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Use of Tetrahedral Finite Element Method for Computing the Gravitation of Irregular-Shaped Asteroid

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Abstract. Asteroids are one of the most important targets for deep space exploration. In previous asteroid missions, the accurate estimate of gravity has proved to have a strong influence on the design of the approach orbit and navigation strategy. The wield gravitational field of an asteroid is mainly caused by the irregular overall shape and possible heterogenous mass distribution the interior. We propose to use the finite element method to compute the gravity of irregularly shaped asteroids; this method combines the advantages of the conventional mascon method and the polyhedral method. The tetrahedral meshes can be generated following the conventional division technique. Taking asteroid 216 Kleopatra as an example, we calculate the exterior gravitational field using the above mentioned methodology. We then compare the results from the finite element method and the polyhedral method under a degenerated case, i.e., with constant density. Then, four different density distribution assumptions are given, and the gravitational fields are calculated respectively. The comparative study and the density distribution assumptions indicate that the proposed method is suitable for modeling an arbitrary asteroid with nonuniform mass distribution. This method is expected to provide reliable gravity data for the design of guidance, navigation, and control systems in future asteroid missions.

1. Introduction

The scientific exploration of asteroids has become a hot research topic in the field of deep space exploration. With the development of deep space exploration technology during the past 30 years, the technology of probing asteroids has been extended from Earth-based observation activities to launching unmanned probes to target asteroid sampling. The United States (NASA), Japan (JAXA), and the European Union have launched multiple probes for the probe of small objects since the 1990s. So far only three probes have successfully landed on asteroids, namely the NEAR Shoemaker [1], Muses-C [1] and Rosetta [1]. The Dawn probe [1] from NASA was launched in 2007 and is currently flying around Ceres. Japan launched Hayabusa 2 [2] in December 2014 and has reached the target asteroid 162173 Ryugu. Hayabusa 2 completed the first landing sampling at 23:06 (UTC) on February 21, 2019 and is expected to return to Earth in 2020. NASA launched OSIRIS-REx in 2016 and reached the target asteroid 101955 Bennu in 2018. OSIRIS-REx plans to land and sample in July 2020 and return to Earth in 2023 [3]. Moreover, the New Horizons New Field Vision was confirmed on

January 1, 2019 to have flown over the asteroid 486958 (Ultima Thule), which is the most distant celestial body ever probed by humans [4].

Compared with planets such as Earth and Jupiter, most asteroids are irregularly shaped; thus, their gravitational field is extremely complicated, which directly affects their orbital dynamics, and the navigation, guidance, and control of asteroid probe. The extended gravitational field of irregularly shaped asteroids is expressed and calculated through three main methods, namely, spherical harmonics, mascon method, and homogeneous polyhedron.

The spherical harmonics method is a classical method of expressing and calculating the gravitational field of any celestial body, and it has been widely used in the field of spacecraft orbital dynamics [5]. It uses infinite series to approximate the gravitational potential function; that is, a series of spherical harmonic functions is superimposed on the basis of the central gravitational term to describe the non-spherical perturbation of the gravitational field. The spherical harmonic function is widely used due to its rapid convergence and high computational efficiency. However, when it is used to describe the irregular gravitational field of asteroids, shortcomings are noticeable. First, the truncation error is enlarged as the distance to the asteroid shortens, and the convergence speed becomes slower and even becomes divergent within the Brillouin sphere. Second, the spherical harmonic method cannot determine whether the probe is inside or outside the asteroid and requires an extraordinary algorithm to determine it; then, its exact spherical harmonic coefficient must be determined by the actual data determined by the orbit. Although a series of improvements for the spherical harmonic function method, such as the ellipsoidal harmonic function method [6], the Brillouin interior/exterior spherical harmonic function [7], and the interior/exterior spherical Bessel function [8], has been proposed, it is the method itself that renders impossible the accurate description of the irregular gravitational field of an asteroid.

The mascon method [9] is an intuitive technique that describes the gravitational field of an asteroid; it uses the volume element (small sphere) to approximate the shape of the target asteroid, and each volume element is approximated as a mass point to calculate the gravitational field of the target asteroid. The mascon method is simple and easy to implement, and the precision of the gravitational field increases with the number of volume elements. The mass weight can be flexibly modified for asteroids with a nonuniform density distribution, and the method can simulate the structure of an asteroid realistically, but the drawback is also obvious. First, the convergence is slow. As the number of volume elements increases, the calculation increases sharply; thus, the error accumulation also becomes a problem. Second, as the calculation point approaches the surface of the asteroid, the error of the calculation increases [10], the spherical harmonic function rises; thus, collision detection is not possible.

The polyhedral method [11] uses a polyhedral model to approximate the shape of an irregular asteroid. With the assumption that the polyhedral density is uniform, the gravitational potential and gravitation are processed and obtained via integral transformation. Although assumed uniform density of the polyhedral method does not conform to the real situation, the gravitational force at the edge of the surface of the polyhedron will produce singularity, and the computational efficiency is lower than that of the spherical harmonic function, the expressions are all closed, thereby ensuring that the calculation error does not change with the relative distance of the asteroid. The error is related only to the model error (shape and density distribution errors), and collision detection can be performed to determine whether the calculation point enters the interior of the polyhedron. Therefore, the method has been used widely in gravitational field analysis, the approach orbit analysis and the orbital design of asteroid missions.

Numerous methods for calculating the gravitational field of irregular asteroids are available; however, when the density distribution is nonuniform, the above methods are not sufficient for accurately calculating the gravitational field. Therefore, by combining the advantages of the mascon and the homogeneous polyhedron methods, this study proposes a tetrahedral finite element method to estimate the gravitational field accurately under the condition of nonuniform density distribution; that is, the polyhedral model is filled with tetrahedrons based on the surface triangles. The gravitational potential and gravitation of each tetrahedral finite element are calculated by the homogeneous polyhedron method, and the total gravitational potential and gravitation are added by each value. Different density distribution model assumptions can be given according to the geological structure, or an accurate density distribution can be inverted by other data. Then, we can accurately obtain the gravitational field of an asteroid.

2. Methods

2.1. Mascon method

The mascon method uses the volume element (small sphere) to approximate the shape of the target asteroid, and each volume element is approximated as a mass point to calculate the gravitational field of the target asteroid.

Equations (1) and (2) show the gravitational potential and the gravitational acceleration

$$U = \sum_{i=1}^{N} \frac{GM_i}{|r - d_i|} \tag{1}$$

$$a = \sum_{i=1}^{N} \frac{GM_i (r - d_i)}{|r - d_i|^3} \tag{2}$$

Each voxel mass M_i satisfies

$$M_A = \sum_{i=1}^{N} M_i \tag{3}$$

where N is the number of volume elements, d_i is the position vector of the ith volume elements, M_i is the mass of the ith volume element, r is the position vector of the calculated point, G is the gravitational constant and M_A is the total mass of the asteroid.

2.2. Polyhedron

The polyhedral method was first applied in the field of geophysical science. In the mid-1990s, Werner and Scheeres [11], assuming the polyhedral density to be uniform, used polyhedrons to approximate the shape of asteroids and derived the expressions of the gravitational potential and the gravitation near the polyhedron.

Equations (4) and (5) show the gravitational potential and the gravitational acceleration.

$$U = \frac{1}{2} G\sigma \sum_{e \in edges} \mathbf{r}_e \bullet \mathbf{E}_e \bullet \mathbf{r}_e \cdot L_e - \frac{1}{2} G\sigma \sum_{f \in faces} \mathbf{r}_f \bullet \mathbf{F}_f \bullet \mathbf{r}_f \cdot \omega_f \tag{4}$$

$$\nabla U = -G\sigma \sum_{e \in edges} \mathbf{E}_e \bullet \mathbf{r}_e \cdot L_e + G\sigma \sum_{f \in faces} \mathbf{F}_f \bullet \mathbf{r}_f \cdot \omega_f \tag{5}$$

where G is the gravitational constant, σ is the density of the asteroid, e is the edge, and f is the face of the polyhedron, $\hat{n}_f$ is the normal vector of the fth face, F_f is the coefficient related to the face, $\hat{n}_e^f$ is the normal vector of the eth edge on the fth face, and E_e is the coefficient related to the edge. $\mathrm{F}_f = \hat{n}_f \hat{n}_f$,

$\mathrm{E}_e = \hat{n}_{f_1} \hat{n}_e^{f_1} + \hat{n}_{f_2} \hat{n}_e^{f_2}$

$$\mathrm{L}_e = \int_e \frac{1}{r} ds = \int_{P_i}^{P_j} \frac{1}{r} ds = \ln \frac{r_i + r_j + e_{ij}}{r_i + r_j - e_{ij}} \tag{6}$$

$$\omega_f = \iint_{triangle} \frac{\Delta z}{r^3} dS = 2 \arctan \frac{\mathbf{r}_i \bullet \mathbf{r}_j \times \mathbf{r}_k}{r_i r_j r_k + r_i (\mathbf{r}_j \bullet \mathbf{r}_k) + r_j (\mathbf{r}_k \bullet \mathbf{r}_i) + r_k (\mathbf{r}_i \bullet \mathbf{r}_j)} \tag{7}$$

2.3. Tetrahedral finite element

The tetrahedral finite element method combines the advantages of the mascon and the homogeneous polyhedron method, filling the polyhedral model with tetrahedrals based on the surface triangles. The gravitational potential and gravitation of each tetrahedral finite element are calculated by the homogeneous polyhedron method, and the total gravitational potential and gravitation is added by each value. Therefore, the expressions of the gravitational potential and the gravitational acceleration are

$$U=\frac{1}{2}G\sum_{l=1}^{n}\sigma_l\left(\sum_{e\in edges}\mathbf{r}_e^l\bullet\mathbf{E}_e^l\bullet\mathbf{r}_e^l\cdot L_e^l-\sum_{f\in faces}\mathbf{r}_f^l\bullet\mathbf{F}_f^l\bullet\mathbf{r}_f^l\cdot\omega_f^l\right) \tag{8}$$

$$\nabla U=-G\sum_{l=1}^{n}\sigma_l\left(\sum_{e\in edges}\mathbf{E}_e^l\bullet\mathbf{r}_e^l\cdot L_e^l-\sum_{f\in faces}\mathbf{F}_f^l\bullet\mathbf{r}_f^l\cdot\omega_f^l\right) \tag{9}$$

where the coefficients are similar to the coefficients of the polyhedral method.

2.4. The Polyhedron-FEM model

The modeling process of the tetrahedral finite element method requires the polyhedron to be filled up by the tetrahedral finite element. Based on a given polyhedral model, a tetrahedral unit of a certain size range is distributed to the interior until the internal space of the entire polyhedron is not empty, as shown in figure 1 (left).

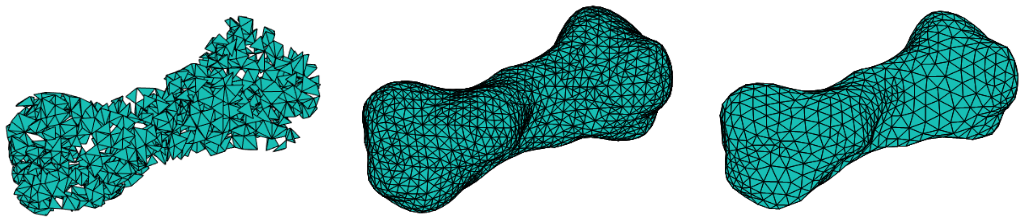

Figure 1. Model of tetrahedral finite element. (Left: tetrahedral distribution, middle: outer surface of original polyhedron model, right: outer surface of tetrahedral finite element model.)

However, the resulting triangular distribution of the outer surface of the polyhedron is not identical with the original polyhedral model, as shown in figure 1 (middle) and (right). Therefore, the data of the grid points and the triangle faces of the outer surface of the tetrahedral finite element model must be extracted to obtain the corresponding polyhedral model.

3. Discussion

In this section, the 216 Kleopatra asteroid is taken as an example for comparision with the polyhedron method to verify the effectiveness of the finite element tetrahedral method. As an example to make a reasonable density distribution hypothesis, the tetrahedral finite element can also calculate the gravitational field of asteroids accurately under the condition of nonuniform density.

The original polyhedral model consists of 2048 vertices, 4096 faces, and the dimensions of the three central inertia main axes are 217 km×94 km×81 km.

3.1. Case I: assumption of uniform density

First, the uniform density hypothesis is used to prove the validity of the tetrahedral finite element method. The average density is assumed to be 3.6 g/cm^3. The gravitational potential and gravitation are calculated at each field point from 150 km to the surface of the asteroid by the tetrahedral finite element and the polyhedron methods. As shown in figure 2, the finite element method divides three grids, with 6885, 37462, and 89029 tetrahedrons.

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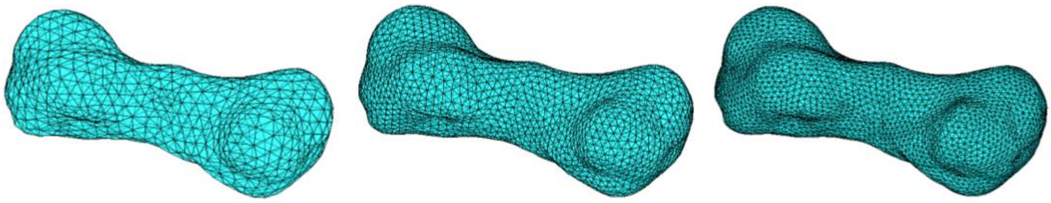

Figure 2. Three grids of finite element model. (Left: 6885 tetrahedrons, middle: 37462 tetrahedrons, right: 89029 tetrahedrons.)

Therefore, we need to know the accuracy of the calculation of the tetrahedral finite element.

We take the polyhedral method as the reference model. First, the accuracy between the tetrahedral finite element model under three kinds of meshes and the polyhedron model with the same outer surface (We call it finite element polyhedron below) are compared. Then the accuracy between the tetrahedral finite element model and the original polyhedron model under three kinds of grids are compared.

For the same field point, the error of the gravitational potential is defined as

$$\varepsilon=\frac{\left|U_t-U_p\right|}{\left|U_p\right|}\times 100\% \tag{10}$$

and the error of gravity is defined as

$$\nu=\frac{\left\|F_t-F_p\right\|}{\left\|F_p\right\|}\times 100\% \tag{11}$$

The direction error of the gravity is defined as

$$\delta=<F_p,F_t> \tag{12}$$

where U_t and F_t are the gravitational potential and gravity calculated by the tetrahedral finite element method, respectively; U_p and F_p are the gravitational potential and gravity calculated by the polyhedral method, respectively; and $<*,*>\in[0°,180°]$ is the angle between two gravitational vectors. Table 1 and table 2 present the error statistics for the two cases.

Figure 3 illustrates the contour of the gravitational potential of the x-y, x-z and y-z planes of the asteroid obtained by the finite element method of the 6885 tetrahedral elements.

Table 1. Gravity and gravitational potential error statistics of 216 Kleopatra calculated via the tetrahedral finite element and polyhedron models.

		Minimum	Maximum	Average	Standard deviation
6885 tetrahedrons	ε(%)	0	2.24×10^{-12}	2.69×10^{-13}	2.31×10^{-13}
	ν(%)	0	4.90×10^{-12}	7.42×10^{-13}	5.17×10^{-13}
	δ(°)	0	1.71×10^{-6}	2.05×10^{-7}	4.11×10^{-7}
37462 tetrahedrons	ε(%)	0	4.82×10^{-12}	5.27×10^{-13}	4.43×10^{-13}
	ν(%)	0	6.58×10^{-12}	1.12×10^{-12}	7.72×10^{-13}
	δ(°)	0	1.71×10^{-6}	2.06×10^{-7}	4.12×10^{-7}
89029 tetrahedrons	ε(%)	0	5.85×10^{-12}	4.11×10^{-13}	4.58×10^{-15}
	ν(%)	0	1.13×10^{-11}	1.28×10^{-12}	1.03×10^{-12}
	δ(°)	0	1.71×10^{-6}	2.04×10^{-7}	4.13×10^{-7}

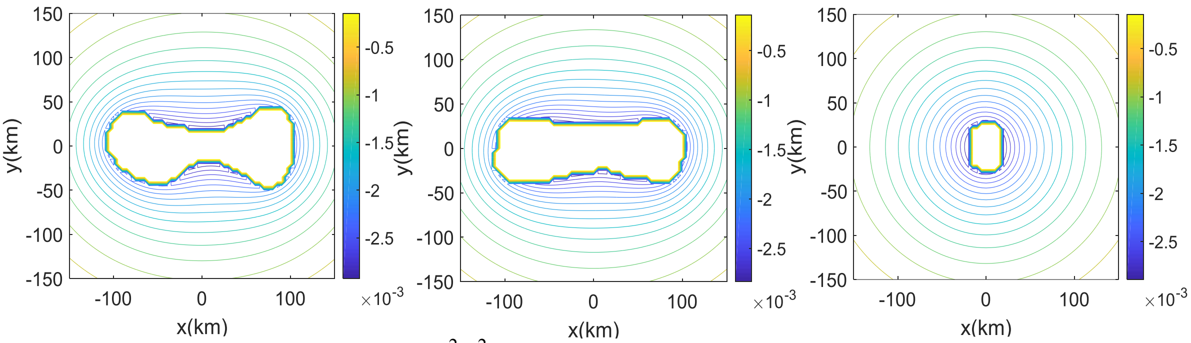

Figure 3. Gravitational potential (km^2/s^2) calculated via tetrahedral finite element. (Left: contour on x-y plane; middle: contour on y-z plane; right: contour on x-z plane)

Table 1 and figure 3 reveal that the results of the tetrahedral finite element method are nearly the same as those of the polyhedral method with the same polyhedron shape. The relative error values of gravitation and potential energy are within 10-12%, and the direction error in gravity is within 10-6° inches. We believe that the error comes from two sources. (1) Truncation error of used 64-bitcomputer: The gravitational value of each small tetrahedron is particularly small; hence, such an influence may occur. (2) Truncation error of the sum of the large and small numbers: The existing error is sufficient to prove the validity of the tetrahedral finite element method; thus, this error source is not further eliminated. In addition, the finer the mesh, the more the error accumulates; therefore, the mesh does not need to be further refined as long as the difference in gravity from the original polyhedron is appropriate.

Table 2 indicates that with the tetrahedral finite element mesh refinement, the average of the relative error of gravity and potential decreases from 1.1217% to 0.2474%, and the average of the direction error of the gravity decreases from 0.0477° to 0.0058°. This table shows that when the original polyhedral model is given, the gravitational field can be calculated by the tetrahedral finite element method. As the tetrahedral mesh becomes finer, the result becomes closer to the gravitational field of the original polyhedron.

Table 2. Gravity and gravitational potential error statistics of 216 Kleopatra calculated via tetrahedral finite element and original polyhedron models.

		Minimum	Maximum	Average	Standard deviation
6885 tetrahedrons	ε(%)	0	1.5259	1.1217	0.1807
	ν(%)	0	3.7216	1.1082	0.2243
	δ(°)	0	1.2591	0.0477	0.0544
37462 tetrahedrons	ε(%)	0	0.5482	0.4088	0.0648
	ν(%)	0	1.4306	0.4060	0.0776
	δ(°)	0	0.3554	0.0141	0.0165
89029 tetrahedrons	ε(%)	0	0.2979	0.2489	0.0388
	ν(%)	0	0.6409	0.2474	0.04376
	δ(°)	0	0.1805	0.0058	0.0088

3.2. Case II: assumption of density distribution model

This study assumes four density distributions that may exist in the asteroids. Their gravity field is calculated by the tetrahedral finite element method to illustrate the influence of different density distributions on the external gravitational field.

The mass of the asteroid 216 Kleopatra is determined to be 4.64×10^{18} kg, and the estimated average density given above is 3.6 g/cm^3. This condition is utilized as a constraint to generate a reasonable assumption of the density distribution.

The constraint of the total mass is

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$$M = \rho_0 V_0 + \sum_{i=1}^{N} (\rho_i - \rho_0) V_i \tag{13}$$

where M is the total mass of the asteroid, V_0 is the volume of the total asteroid shape, ρ_0 is the assumed nominal density of the asteroid, V_i is the volume of the ith internal addition, and ρ_i is the density of the ith internal addition.

216 Kleopatra is an M-type asteroid; thus, the part with the largest density can be assumed to be iron with a density of 7.5 g/cm^3, and the rest are assumed to be uniform density, which is constrained by mass (this study aims to verify the finite element method; thus, the total mass does not necessarily need to be strictly equal). And a simple way to determine the density of the tetrahedron is based on the centroid coordinates of the tetrahedron. The following four assumptions are made about the internal density distribution of 216 Kleopatra, as shown in figure 4.

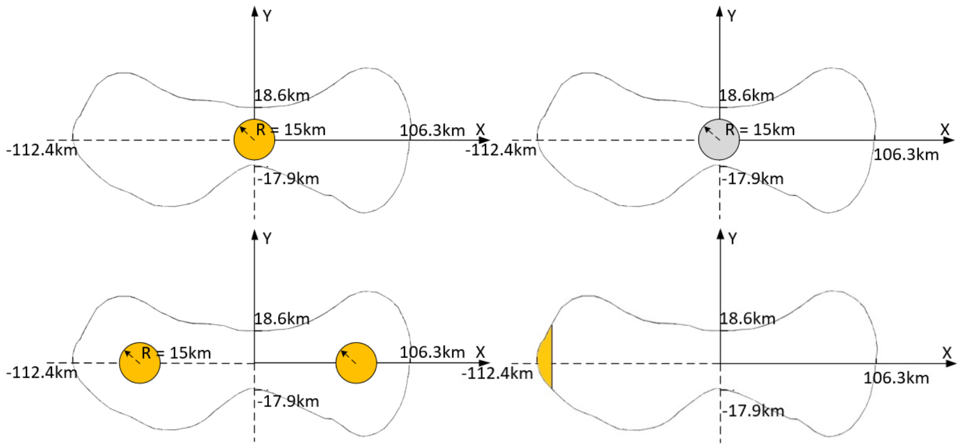

Figure 4. Single core model (top-left), air core model (top-right), binary core model (bottom-left), heavy mass model (bottom-right).

3.2.1. Single core model. Assume that the asteroid has a "nucleus" composed of iron with a density of 7.5 g/cm^3. As shown in the yellow part in figure 4 (top-left), the range of the core is assumed to be a sphere with a radius of 15 km, and the density of other portions is estimated to be 3.5 g/cm^3. Therefore, we can obtain the contour map of the gravitational potential on the equatorial plane in figure 5 and percentage contour map of the relative error compared with the normal density in figure 6. The figures show that the part with the largest relative error is approximately 4% near the core, whereas the error away from the core is within 0.5%, since the core density is significantly higher than the original density, which has a greater influence on the gravitational potential around it; however, the density concentration area in the core has a reduced influence on its surroundings.

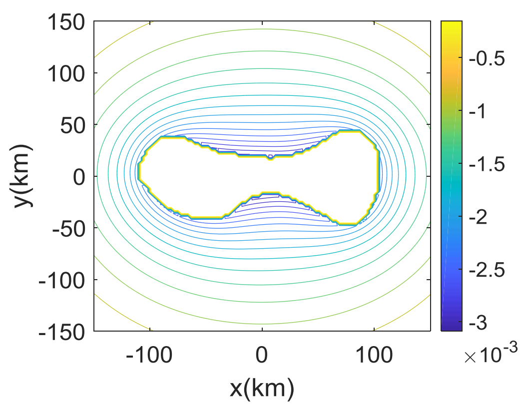

Figure 5. Potential contour map of single core model.

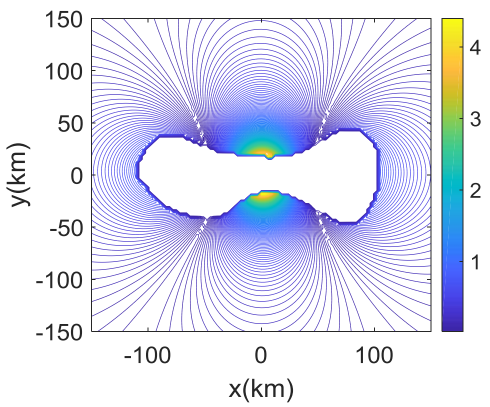

Figure 6. Error contour map of single core model.

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3.2.2. Air core model. Assume that there is an air core exists inside the asteroid with a density of 0 g/cm^3, and the range of the air core is also assumed to be a sphere with a radius of 15 km. Then, the density of the other parts is estimated to be 3.7 g/cm^3, as shown in the gray part in figure 4 (top-right).Therefore, we can obtain the contour map of the gravitational potential on the equatorial plane in figure 7 and the percentage contour map of the relative error compared with the normal density in figure 8. It can be seen from the figures that the part with the largest relative error is near the air core approximately 3% and at both ends at about 2%, and the error away from the core is within 0.5%.It can be understood that the air core greatly influences the gravitational field near itself and has a certain influence overall. Unlike the mass under uniform density, the mass of the two ends accounts for a larger proportion of the total mass; thus, the difference in uniform density will also increase.

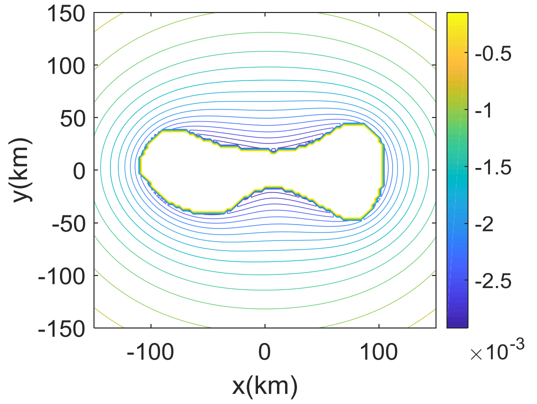

Figure 7. Potential contour map of air core model.

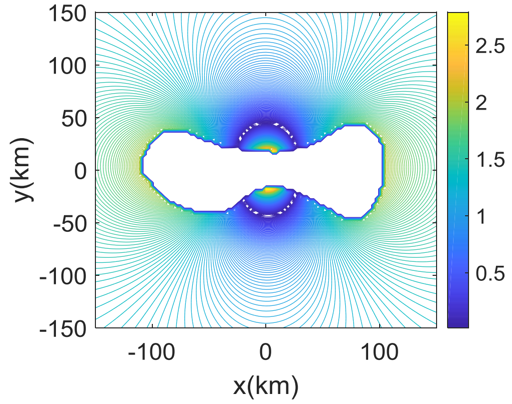

Figure 8. Error contour map of air core model.

3.2.3. Binary core model. Assume that the asteroid has two cores of iron on both sides, and its density is 7.5 g/cm^3. The range of each core is assumed to be a sphere with a radius of 15 km. As shown in the yellow part in figure 4 (bottom-left), the density of the other parts is estimated to be 3.5 g/cm^3.Therefore, we can obtain the contour map of the gravitational potential on the equatorial plane in figure 9 and the percentage contour map of the relative error compared with the normal density in figure 10 .The figures show that the part with the largest relative error is approximately 2.5% at both ends, and it can increase the error on the other parts. It can be understood that the "binary core" at both ends will not only have a great impact on the gravitational field near itself but will also have a certain impact overall. Nevertheless, the influence near the nucleus is relatively weaker than that of the single core, because part of the influence moves to other structures.

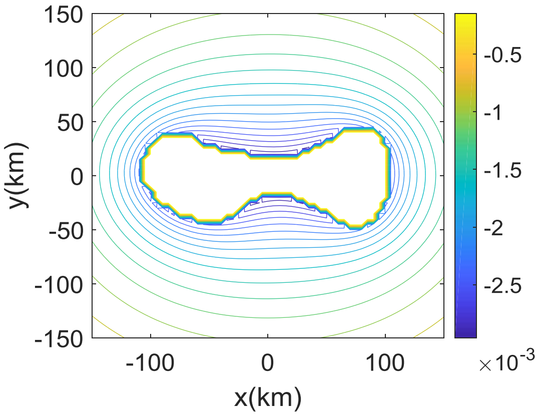

Figure 9. Potential contour map of binary core model.

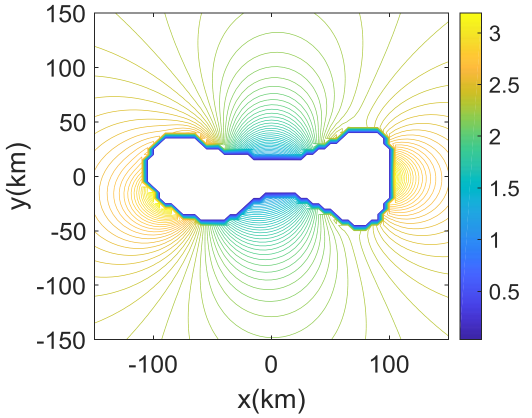

Figure 10. Error contour map of binary core model.

3.2.4. Heavy mass model. Assume that a heavy mass composed of iron exists on the surface of the asteroid, the density is 7.5 g/cm^3, and the heavy mass is located outside the −100km of the x-axis of the asteroid body coordinate system, as shown in the yellow part in figure 4 (bottom-right). The density of the other part is estimated to be 3.5 g/cm3. The contour map of the gravitational potential on its equatorial plane and the percentage contour map of the relative error compared with the normal density are obtained as shown in figures 11 and 12. The figures show that the part with the largest relative error of about 8% is near the heavy mass of the asteroid, whereas the relative error in other places is approximately 1%.

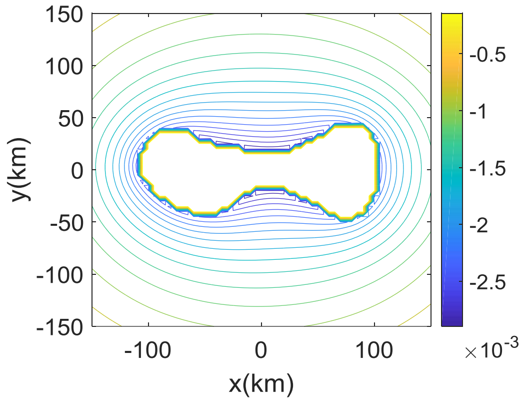

Figure 11. Potential contour map of heavy mass model.

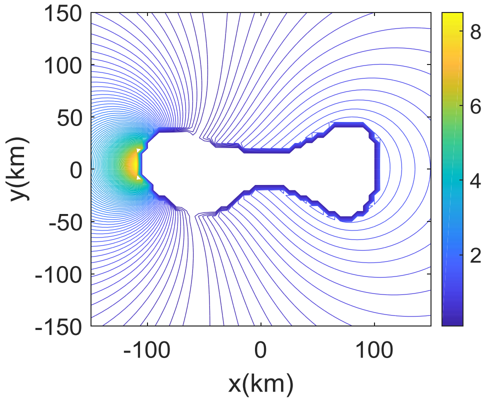

Figure 12. Error contour map of heavy mass model.

A large mass exists near the north pole of Ryugu, an asteroid being probed by JAXA [12]. It can be understood that when a heavy mass exists on the surface of the asteroid, the gravitational field near the mass will be highly different from the gravitational field with the uniform density. However, the mass only has a large effect on its vicinity. If the polyhedral method was used to calculate its gravitational field, it can only be assumed that its density is uniform. While the probe is approaching this asteroid, the true gravitational field will be different from the assumed gravitational field near this large mass. A mutation, which is highly dangerous for the probe, would occur in this field.

Therefore, if the density of the mass can be obtained, the tetrahedral finite element method can be used to obtain an accurate gravitational field of the Ryugu.

4. Conclusion

A tetrahedral finite element method is proposed to compute the gravity of irregularly shaped asteroids. Under the condition of constant density distribution, the tetrahedral finite element can be regarded as a refinement of the polyhedral method, and its essence is to calculate the gravitational field of the same polyhedral model. Therefore, in this case, the tetrahedral finite element method has the same advantages as the polyhedral method, but the calculation amount is slightly larger. The tetrahedral finite element model is unique advantageous when the density distribution is not uniform, because it can accurately calculate the external gravitational field; whereas the polyhedral model is not applicable, and the mascon and spherical harmonic methods have many limitations. The tetrahedral finite element model can also reflect the density distribution of the asteroid, which cannot be achieved by other methods.

When the density distribution inside the asteroid is not uniform, the part with the higher density will have a great influence on the gravitational field near itself, and it may also have a certain influence on the gravitational field of the whole asteroid. In particular, when the surface of an asteroid has a heavy mass, the gravitational field must only be accurately estimated by the tetrahedral finite element method.

Acknowledgments

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