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# An Improved Regression Analysing Method for Multi-position Calibration Test of Gyrowheel System

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**Abstract.** Gyrowheel is an innovative rate sensing and momentum management instrument. It could measure two-axes external angular rates and provide three-axes control torques at the same time. In order to improve the rate sensing performance, calibration tests are necessary to identify error terms and acquire compensation equations. However, there are too many possible error terms which make the calibration tests too complex to carry out. Considering that some of the error terms might be neglected without decreasing the compensation accuracy, proper test procedure to identify and neglect insignificant error terms is of great help to simplifying calibration tests and compensation equations. In this paper, a method to figure out and exclude insignificant error terms based on variance analysis is proposed. Test result shows that this method can simplified the form of the compensation equations without decreasing the compensation accuracy.

## 1. Introduction

Gyrowheel is an innovative attitude determination and control instrument, it provides control torques about the whole three axes and measures the angular rates about two axes of a spacecraft at the same time. Because of the multi-role ability of Gyrowheel, it can remarkably reduce the size, mass and power of attitude control system of a spacecraft, leading to the reduction of the ultimate costs for the spacecraft[1-4].

When acting as an angular rate sensor, Gyrowheel system operates like a Dynamic Tuned Gyroscope (DTG). Because of the imperfections of the instrument, angular rate output could be affected by gravity acceleration. Multi-position calibration tests have been widely used to identify and eliminate such errors[5-7]. These calibration tests focus on how to identify different error terms related to gravity acceleration input. However, some of these error terms may have comparingly slight influence on the accuracy of compensation equations. Considering the complexity of tilt-condition tests for Gyrowheel system, proper method to simplify the procedure of multi-position tests is required.

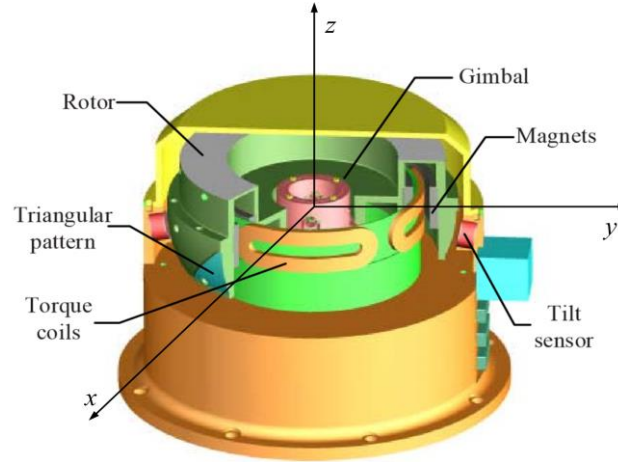
In this paper, the resource of the error terms of Gyrowheel system about gravity acceleration is analysed. A method to identify these error terms based on multi-position tests is studied, and the way to figure out insignificant error terms is proposed. Test results show that this process can help to neglect insignificant error terms effectively without decreasing the compensation accuracy.

## 2. Rate Sensing Principle of the Gyrowheel System

Figure.1 shows a cut-away view of the Gyrowheel. The basic structure of Gyrowheel system is similar to a DTG, except that its rotor is able to tilt in a range of 5 degrees and change the spinning speed. The spinning axis of the rotor is defined as the z axis and x and y axes are the two sensitive axes. The rotor



spins along the z-axis and torque coils could apply control torques along x and y axes. By altering the tilt angle and spinning speed of the rotor, Gyrowheel could provide three-axes control torques. And when there is angular rate input along x or y axis, the currents of the torquers would be altered to maintain the tile angle. As a result, the angular rate input could be measured by processing the current data of two-axes torquers.



**Figure 1.** Cut-away view of the Gyrowheel.

The rate sensing equations can be written as (1) and (2) after reasonable simplification. Where  $\omega_{cX}$ ,  $\omega_{cY}$  represent angular input along two axes.  $\Phi_x$ ,  $\Phi_y$  are tilt angles of the rotor,  $\omega_z$  is the spinning speed and  $i_x$ ,  $i_y$  represent the torque coils current.  $k_{scale,x}$ ,  $k_{scale,y}$  are the scale factors and  $I_{t,x}$ ,  $I_{t,y}$ ,  $c_{g,x}$ ,  $c_{g,y}$  could be just regarded as factors determined by parameters of the prototype.

$$\omega_{cX} = \frac{1}{\omega_z} [k_{scale,x} i_x - c_{g,y} \dot{\Phi}_y - K_{dyn,y} \Phi_y] - I_{t,x} \dot{\Phi}_x + 2c_{g,x} \Phi_x \quad (1)$$

$$\omega_{cY} = \frac{1}{\omega_z} [k_{scale,y} i_y - c_{g,x} \dot{\Phi}_x - K_{dyn,x} \Phi_x] - I_{t,y} \dot{\Phi}_y - 2c_{g,y} \Phi_y \quad (2)$$

Design limitations and constructional deficiencies could lead to disturb torques which produce errors when measuring angular rates. The negative effect of these disturb torques might be reduced by compensation equations which could be obtained by implementing calibration tests. Similar to the DTG, the drifting errors could be categorized as: g insensitive terms, g sensitive terms and g-squared sensitive terms. It has been widely accepted that the g-squared sensitive terms are in most cases negligible. However, in order to function as a momentum actuator, the Gyrowheel have significantly larger rotor mass and inertia compared to DTGs. That leads to comparingly larger g-squared sensitive terms which may not be neglected entirely. Thinking of all these error terms, the Gyrowheel error model for x axis can be given as (3), and equations for y axis have the same form.

$$\begin{aligned} \omega_{dX} = & D(x)_0 + D(x)_x g_x + D(x)_y g_y + D(x)_z g_z \\ & + D(x)_{xy} g_x g_y + D(x)_{yz} g_y g_z + D(x)_{xz} g_x g_z \\ & + D(x)_{xx} g_x^2 + D(x)_{zz} g_z^2 \end{aligned} \quad (3)$$

Where  $\omega_{dX}$  is drifting angular rates along the two sensitive axes.  $D(x)_0$ , represents the g insensitive error terms,  $D(x)_i$ ,  $i = x, y, z$  are the g sensitive error coefficients,  $D(x)_{ij}$ ,  $ij = xx, zz, xy, yz, xz$ , are all the possible g-squared sensitive error coefficients, and  $g_x, g_y, g_z$  represent components of gravity vector along the three axes.

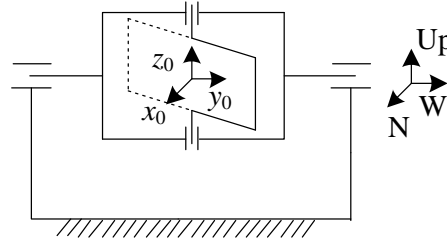
In the early process of the calibration tests, the Gyrowheel is actualized based on the null tilt condition in order to maintain a relatively high accuracy. The research in this paper is also done under this condition. In the null tilt condition, after incorporating error terms into the rate sensing equation, the equation becomes (4), which is the original form of the compensation equation for x axis.

The model listed above considered every possible g sensitive and g-squared sensitive error terms. However, the importance of each error terms is different and some terms may be negligible. The final equations used to compensate the error terms may have a simpler form. A method to work out the insignificant error terms based on multi-position tests will be discussed in following sections.

$$\begin{aligned} \omega_{cX} = & k_x i_x - D(x)_0 - D(x)_x g_x - D(x)_y g_y - D(x)_z g_z \\ & - D(x)_{xy} g_x g_y - D(x)_{yz} g_y g_z - D(x)_{xz} g_x g_z \\ & - D(x)_{xx} g_x^2 - D(x)_{zz} g_z^2 \end{aligned} \quad (4)$$

### 3. Multi-Position Test

In order to identify different error coefficients in (4), multi-position tests should be carried out. Multi-position tests use a two-axes motion table to provide precise position reference. The rotation rate of the earth and components of gravity vector are regarded as nominal inputs of the Gyrowheel. Before carrying out a test, the Gyrowheel y axis and z axis are aligned with the table elevation and azimuth axes respectively. The motion table is implemented that its elevation axis is in the west direction and azimuth axis directs upward, as shown in Figure.2.



**Figure 2.** Diagram of the two-axis motion table.

Defining that  $\psi$ ,  $\Phi$  are the azimuth and elevation angles respectively,  $\omega_e$  is the earth's rotation rate, and  $\lambda$  is the latitude, the gravitational acceleration components and the earth rate component can be expressed as:

$$\begin{cases} g_x = g \sin \Phi \cos \psi \\ g_y = -g \sin \Phi \sin \psi \\ \omega_{cX} = \omega_e \cos \psi \cos(\lambda + \Phi) \\ \omega_{cY} = -\omega_e \sin \psi \cos(\lambda + \Phi) \end{cases} \quad (5)$$

Considering that mathematical expressions for the two axes are similar, now take equation of x axis for analyse purpose. (4) could be rewritten in a matrix form:

$$\mathbf{G}_x^T \mathbf{D}(\mathbf{x}) = k_x i_x - \omega_{cX} \quad (6)$$

Where

$$\mathbf{G}_x = [1 \quad g_x \quad \cdots \quad g_z^2]^T \quad (7)$$

$$\mathbf{D}(\mathbf{x}) = [D(x)_0 \quad D(x)_x \quad \cdots \quad D(x)_{zz}]^T \quad (8)$$

$$\mathbf{Y} = \mathbf{A}\boldsymbol{\beta} + \boldsymbol{\eta} \quad (9)$$

Equations in the form of (6) could be obtained at each position by changing the azimuth and elevation angle of the motion table. Different azimuth and elevation angles lead to different  $\mathbf{G}_x$  and different torquer current data. Considering that random error is unescapable when carrying out the test, the equation of data is in the form of (9). Where  $\mathbf{A}$  is the structural matrix of the regression equations. It's determined by how the positions are configured. Elements in vector  $\mathbf{Y}$  are drifting angular rates. The regression method finds a specific  $\boldsymbol{\beta}$ , which minimize the variance of  $\boldsymbol{\eta}$ . This can be done by solving the regression equation (10).

$$A^T A \hat{\beta} = A^T Y \quad (10)$$

#### 4. Regression Analysing Method

Except for identifying error coefficients, multi-position tests could also be used to figure out the importance of different error terms. Considering that different error coefficients might have different dimensions, it's not reasonable to evaluate importance by simply comparing magnitudes of different coefficients. Instead, some variance analysis methods should be applied. Because of the similarity of two axes, analysis is done with x axis.

Each value in  $Y$  in (10) is acquired from a certain position. Suppose that:

$$\hat{Y} = [\hat{y}_1 \quad \hat{y}_2 \quad \cdots \quad \hat{y}_N]^T \quad (11)$$

Where

$$\hat{Y} = A \hat{\beta} \quad (12)$$

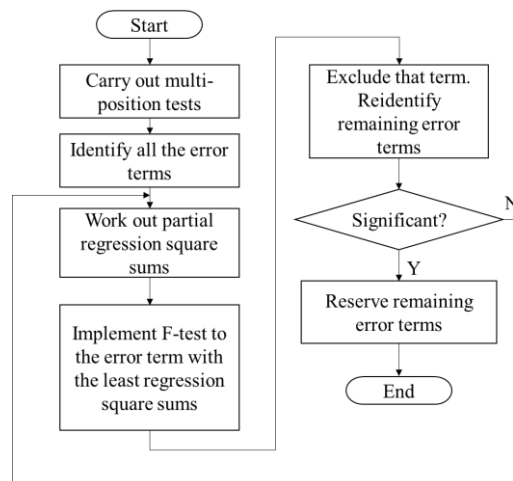
Regression square sum could be calculated by:

$$U_x = \sum_{i=1}^N (\hat{y}_i - \bar{y})^2 \quad (13)$$

Where  $\bar{y}$  is the average value of vector  $Y$ .  $U_x$  evaluates the total effect of different error terms on the torquer current. The more error terms are included, the larger the regression square sum would be. If one of the error terms is excluded, the regression square sum could only decrease. The larger the regression square sum decreases after an error term is excluded, the more important that specific term is. The reduced value after excluding a specific error term is called partial regression square sum. Suppose that  $\hat{\beta}_i$  is the number  $i$  value of vector  $\hat{\beta}$  and the partial regression square sum of it is  $P_i$ , it could be proven that:

$$P_i = \frac{\hat{\beta}_i^2}{c_{ii}} \quad (14)$$

Where  $c_{ii}$  is the number  $i$  diagonal elements of matrix  $A^T A$ .



**Figure 3.** Procedure to exclude insignificant error terms.

The process of excluding insignificant error terms could be expressed by the flowchart in Figure.3. In which F-test is implemented by calculating the statistic quantity:

$$F_i = \frac{P_i}{Q / (N - M - 1)} \quad Q = \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad (15)$$

and comparing it with values in F distribution table.

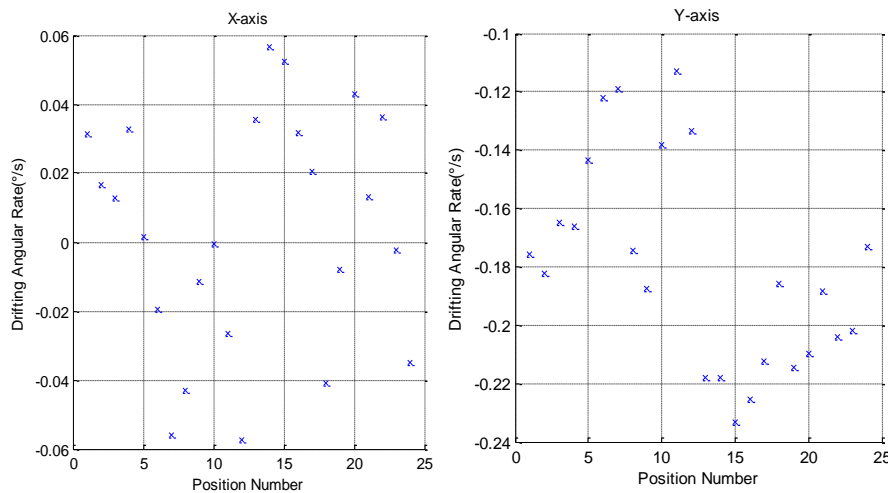
To evaluate whether this process decreases the accuracy, compensation accuracy could be worked out as  $\sigma$ , which is calculated by (16).

$$\sigma = \sqrt{\frac{Q}{N-M-1}} \quad (16)$$

This value is the residual standard deviation of the compensation equation, and could be used to evaluate the error of the compensation equation. The smaller the  $\sigma$ , the higher the accuracy of the compensation equation.

## 5. Test Results

A series of 24-position tests have been carried out. The relationship between drifting angular rates and test positions is shown in Figure.4. The identification of all the possible term are listed in Table.1. And the compensation accuracy values of the equations are listed in (17) and (18).



**Figure 4.** Drifting angular rate of two axes in different positions.

**Table 1.** Identification of all the possible error terms.

x-axis Error Coefficients	Values	y-axis Error Coefficients	Values
$D(x)_0(^\circ/\text{s})$	-0.0272	$D(y)_0(^\circ/\text{s})$	-0.1599
$D(x)_x((^\circ/\text{s})/\text{g})$	-0.0365	$D(y)_x((^\circ/\text{s})/\text{g})$	0.0165
$D(x)_y((^\circ/\text{s})/\text{g})$	0.0054	$D(y)_y((^\circ/\text{s})/\text{g})$	0.0200
$D(x)_z((^\circ/\text{s})/\text{g})$	-0.0067	$D(y)_z((^\circ/\text{s})/\text{g})$	0.0138
$D(x)_{xy}((^\circ/\text{s})/\text{g}^2)$	0.0098	$D(y)_{xy}((^\circ/\text{s})/\text{g}^2)$	-0.0144
$D(x)_{yz}((^\circ/\text{s})/\text{g}^2)$	-0.0006	$D(y)_{yz}((^\circ/\text{s})/\text{g}^2)$	-0.0035
$D(x)_{xz}((^\circ/\text{s})/\text{g}^2)$	-0.0217	$D(y)_{xz}((^\circ/\text{s})/\text{g}^2)$	-0.0078
$D(x)_{xx}((^\circ/\text{s})/\text{g}^2)$	0.0004	$D(y)_{yy}((^\circ/\text{s})/\text{g}^2)$	0.0189
$D(x)_{zz}((^\circ/\text{s})/\text{g}^2)$	0.0606	$D(y)_{zz}((^\circ/\text{s})/\text{g}^2)$	-0.0477

$$\sigma_x = 0.0131^\circ/\text{s} \quad (17)$$

$$\sigma_y = 0.0253^\circ/\text{s} \quad (18)$$

After implementing the process of excluding insignificant error terms the remaining error terms are listed in Table.2. And the compensation accuracy values after simplification are in (19) and (20).

$$\sigma_x = 0.0123 \text{ } ^\circ/\text{s} \quad (19)$$

$$\sigma_y = 0.0245 \text{ } ^\circ/\text{s} \quad (20)$$

**Table 2.** Remaining error terms.

<b>x-axis Error Coefficients</b>	<b>Values</b>	<b>y-axis Error Coefficients</b>	<b>Values</b>
$D(x)_0(^\circ/\text{s})$	-0.0269	$D(y)_0(^\circ/\text{s})$	-0.155
$D(x)_x((^\circ/\text{s})/\text{g})$	-0.0365	$D(y)_y((^\circ/\text{s})/\text{g})$	0.0200
$D(x)_z((^\circ/\text{s})/\text{g})$	-0.0067	$D(y)_z((^\circ/\text{s})/\text{g})$	0.0138
$D(x)_{xx}((^\circ/\text{s})/\text{g}^2)$	-0.0218	$D(y)_{zz}((^\circ/\text{s})/\text{g}^2)$	-0.0571
$D(x)_{zz}((^\circ/\text{s})/\text{g}^2)$	0.0604		

After excluding these insignificant error terms, the compensation accuracy doesn't decrease, which means the simplification of the compensation equations is reasonable.

## 6. Conclusion

In this paper, an improved regression analysing method for Gyrowheel system has been proposed. Based on Multi-position tests, variance analysis is used to exclude insignificant error terms. Partial regression square sums are worked out to determine the importance of error terms and F-tests are carried out to determine whether the insignificant error term is negligible. Test results indicated that this method can significantly simplify the form of the compensation equations and further test procedure while at the same time maintaining the compensation accuracy. This will be of good help to Gyrowheel calibration tests, especially when tests for tilt condition are implemented.

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