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Stability Area of a Spacecraft's Partially Invariant Centre of Mass Motion Stabilization System

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Abstract. In practice, a spacecraft's stabilization system, which is partially invariant to the disturbing moment, is the easiest to implement. The velocity performance limitation of the control actuator has the most influence upon stability of the system among all the nonlinearities considered in the mathematical modelling. Therefore, we shall consider a model of an invariant stabilization system with due regard to nonlinearity. For the system under consideration, it is possible to construct lines of equal values of the auto oscillation amplitude in a two-parameter plane. The study of the stability of the proposed partially invariant stabilization system revealed that it is possible to ensure sufficient stability margins in the system under consideration by choosing parameters for the stabilization controller. At the same time, it allows to provide high quality of the transition process.

Keywords: *accuracy, spacecraft, stabilization, controller, nonlinearity.*

1. Introduction

Some publications [1-6] included synthesis of an invariant centre of mass motion stabilization system of a spacecraft and a comparative analysis of accuracy and quality of transition processes when conventional stabilization system is used. They demonstrated advantages of the proposed algorithms in terms of accuracy improvement.

An essential criterion determining practical application of the control system in addition to the control accuracy is a possibility to provide proper stability margins of the system. Consequently, the challenge is to select appropriate parameters for a stabilization controller. Therefore, a detailed study of the stability of the partially invariant algorithm proposed in [4-6] as well as selection of the proper parameters for the stabilization controller based on conducted studies, which shall provide sufficient stability margins for the system and the required quality characteristics for the transition process shall be the task to be solving herein.

The selection of stabilization controller parameters shall be more convenient and more informative if we build the system stability area in the stabilization controller parameter plane: for the angular stabilization channel and for the centre of mass motion stabilization channel. In practice, sufficient stability margins of the system shall be 1.5 – 2.0 time decrease (increase) margin of the system parameters with respect to their boundary stability conditions [7]. Therefore, ensuring of the specified stability margins is an essential criterion in the selection of parameters for automatic system in this research.

2. Problem formulation



Let us examine the stability of the proposed partially invariant stabilization system considering the influence of nonlinear elements of the system. As regards stability, the velocity performance limitation of the control actuator [8] has the most influence upon stability of the system among all the nonlinearities considered in the mathematical modelling [9, 10]. Therefore, we shall consider a model of an invariant stabilization system with due regard to nonlinearity. A block diagram corresponding to a model of the system allowing for nonlinearity is showing in Figure 1.

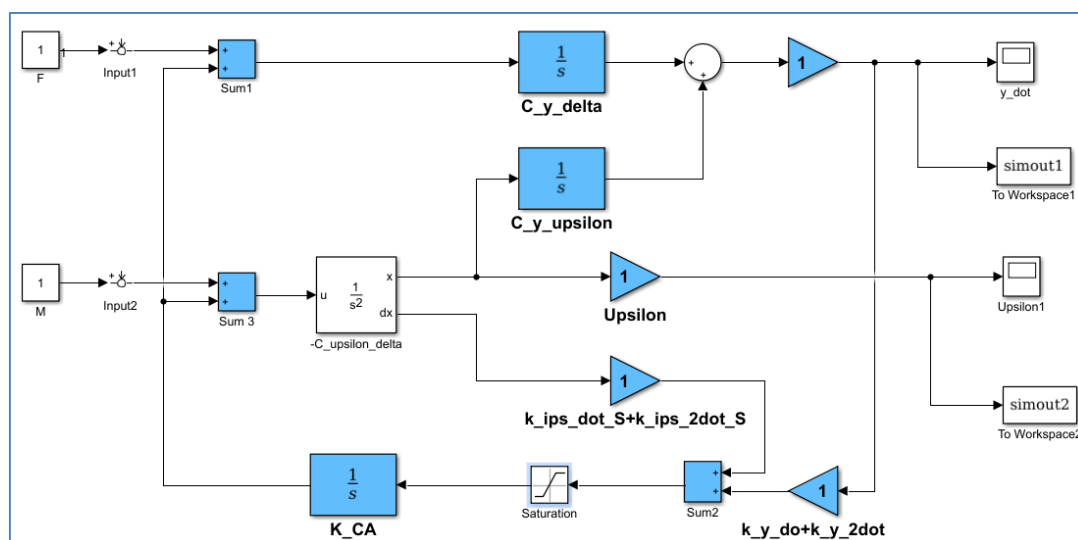


Figure 1. Block diagram of an invariant centre of mass motion stabilization system considering nonlinear velocity performance of the control actuator [6].

In order to study the nonlinear system we shall apply the method of harmonious balance [7]. For this purpose, we shall open the system at input of the linear element (Figure 2).

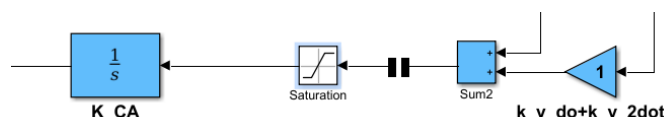


Figure 2. Break element at input of the nonlinear element.

We shall define the transfer functions for the linear and nonlinear parts of the system. According to the block diagram, the transfer function of the linear part of the system is as follows:

$$W_L(s) = \frac{K_{OD}}{s^4} \left[s^3 (C_{y\theta} k_{\ddot{y}} - C_{\theta\theta} k_{\ddot{\theta}}) + s^2 (C_{y\theta} k_{\dot{y}} - C_{\theta\theta} k_{\dot{\theta}}) - s C_{\theta\theta} C_{y\theta} k_{\ddot{y}} - C_{\theta\theta} C_{y\theta} k_{\ddot{\theta}} \right]. \quad (1)$$

For nonlinearity with a saturation zone, the expression for the coefficient of harmonic linearization is [9]:

$$\begin{cases} q = \frac{2}{\pi} \left(\arcsin \frac{I_N}{i_{\max}} + \frac{I_N}{i_{\max}} \left(1 - \frac{I_N^2}{i_{\max}^2} \right)^{1/2} \right), \text{when } i_{\max} > I_N; \\ q = 1, \text{when } i_{\max} \leq I_N. \end{cases} \quad (2)$$

where i_{\max} – the amplitude of the input is signal for the control actuator; I_N – is velocity performance saturation current for the control actuator.

According to the above formulas, the dependence of the coefficient of harmonic linearization q from the amplitude of the input signal i_{\max} is showing in Figure 3.

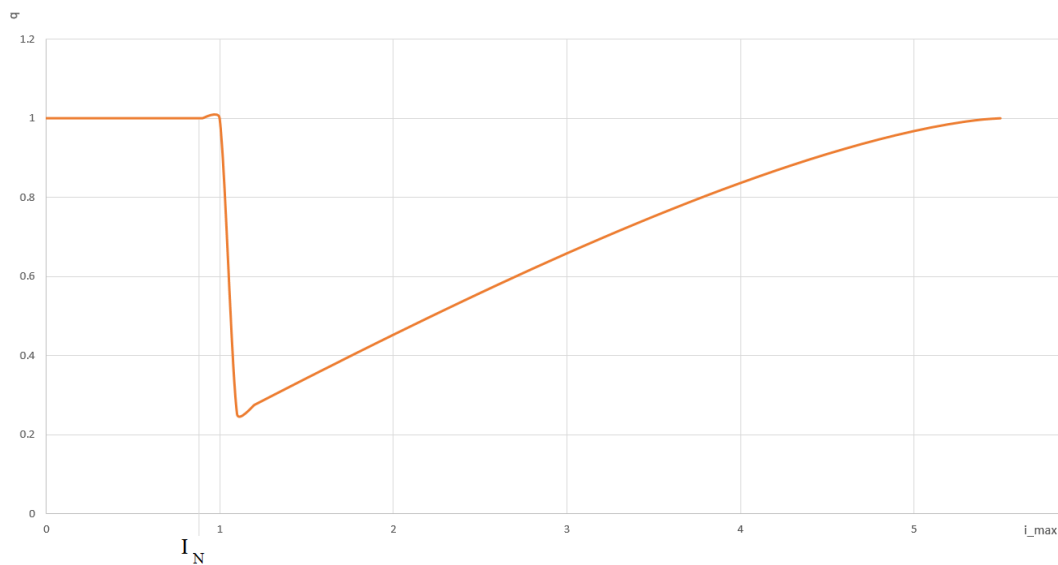


Figure 3. Dependence of the coefficient of harmonic linearization q from the signal amplitude i_{\max} .

3. Stability area of a spacecraft's partially invariant centre of mass motion stabilization system with due regard to velocity performance saturation zone of the control actuator

To analyse periodic solutions we shall use the graphical method [11]. For this purpose, we shall build a frequency hodograph for the transfer function of the linear part of the system. By substituting Laplace operator s for $j\omega$, in the expression $W_L(s)$ we obtain:

$$W_L(j\omega) = \frac{K_{OD}}{\omega^4} \left[j\omega^3 (-C_{y\delta}k_{\ddot{y}} + C_{g\delta}k_{\dot{g}}) + \omega^2 (-C_{y\delta}k_{\dot{y}} + C_{g\delta}k_{\dot{g}}) - j\omega C_{g\delta}C_{y\delta}k_{\ddot{y}} - C_{g\delta}C_{y\delta}k_{\dot{y}} \right]. \quad (3)$$

Let us introduce the following notations:

$$\alpha_1 = (C_{g\delta}k_{\dot{g}} - C_{y\delta}k_{\dot{y}}); \quad \alpha_2 = (C_{g\delta}k_{\ddot{g}} - C_{y\delta}k_{\ddot{y}}); \quad \alpha_3 = C_{y\delta}C_{g\delta}k_{\ddot{y}}; \quad \alpha_4 = C_{y\delta}C_{g\delta}k_{\dot{y}}. \quad (4)$$

Given the above notations, the expression (1) will be as follows:

$$W_L(j\omega) = \frac{K_{OD}}{\omega^4} \left[j\omega^3 \alpha_1 + \omega^2 \alpha_2 - j\omega \alpha_3 - \alpha_4 \right]. \quad (5)$$

We shall separate the real and imaginary parts from the expression (5):

$$\text{Re}(W_L) = \frac{K_{OD}}{\omega^4} (\omega^2 \alpha_2 - \alpha_4) \quad (6)$$

$$\text{Im}(W_L) = j \frac{K_{OD}}{\omega^4} (\omega^3 \alpha_1 - \omega \alpha_3). \quad (7)$$

The characteristic equation for the linear part of the open-loop system has four zero roots, so when the frequency ω changes between -0 and $+0$ the hodograph of the transfer function $W_L(j\omega)$ will move along a circle with infinitely large radius with $\omega_1 = +180^\circ$ till $\omega_2 = -180^\circ$.

If $\omega = 0$ the expressions (6) and (7) will take on values $-\infty: \text{Re}(W_L)_{\omega=0} = -\infty; \text{Im}(W_L)_{\omega=0} = -\infty$. In this case, the phase shift of frequency hodograph will be:

$$\text{tg} \varphi_{\omega=0} = \frac{\text{Im}(W_L)_{\omega=0}}{\text{Re}(W_L)_{\omega=0}} = \frac{\omega(\omega^2 \alpha_1 - \alpha_3)}{\omega^2 \alpha_2 - \alpha_4} \bigg|_{\omega=0} = 0; \varphi_{\omega=0} = -180^\circ. \quad (8)$$

If $\omega = +\infty$ the expressions (6) and (7) will take on values $0: \text{Re}(W_L)_{\omega=+\infty} = 0; \text{Im}(W_L)_{\omega=+\infty} = 0$.

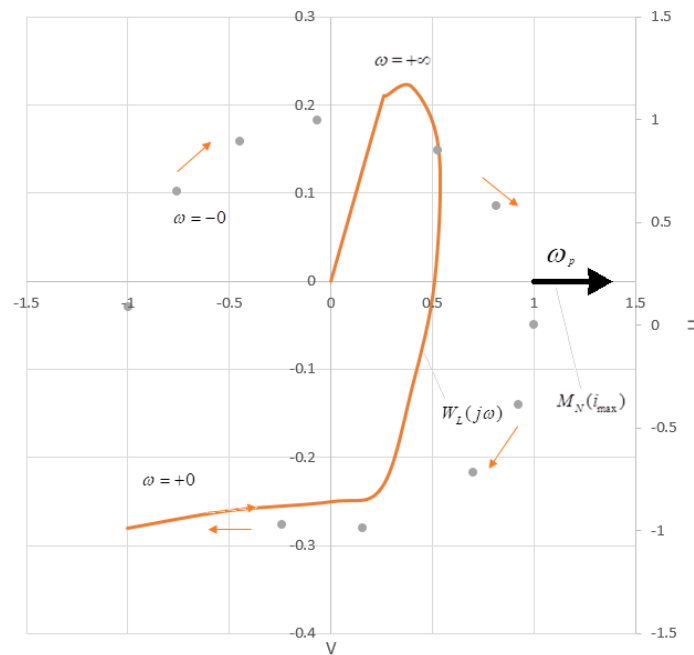


Figure 4. The amplitude-frequency characteristics of the partially invariant spacecraft centre of mass stabilization system.

Thus, if frequency ω changes from 0 to $+\infty$, the transfer function hodograph $W_L(j\omega)$ beginning in the third quadrant with the phase shift $\varphi = -180^\circ$, while moving through the 4th and 1st quadrants reaches the origin of coordinates. In this case, since closure of the system was made through +1 and not through -1, it is necessary that the branch of the phase and gain performance corresponding to the positive frequencies, together with a fragment of the circle beginning with a positive real semi axis doesn't cover the point $(+1; 0j)$ to provide stability of the closed-loop system [12].

The existence of periodic solutions in the system can be determined from the condition $W_L(j\omega)W_N(i_{\max}) = 1$, where $W_N(i_{\max})$ a transfer function of the nonlinear element is, or otherwise:

$$W_L(j\omega) = \frac{1}{q(i_{\max})}. \quad (9)$$

According to (6) and (7), condition (8) shall be split into two conditions:

$$\text{Im}(W_L) = 0 \quad (10)$$

$$\text{Re}(W_L) = \frac{1}{q(i_{\max})}. \quad (11)$$

By substituting the expression for the imaginary part of the transfer function of the linear part of the system into the condition (8), we get:

$$\frac{K_{OD}}{\omega^4} (\omega^3 \alpha_1 - \omega \alpha_3) = 0, \quad (12)$$

where from $\omega^2 \alpha_1 - \alpha_3 = 0$, or

$$\omega = \left(\frac{\alpha_3}{\alpha_1} \right)^{1/2}. \quad (13)$$

The determined frequency is the frequency of a periodic solution for the given nonlinear system. Further, we will denote it as ω_p . The self-oscillation amplitude in the system under consideration i_{\max_p} shall be found from the expression (13):

$$\frac{K_{OD}}{\omega_p^4}(\omega_p^2 \alpha_2 - \alpha_4) = \frac{1}{q(i_{\max_p})}, \quad q(i_{\max_p}) = \frac{\omega_p^4}{K_{OD}(\omega_p^2 \alpha_2 - \alpha_4)}. \quad (14)$$

By substituting the expression (2) for $q(i_{\max_p})$ into the last formula, we finally obtain:

$$\frac{2}{\pi} \arcsin \left(\frac{I_N}{i_{\max_p}} \right) + \frac{I_N}{i_{\max_p}} \left(1 - \frac{I_N^2}{i_{\max_p}^2} \right)^{1/2} = \frac{\omega_p^4}{K_{OD}(\omega_p^2 \alpha_2 - \alpha_4)}. \quad (15)$$

The expression (15) is transcendental relative i_{\max_p} , therefore, it can be solved only numerically.

The value i_{\max_p} can be approximately determined from the diagram $q(i_{\max})$ (Figure 4).

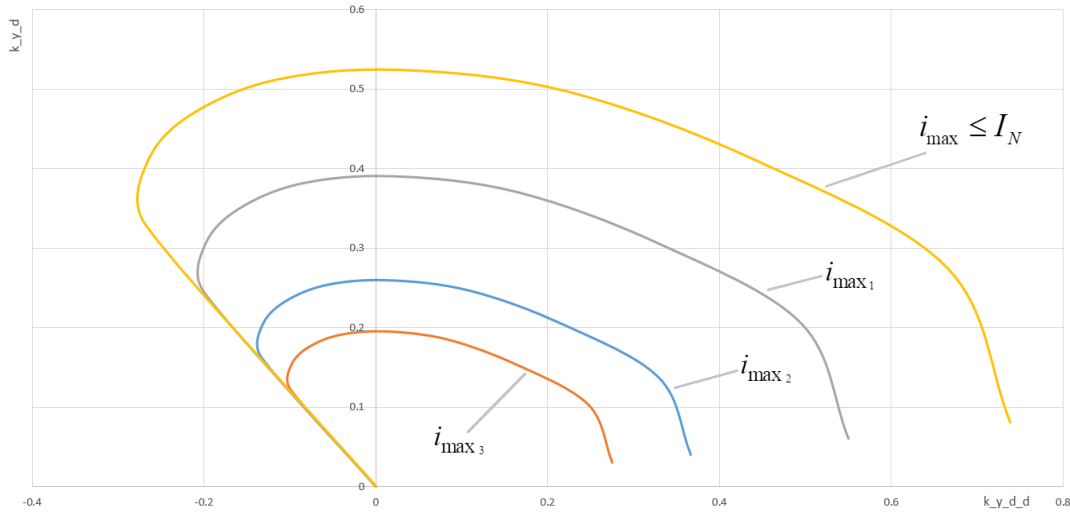


Figure 5. Stability area limits of a partially invariant stabilization system in the parameter plane k_y, k_d with due regard to velocity performance saturation zone of the control actuator.

Now we shall consider stability of periodic solution. To this end, we shall build a hodograph of the function $M_N = \frac{1}{q(i_{\max})}$ in the plane UV (Figure 5). According to (2), when $0 \leq i_{\max} \leq I_N, M_N = 1$, if

$i_{\max} > I_N, M_N$ monotonically increases to $+\infty$. Since hodograph M_N crosses the amplitude-frequency characteristic of the linear part of system “from the outside to the inside” [13], then the periodic solution of the system under study (i_{\max_p}, ω_p) will be unstable. Therefore, when $i_{\max} < i_{\max_p}$ the system solution will converge to 0, that is, the system will be stable, and if $i_{\max} > i_{\max_p}$ we will observe a divergent transient process, that is, the system will be unstable.

Thus, the stability condition for the system in general shall be determined by the initial conditions of the transition process, at which the amplitude i_{\max} at the input of the nonlinear element is less than the value i_{\max_p} :

$$i_{\max} < i_{\max_p}. \quad (16)$$

For the system under consideration, it is possible to construct lines of equal values of the auto oscillation amplitude in a two-parameter plane. Such parameters will be the gain of the angular

velocity and angular acceleration stabilization controller $k_{\dot{\theta}}, k_{\ddot{\theta}}$. The following can be obtained from (15) and (16):

$$\omega_p = \left(\frac{\alpha_3}{\alpha_1} \right)^{1/2} = \left(\frac{C_{\theta\delta} C_{y\delta} k_{\ddot{\theta}}}{C_{\theta\delta} k_{\ddot{\theta}} - C_{y\delta} k_{\ddot{\theta}}} \right)^{1/2} = \left(\frac{C_{\theta\delta}}{\frac{C_{\theta\delta}}{C_{y\delta}} - 1} \right)^{1/2}. \quad (17)$$

Let us set $c_1 = \left(\frac{C_{\theta\delta}}{C_{y\delta} k_{\ddot{\theta}}} \right)^{1/2}$, then $\omega_p = \left(\frac{C_{\theta\delta}}{c_1 k_{\ddot{\theta}} - 1} \right)^{1/2}$. From $\frac{K_{OD}}{\omega_p^4} (\omega_p^2 \alpha_2 - \omega \alpha_4) = \frac{1}{q(i_{\max_p})}$, the expression (11):

$$\frac{K_{OD}}{\omega_p^4} (\omega_p^2 \alpha_2 - \omega \alpha_4) = \frac{1}{q(i_{\max_p})}, \quad (18)$$

from which

$$\alpha_2 = \left[\frac{\omega_p^4}{K_{OD} q(i_{\max_p})} + \alpha_4 \right] \frac{1}{\omega_p^2}. \quad (19)$$

By expanding the expression for α_2 , we get after transformation:

$$k_{\dot{\theta}} = \frac{\frac{1}{\omega_p^2} \left[\frac{\omega_p^4}{K_{OD} q(i_{\max_p})} + \alpha_4 \right] + C_{y\theta} k_{\ddot{\theta}}}{C_{\theta\delta}}. \quad (20)$$

After substituting in the expression for ω_p :

$$k_{\dot{\theta}} = \frac{\frac{C_1 k_{\ddot{\theta}} - 1}{C_{\theta\delta}} \left[\frac{C_{\theta\delta}^2}{K_{OD} q(i_{\max_p}) (C_1 k_{\ddot{\theta}} - 1)^2} + \alpha_4 \right] + C_{y\theta} k_{\ddot{\theta}}}{C_{\theta\delta}}. \quad (21)$$

After transformation, we get:

$$k_{\dot{\theta}} = \frac{1}{K_{OD} q(i_{\max_p}) (C_1 k_{\ddot{\theta}} - 1)} + \alpha_4 \frac{C_1 k_{\ddot{\theta}} - 1}{C_{\theta\delta}^2} + \frac{C_{y\theta}}{C_{\theta\delta}} k_{\ddot{\theta}}. \quad (22)$$

By expanding the expressions for α_4 and c_1 , we finally have:

$$k_{\dot{\theta}} = \frac{C_{y\theta} k_{\ddot{\theta}}}{K_{OD} q(i_{\max_p}) (C_{\theta\delta} k_{\ddot{\theta}} - C_{y\theta} k_{\ddot{\theta}})} + \frac{k_{\ddot{\theta}}}{k_{\ddot{\theta}}}. \quad (23)$$

It should be noting that the right-hand side of expression (23) is different from the right side of the condition (10), which determines the stability limit for the linear systems only by the presence of harmonic linearization coefficient in the denominator of the first term. Since $q(i_{\max}) \leq 1$, then the presence of nonlinearity $I_N < i_{\max_1} < i_{\max_2} < i_{\max_3}$ will lead to narrowing of the stability area according to the parameters $k_{\dot{\theta}}, k_{\ddot{\theta}}$.

Accordingly, the expression for the stability limits according to the parameters $k_{\dot{\theta}}, k_{\ddot{\theta}}$ will be

similar to the right side of inequality (16) taking into account the coefficient of harmonic linearization $q(i_{\max})$:

$$k_{\ddot{y}} = -\frac{C_{y,g}k_{\ddot{y}}^2}{K_{OD}q(i_{\max_p})(C_{g,\delta}k_{\ddot{y}} - C_{y,g}k_{\ddot{y}})k_{\ddot{y}}} + \frac{k_{\ddot{y}}}{k_{\ddot{y}}}k_{\ddot{y}}. \quad (24)$$

A general view of the stability area in the parameter plane $k_{\ddot{y}}, k_{\ddot{y}}$ for different amplitude values at the input of the nonlinear element is shown in Figure 5.

As can be seen from the expressions (13)-(14) and Figure 5, stability of the invariant stabilization system considering nonlinearity and having a saturation zone, will directly depend on the value of the input signal in the nonlinear element, and ultimately on the disturbances acting on the spacecraft [14]. Therefore, in order to use such a stabilization system it is necessary to choose in practice such parameter values of the control action under which the system will be stable under possible perturbations. On the other hand, we will need to determine for the system under study conditions for its absolute stability regardless of perturbation.

According to [10] the absolute stability area shall be determined as an area of system parameters, which obviously escapes periodic solutions, and in which any transition process fades under any initial conditions to the value $x = 0$.

To determine an absolute stability area we can apply any linear stability criterion to the harmonically linearized system.

Let us apply the Hurwitz criterion to the system. According to the block diagram (Figure 1) and the expression for the transfer function of the linear part of the system (1), a characteristic equation for a harmonically linearized system will be:

$$s^4 + K_{OD} = \left[s^3 (C_{g,\delta}k_{\ddot{y}} - C_{y,g}k_{\ddot{y}}) + s^2 (C_{g,\delta}k_{\ddot{y}} - C_{y,g}k_{\ddot{y}}) + sC_{g,\delta}C_{y,g}k_{\ddot{y}} + C_{g,\delta}C_{y,g}k_{\ddot{y}} \right] q(i_{\max}) = 0. \quad (25)$$

Let us introduce the following notations:

$$\beta_1 = (C_{g,\delta}k_{\ddot{y}} - C_{y,g}k_{\ddot{y}})K_{OD}; \quad \beta_2 = (C_{g,\delta}k_{\ddot{y}} - C_{y,g}k_{\ddot{y}})K_{OD}; \quad \beta_3 = C_{y,g}C_{g,\delta}k_{\ddot{y}}K_{OD}; \quad \beta_4 = C_{y,g}C_{g,\delta}k_{\ddot{y}}K_{OD}. \quad (26)$$

Then the characteristic equation for the linearized system can be written as:

$$s^4 + s^3\beta_1q + s^2\beta_2q + s\beta_3q + \beta_4q = 0. \quad (27)$$

The Hurwitz conditions for the characteristic equation of the fourth order [8]:

$$q\beta_1 > 0; q\beta_2 > 0; q\beta_3 > 0; q\beta_4 > 0; \beta_3q(\beta_1\beta_2q^2 - \beta_3q) - \beta_4\beta_1^2q^3 > 0. \quad (28)$$

To ensure absolute stability of the system it is necessary that the above conditions be satisfied for any admissible values of the coefficient of harmonic linearization q . For the system under study $0 \leq q \leq 1$. Obviously, with the boundary condition $q = 0$ of the Hurwitz conditions for this system shall be not performed. Therefore, this system escapes absolute stability with any parameters. This is also confirmed by the graphic method of periodic solutions investigation. Let us refer to Figure 4. To ensure stability of a linear system it is necessary that the frequency hodograph for the transfer function of the linear part of the system $W_L(j\omega)$ should not cover point $(+1; 0j)$. When frequency changes from

0 to $+\infty$. On the other hand, the hodograph of the function $\frac{1}{q(i_{\max})}$ when q changes from 0 to 1 goes

through a real positive semi axis from $+\infty$ to $+1$. According to the above studies, the point of intersection of the frequency hodograph $W_L(j\omega)$ and the hodograph of the function $\frac{1}{q(i_{\max})}$ is a

point of unstable periodic solutions and provided the conditions for the stability of the nonlinear

system fulfilled (when the hodograph does not cover point $(+1;0j)$), there will be always an area of unstable periodic solutions.

4. Conclusion

Based upon research results we can conclude the following:

- 1) It is impossible to implement a centre of mass stabilization system, which is invariant regarding both the disturbing force and disturbing moment.
- 2) In practice, a stabilization system, which is partially invariant to the disturbing moment, is the easiest to implement. In order to comply with the invariance conditions, there must be a positive control actuator feedback with a gain equal to the object's angular deflection gain of in the angular stabilization channel. Stability of the system shall be ensuring by introduction of an additional second derivative action from the object's deflection angle into the action as well as by introduction of an equivalent delay loop in the feedback of control actuator in order to compensate for the dynamic delay of the stabilization controller.
- 3) It is also possible to synthesize a stabilization system, which shall be partially invariant fewer than two disturbances simultaneously. Open feedback of the control actuator and exclusion of control according to object's deflection angle and of the spacecraft centre of mass drift coordinate from the angular stabilization channel are the invariance conditions in this case. Such a stabilization system has obvious advantages over a system, which is invariant under disturbing moment M , and therefore it is more suitable for practical implementation.
- 4) A partially invariant under two disturbances stabilization system provides a significant increase (several times) in the accuracy of the centre of mass tangential stabilization velocities as compared to known stabilization systems.
- 5) The tangential velocity transition process in a partially invariant stabilization system has a significantly shorter (several times) decay time as compared to known stabilization systems.
- 6) Employment of additional self-regulation elements in a partially invariant stabilization system reveals the significant advantages of such a system in terms of greater accuracy when compared to known stabilization systems.
- 7) The study of the stability of the proposed partially invariant stabilization system revealed that it is possible to ensure sufficient stability margins in the system under consideration by choosing parameters for the stabilization controller. At the same time, it allows to provide high quality of the transition process.
- 8) The system under consideration has an unstable limit cycle due to "saturation zone" nonlinearity, which cannot be excluding by system parameter adjustment. It means the system is missing the absolute stability area.

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