontal boundaries on two sides of the hill when $\alpha = 90^\circ$. In the figure, $R/a = 0$ indicate the radius of fixed rigid inclusion is zero, which means there is only a semi-cylindrical hill in half space. It is can be seen that the surface displacement in figure 3 when $R/a = 0$ is consistent with that in reference [6]. When

$\eta = 0.25$, $R/a = 0.5$, compared with the case of no inclusion, the displacement amplitude of the hill peak is reduced by 53%, and which decreases by 40% when $\eta = 1.25$. In higher frequency, for the points on the horizontal boundary, when the distance of them to the intersection points between the hill and the horizontal boundaries is twice the radius of the hill, the existence of fixed rigid inclusion magnified the displacement of these points in varying degrees. Variation of the displacement of the hill peak with h/a is shown in figure 4, it is clearly that the fixed rigid inclusion reduces the displacement of the hill peak.

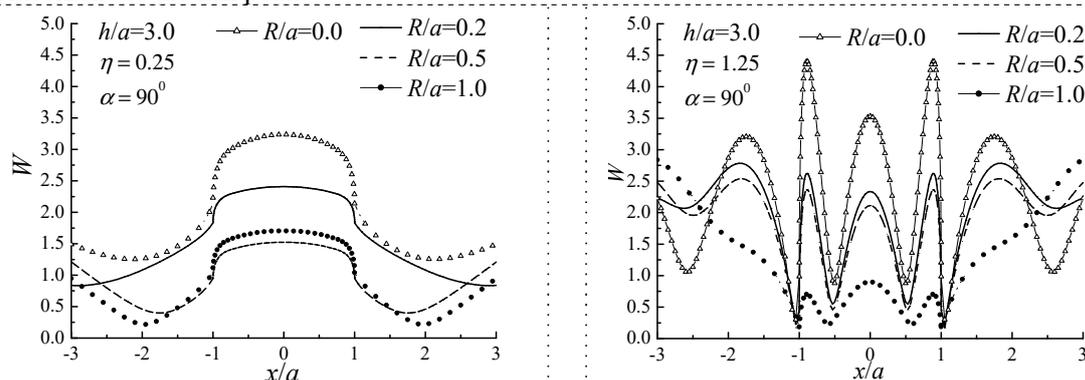


Figure 3. Variation of surface displacement with R/a .

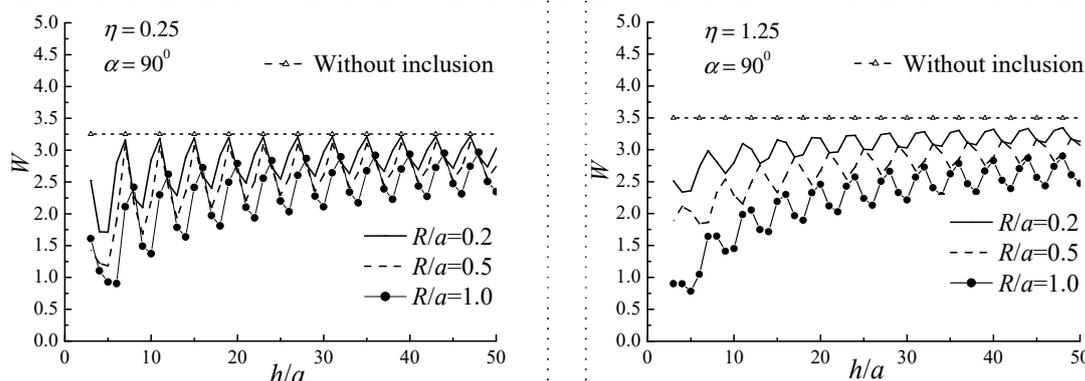


Figure 4. Variation of the displacement of the hill peak with h/a .

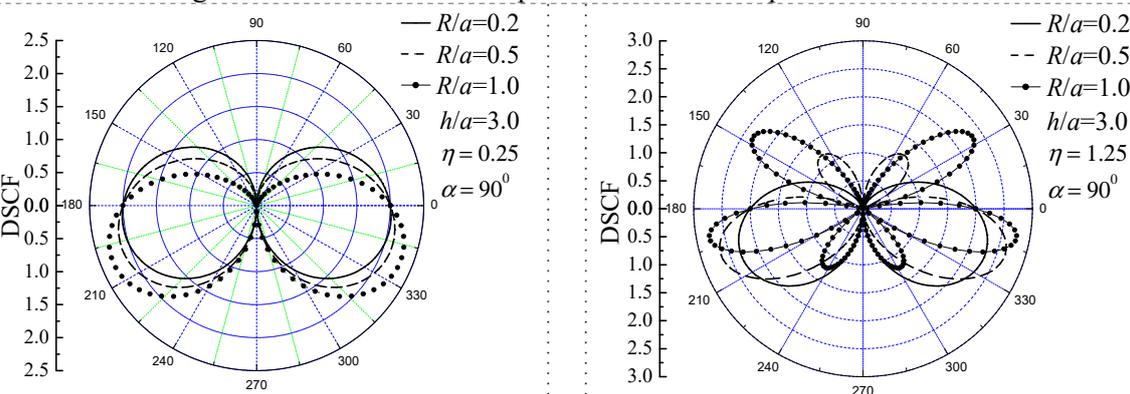


Figure 5. Distribution of DSCF on the edge of fixed rigid inclusion

Figure 5 illustrates the influence of R/a on the distribution of dynamic stress concentration factor (DSCF) on the edge of inclusion by the incident SH wave vertically. At this time, distribution of DSCF shows obviously symmetry, and with the incidence of high frequency SH wave, it shows remarkable dynamic characteristics especially for the bigger R/a .

6. Conclusion

The displacement amplitude of the semi-cylindrical hill surface is significantly reduced by the presence of fixed rigid inclusion. When the SH wave is incident at high frequency, the larger radius of the fixed rigid inclusion is, the greater decrease of the surface displacement amplitude is.

Acknowledgments

This work was supported by key project of natural science research of Universities in Anhui province (KJ2016A599), key project of university excellent young scholars for domestic and overseas visit in Anhui province (gxfzD2016217), Major project of natural science research of Universities in Anhui province (KJ2018ZD054), and development fund of Hefei University (2018ZR14ZDA).

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