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# Study on the Determination of Attitudes and Orbits of Micro-satellites only by 3-axis Magnetometer

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**Abstract.** This paper explains a method to determine the attitude and orbit only using onboard magnetometer. This method, only using 3-axis magnetometer, makes it efficient and possible to get the attitude and orbit information, without high-priced and complicated sensors. This method comprises 2 parts- one is the attitude determination and the other is the orbit determination by the magnetometer. In the attitude determination, 2-stage filter (Linear Kalman filter and Unscented Kalman filter) is applied and in the orbit determination, the Extended Kalman filter. The attitude determination accuracy is below 1°, the attitude angular rate determination accuracy below 0.005°/s and the orbital position determination accuracy below 10 kms when the measurement error of magnetometer is about 200nT. Thus, this study contributes to the mission of the micro satellites.

## 1. Introduction

In order to decrease their total mass and development cost as much as possible, the general trend is not to use the high cost and complicated sensors and actuators but the high reliable and low cost ones. For this reason, nowadays, many countries are only using onboard 3-axis magnetometer to determine the attitudes and orbits of microsatellites.

## 2. The attitude determination of micro-satellite only using the magnetometer data

The attitude determination comprises 2-stage filter, firstly the LKF (Linear Kalman Filter) estimates the derivatives of the geomagnetic field. As rates of satellite are unknowns, Markov process is used to model the geomagnetic field.

$$\frac{d^3}{dt^3}\mathbf{B} = \mathbf{w}, \quad (1)$$

where  $\mathbf{B}$  is the geomagnetic field vector,  $\mathbf{w}$  is the white Gaussian process noise. If the states are defined as

$$\mathbf{x} = [B_x, B_y, B_z, \dot{B}_x, \dot{B}_y, \dot{B}_z, \ddot{B}_x, \ddot{B}_y, \ddot{B}_z]^T \quad (2)$$

the systematic equations can be written as follows.

$$\dot{\mathbf{x}} = \frac{d}{dt} \begin{bmatrix} B_x, B_y, B_z, \dot{B}_x, \dot{B}_y, \dot{B}_z, \ddot{B}_x, \ddot{B}_y, \ddot{B}_z \end{bmatrix}^T = \begin{bmatrix} \dot{B}_x, \dot{B}_y, \dot{B}_z, \ddot{B}_x, \ddot{B}_y, \ddot{B}_z, 0, 0, 0 \end{bmatrix}^T \quad (3)$$



$$y = \begin{bmatrix} B_x, B_y, B_z \end{bmatrix}^T \quad (4)$$

Therefore, the state space model for estimating the derivatives of the geomagnetic field is as follows.

$$\dot{x} = Fx + w, \quad y = Hx + v \quad (5)$$

where  $w$  is the process noise, and  $v$  is the measurement noise.

Apply the LKF algorithm to the system above. After the determination of the magnetic field vector and its derivatives(pseudo-measurements), and the reference vector in inertial frame by the linear Kalman filter and the geomagnetic field model, only its attitudes and angular rates are remained unknown. If the state vector and measurement vector are selected as follows

$$x = \begin{bmatrix} q^i, \omega_{ib}^b \end{bmatrix}^T = [q_1, q_2, q_3, q_4, \omega_x, \omega_y, \omega_z]^T \quad (6)$$

$$y = \begin{bmatrix} B^b, \dot{B}^b \end{bmatrix}^T = [B_x, B_y, B_z, \dot{B}_x, \dot{B}_y, \dot{B}_z]^T \quad (7)$$

the system model for estimating attitude angles and rates is as follows.

$$\begin{aligned} \dot{x} &= f(x) + w \\ y &= h(x) + v \end{aligned} \quad (8)$$

$$f(x) = \left[ \frac{1}{2} \Omega q^i, \left( I^b \right)^{-1} \left( T_c - \omega_{ib}^b \times \left( I^b \omega_{ib}^b \right) \right) \right]^T \quad (9)$$

$$h(x) = \left[ R_i^b B^i, R_i^b \dot{B}^i - \omega_{ib}^b \times \left( R_i^b B^i \right) \right]^T \quad (10)$$

where

$$\Omega = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_1 \\ -\omega_z & 0 & \omega_1 & \omega_y \\ \omega_y & -\omega_1 & 0 & \omega_z \\ -\omega_1 & \omega_y & -\omega_z & 0 \end{bmatrix} \quad (11)$$

$$R_i^b = \begin{pmatrix} q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & q_4^2 - q_1^2 - q_2^2 + q_3^2 \end{pmatrix} \quad (12)$$

$T_c$  : torque vector applied to the satellite

$I^b$  : inertia tensor of the satellite

$R_i^b$  : attitude transformation matrix from inertial frame to body frame

$B^i, \dot{B}^i$  : geomagnetic vector and its derivatives in inertial frame

The UKF (Unscented Kalman Filter) is applied to this non-linear system to estimate the states, the attitude quaternions and angular rates. [1,2]

### 3. The orbit determination only using the magnetometer data

For the orbit determination by the magnetometer data, the EKF (Extended Kalman Filter) is used. [3,4]  
The state variable  $X$  comprises 6 positions and velocities.

$$X = (\mathbf{r}, \mathbf{V}) = (\mathbf{r}, \dot{\mathbf{r}}) \quad (13)$$

The state transition model is

$$\dot{X} = f(X(t), t) + w(t) \quad (14)$$

where the process noise  $w(t)$  can be related to the time, but the expectation value of its distribution is supposed zero at every moment.

$$E(w(t)) = 0 \quad (15)$$

In addition, the covariance of process noise can also be related to time, but it is supposed that the time correlation between observations does not exist.

$$E\left(w(t_j)w^T(t_k)\right) = Q(t_j)\delta_{jk} \quad (16)$$

where  $Q(t_j)$  is the covariance of process noise at  $t_j$ .

The state transition model for the satellite movement is the dynamics model, where the zonal harmonics is considered up to 4th order for its modelling. [5] Considering the atmospheric drag is very small at the height of 700km, this item is ignored. If the dynamics model is configured according to Newton's law, the state transition equation can be written as follows.

$$\dot{\mathbf{r}} = \mathbf{V}, \dot{\mathbf{V}} = \nabla U \quad (17)$$

where

$$U = \frac{GM_e}{r} \left[ 1 - J_2 \left( \frac{a_e}{r} \right)^2 \left( \frac{3}{2} \sin^2 \varphi - \frac{1}{2} \right) - J_3 \left( \frac{a_e}{r} \right)^3 \left( \frac{5}{2} \sin^3 \varphi - \frac{3}{2} \sin \varphi \right) - J_4 \left( \frac{a_e}{r} \right)^4 \left( \frac{35}{8} \sin^4 \varphi - \frac{15}{4} \sin^2 \varphi + \frac{3}{8} \right) \right] \quad (18)$$

And  $\varphi$  is the latitude of space position,  $M_e$  the mass of the earth and  $r$  the distance from the center of the earth to a particular point of space. In standard earth model WGS84-EGM96,  $GM_e = 3.986004418 \times 10^5 \text{ km}^3/\text{s}^2$ ,  $a_e = 6378.137 \text{ km}$ . Using the relation  $z = r \sin \varphi$  and expressing the earth gravitational potential by harmonic item,

$$U = U_0 + U_2 + U_3 + U_4 \quad (19)$$

$$U_0 = \frac{GM_e}{r} \quad (20)$$

$$U_2 = \frac{GM_e}{r} \left[ -J_2 \left( \frac{a_e}{r} \right)^2 \left( \frac{3}{2} \left( \frac{z}{r} \right)^2 - \frac{1}{2} \right) \right] \quad (21)$$

$$U_3 = \frac{GM_e}{r} \left[ -J_3 \left( \frac{a_e}{r} \right)^3 \left( \frac{5}{2} \left( \frac{z}{r} \right)^3 - \frac{3}{2} \frac{z}{r} \right) \right] \quad (22)$$

$$U_4 = \frac{GM_e}{r} \left[ -J_4 \left( \frac{a_e}{r} \right)^4 \left( \frac{35}{8} \left( \frac{z}{r} \right)^4 - \frac{15}{4} \left( \frac{z}{r} \right)^2 + \frac{3}{8} \right) \right] \quad (23)$$

The calculation results of gradients of  $U$  are given by items below.

$$\nabla U_0 = -GM_e \frac{1}{r^3} (x, y, z)^T \quad (24)$$

$$\nabla U_2 = GM_e A_2 \left( \frac{15xz^2}{r^7} - \frac{3x}{r^5}, \frac{15yz^2}{r^7} - \frac{3y}{r^5}, \frac{15z^3}{r^7} - \frac{9z}{r^5} \right)^T \quad A_2 = \frac{1}{2} J_2 a_e^2 \quad (25)$$

$$\nabla U_3 = GM_e A_3 \left( \frac{35xz^3}{r^9} - \frac{15xz}{r^7}, \frac{35yz^3}{r^9} - \frac{15yz}{r^7}, \frac{35z^4}{r^9} - \frac{30z^2}{r^7} + \frac{3}{r^5} \right)^T \quad A_3 = \frac{1}{2} J_3 a_e^3 \quad (26)$$

$$\nabla U_4 = GM_e A_4 \times \left( \frac{63xz^4}{r^{11}} - \frac{42xz^2}{r^9} + \frac{3x}{r^7}, \frac{63yz^4}{r^{11}} - \frac{42yz^2}{r^9} + \frac{3y}{r^7}, \frac{63z^5}{r^{11}} - \frac{70z^3}{r^9} + \frac{15z}{r^7} \right)^T \quad (27)$$

$$A_4 = \frac{5}{8} J_4 a_e^4$$

The measurement model to decide the measurement  $Z$  from the state vector is

$$Z = h(X(t)) + v(t) \quad (28)$$

where  $v(t)$  is zero-mean white noise.

$$E(v(t_j)) = 0, \quad E(v(t_j)v^T(t_k)) = \mathbf{I} R^2 \delta_{jk}$$

When using the magnetometer data, the magnitude(measurement) of the geomagnetic field is

$$B_{mes} = \left( B_{bx}^2 + B_{by}^2 + B_{bz}^2 \right)^{1/2} \quad (29)$$

where  $B_{bx}$ ,  $B_{by}$ ,  $B_{bz}$  are the axial components of 3-axis magnetometer. Defining the magnitude of the geomagnetic field as measurement, the measurement model is as follows.

$$h(X(t)) = B(\mathbf{r}) = \left( B_r^2 + B_\theta^2 + B_\phi^2 \right)^{1/2} \quad (30)$$

where  $B_r$ ,  $B_\theta$ ,  $B_\phi$  are the spherical coordinate components in ECEF frame.

$$B_r = \sum_{n=1}^k \left( \frac{a}{r} \right)^{n+2} (n+1) \sum_{m=0}^n (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \quad (31)$$

$$B_\theta = - \sum_{n=1}^k \left( \frac{a}{r} \right)^{n+2} \sum_{m=0}^n (g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) \frac{\partial P^{n,m}(\theta)}{\partial \theta} \quad (32)$$

$$B_\phi = - \frac{1}{\sin \theta} \sum_{n=1}^k \left( \frac{a}{r} \right)^{n+2} \sum_{m=0}^n m (-g^{n,m} \cos m\phi + h^{n,m} \sin m\phi) P^{n,m}(\theta) \quad (33)$$

The meanings of symbols are as follows.

$(r, \theta, \phi)$ : earth centered spherical coordinates

$r$ : geocentric distance

$\theta$ : coangle of latitude

$\phi$ : longitude

$a$ : standard radius(6371.2km)

$g^{n,m}$ : Gaussian coefficients

$P^{n,m}$ : Schmidt semi-orthogonalized Legendre polynomial

Gaussian coefficients in IGRF (International Geomagnetic Reference Field) are varied with time and it is supposed that they are varied at constant rate for 5 years. The state transition model and measurement model are as follows.

$$\dot{X} = f(X(t), t) + w(t) \quad (34)$$

$$Z = h(X(t)) + v(t) \quad (35)$$

where  $w(t)$ : zero-mean white process noise,  $v(t)$  : zero-mean white measurement noise.

The EKF is applied to the state transition model and measurement model above.

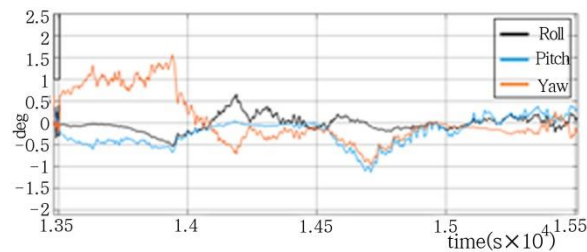
#### 4. Simulations

The micro-satellite circling the sun synchronous orbit at the height of 700km is supposed for simulation. The standard orbit data are as follows.

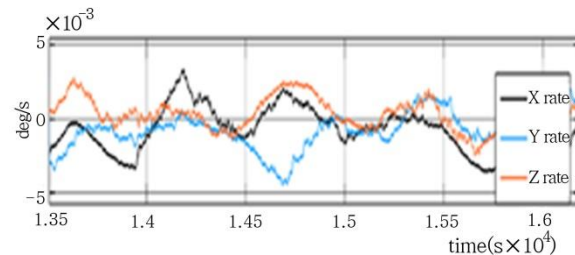
Epoch		
	(UTC)	: 2017/10/10 02:33:33.6286
	(TLE)	: 17283.106639219914
Orbital elements(TLE)		
Semi-major radius		: 7064.80364867094 km
Eccentricity		: 1.04530705730871E-03
Inclination		: 98.1581910505179°
Right ascension of ascending node		: 356.39561764568°
Argument of perigee		: 90°
Mean anomaly		: 50.4966213381126°
Mean motion		: 14.6110967250392 r/d
Ballistic drag coefficient		: 0

##### 4.1 The simulation result of attitude determination

The attitude determination algorithm of the micro-satellite using UKF is simulated for 20000s, and the effectiveness of this method is verified. Firstly, consider the estimation features of geomagnetic field vector and its derivatives.



**Figure 1.** The micro-satellite attitude estimation error feature by UKF.



**Figure 2.** The micro-satellite rate estimation error feature by UKF.

Fig.1 shows the attitude estimation error of micro-satellite. The black, blue and red lines represent roll, pitch and yaw, respectively. Fig.2 shows the rate estimation error of micro-satellite by UKF. Black, blue and red lines represent rates of x, y and z axes respectively.

As shown above, one can know that the attitude determination error is below  $1^\circ$  and the rate determination error is below  $0.005^\circ/\text{s}$  at  $1\sigma$ , thus this method is very accurate.

##### 4.2 Orbit determination simulation result

The satellite position is obtained by propagating the satellite orbit at a second intervals on the standard orbit, and the geomagnetic field is calculated based on the IGRF model, and then the noise of 200nT is added for simulating the observation data. The covariance matrix of initial error is supposed for the position and velocity, respectively, as follows.

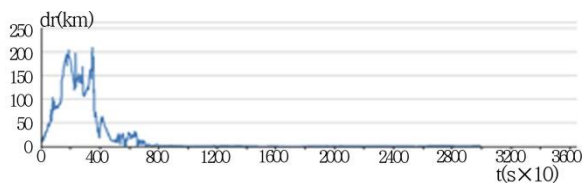
$$\sigma_r^2 = 100\text{km}^2$$

$$\sigma_{\dot{r}}^2 = 0.0001(\text{km/s})^2$$

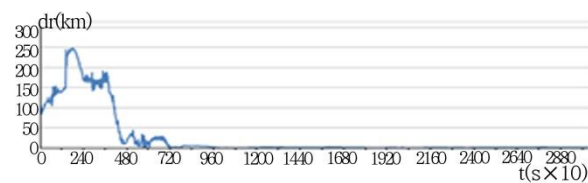
$$P_0 = \begin{pmatrix} \sigma_r^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_r^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_r^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\dot{r}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\dot{r}}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{r}}^2 \end{pmatrix}$$

The covariance matrix of process noise is set as  $Q = 0.0001P_0$  at the beginning and after 2000s, it is forced to decrease by 1/10 with every 1000 seconds in the duration of 100 minutes. Different deviations are given to the standard orbit to simulate the initial position and velocity.

Fig.3 shows the positional deviation between the standard orbit and estimation orbit, when the initial position and velocity deviation compared with standard orbit is 10km and 10m/s. The observation time is about 114 minutes, when the positional deviation is less than 10km.



**Figure 3.** Positional deviation between standard orbit and estimation orbit (initial position and velocity deviation 10km, 10m/s).



**Figure 4.** Positional deviation between standard orbit and estimation orbit (initial position and velocity deviation 100km, 10m/s).

Fig.4 shows the positional deviation between the standard orbit and estimation orbit when the initial position and velocity deviation with standard orbit is 100km and 10m/s, respectively. The observation time is about 116 minutes, when the positional deviation is less than 10km.

As shown in simulation, when the noise is less than 200nT, initial positional deviation is below 100km and velocity deviation is below 10m/s, the final positional deviation converges to below 10km after about 120 minutes.

## 5. Conclusion

In this paper, a method, only using the magnetometer data is presented to determine the attitudes and orbits. Different kinds of Kalman filters are used to solve these problems. Simulation results show that the attitudes and orbits determination accuracies are assured as expected for the micro-satellites.

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