

PAPER • OPEN ACCESS

The determination of workspace and the performance evaluation of PRoM-120 with 3 and 4 kinematic constants

To cite this article: Adriyan and Sufiyanto 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **602** 012064

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the [collection](#) - download the first chapter of every title for free.

The determination of workspace and the performance evaluation of PRoM-120 with 3 and 4 kinematic constants

Adriyan and Sufiyanto

Department of Mechanical Engineering, Sekolah Tinggi Teknologi Nasional, Jambi, Indonesia

E-mail: adriyan0686@gmail.com

Abstract. This paper presented a parallel robotic manipulator having combined DOF, i.e., one translational and two rotational DOF, called PRoM-120. PRoM-120 is constructed by using 2PRU/PRS kinematic chains (KCs) and arranged in an asymmetric configuration. The objective of the research is to determine the workspace and to evaluate the performance of PRoM-120 for three cases based on the numerical values of its kinematic constants. The workspace is calculated by applying the discretization method. The inverse kinematic solution, direct singularity, inverse singularity, and actuators limit are taken as the significant factor to determine the workspace. Meanwhile, the performance of PRoM-120 is evaluated by applying conditioning index and transmission based index. It shows the conditioning index cannot be used for PRoM-120, or parallel manipulator having combined DOF. Assessing the performance of PRoM-120 using transmission based index exposes only less than 20% of the workspace be in the good transmission workspace.

1. Introduction

Lower DOF parallel manipulators (PMs) – PMs are having DOF less than six – is more attractive since the needs for manipulation tasks can be fulfilled adequately with several DOFs. It has been applied in the diverse area such as machine tools [1], vibration testing [2], telescopes [3], and even medical devices [4]. The application of PMs in the diverse area is due to advantages offered by PMs compared to its counterpart, the serial manipulators. These advantages are high stiffness, high payload to weight ratio, high acceleration, high precision, and lower inertia – because actuators are commonly installed on the base [5]. Meanwhile, the smaller workspace is the prominent disadvantage of PMs.

The first study of PMs having combined motion especially 1T2R DOF was performed by Hunt which utilized 3PRS kinematic chain [6]. Afterward, Carretero investigated the manipulator constructed by 3PRS kinematic chains showed a small motion in some direction while the platform is actuated [3]. This small motion is not the actual DOF had by the manipulator and later called as a parasitic motion. A work of Li, et al. presented a comprehensive list of several kinematic chains and their arrangements that produce the parasitic motion [7].

While dealing with the optimal design of PMs, the workspace is the first parameter which must be considered. The performance of PMs has to be evaluated within the workspace to show its capabilities while doing the given task. In general, the workspace of PMs can be determined by applying one of the following methods, namely, the geometrical method, the discretization method, and the numerical method. The simple method applied to compute the workspace is the discretization method, but it



costs high memory in calculation [8]. Meanwhile, there are several indices utilized to evaluate the performance of PMs. Conditioning index is the common way to be applied because it is used a Jacobian matrix of PMs. However, it cannot be employed for PMs having combined DOF because of its inconsistencies [9]. Liu, et al. proposed a transmission based index as a uniform metric to assess the performance of PMs having combined DOF [10].

This paper presents a determination of workspace and performance evaluation of PMs with combined motion (1T2R DOF) constructed by 2PRU/PRS kinematic chain which later referred to as PRoM-120. To achieve the objective, an inverse kinematic problem (IKP) is solved, and the Jacobian matrix is derived from the IKP. The screw system is set up to establish motion/force transmission system existed in PRoM-120. The workspace of PRoM-120 is determined by applying the discretization method. Then, the conditioning index and transmission based index are applied to evaluate the performance of PRoM-120.

2. Kinematic of PRoM-120

2.1. Configuration of PRoM-120

PRoM-120 is constructed by three limbs organized in an asymmetric configuration which connecting base to platform as shown in figure 1. The first two limbs have the same kinematic chains, prismatic-revolute-universal (P-R-U) chain. Both limbs are arranged oppositely to each other on the same plane, XZ -plane.

Meanwhile, the prismatic-revolute-spherical (P-R-S) chain is used in the last limb which configured perpendicularly to the plane of the first two limbs. All prismatic joints installed on the base are active joints and can move linearly in the Z -direction. Two reference frames are assigned to the manipulator, i.e., the fixed frame $O-XYZ$ and the moving frame $P-xyz$ attached to the base and the platform, respectively.

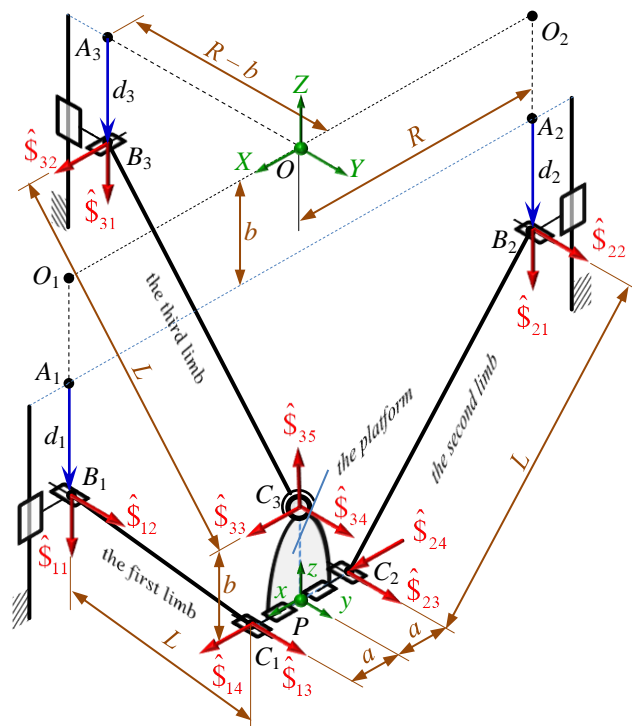


Figure 1. The kinematic structure of PRoM-120 and its unit screw assigned at each joint.

There are four kinematic constants of PRoM-120 as depicted in figure 1 which are R , L , a , and b . These four constants R , L , a , and b represent a radius of the base, a length of the link, a half distance

between point C_1 and C_2 , and a distance between point P and C_3 , respectively. Thus, PRoM-120 with four and three kinematic constants is named as PRoM-120-4 and PRoM-120-3, respectively. The later is the simplified version of PRoM-120 with four kinematic constants when b is equal to a .

2.2. The inverse kinematic problem of PRoM-120

As referenced previously that PRoM-120 is having two rotational DOF about x and y -axis. The geometry of the manipulator has been described in detail as depicted in figure 1. Thus, it can be set the loop vector equation of PRoM-120 by utilizing the given information which is stated mathematically as

$$\overline{B_i C_i} = -\overline{OB_i} + \overline{O_i C_i} = -\overline{OB_i} + \overline{OP} + \mathbf{R}(\vartheta, \phi) \cdot \overline{PC_i}; \text{ for } i = 1, 2, 3. \quad (1)$$

$\overline{OB_i}$ And $\overline{PC_i}$ can be known from the geometry given in figure 1. \overline{OP} is a vector of point P in the platform that defined by a vector of $(x \ y \ -z)^T$. Finally, $\mathbf{R}(\vartheta, \phi) = \mathbf{R}_{y,\phi} \mathbf{R}_{x,\vartheta}$ represents the rotational matrix that relates the orientation of the platform concerning the base. A parasitic motion belonging to PRoM-120 can be obtained by applying a dot product between $\overline{O_i C_i}$ and a unit vector which perpendicular to the plane of each limb. Thus, it produces one parasitic motion in X -direction, i.e., $x = -bc_{\vartheta}s_{\phi}$ and $y = 0$, and the point P can be rewritten as $(-bc_{\vartheta}s_{\phi} \ 0 \ -z)^T$.

Finally, a solution of the inverse kinematic problem (IKP) can be found by taking the dot product of equation (1) to itself and solves it for the actuated joint space (d_1, d_2, d_3) while the platform space (z, ϑ, ϕ) is given. Hence, it gives

$$\begin{aligned} d_1 &= z + as_{\phi} - b \pm \lambda_1 \\ d_2 &= z - as_{\phi} - b \pm \lambda_2 \\ d_3 &= z - bc_{\vartheta}c_{\phi} \pm \lambda_3 \end{aligned} \quad (2)$$

with $\lambda_i = \sqrt{L^2 - k_i^2}$; for $i = 1, 2, 3$; $k_1 = R - ac_{\phi} + bc_{\vartheta}s_{\phi}$, $k_2 = R - ac_{\phi} - bc_{\vartheta}s_{\phi}$, and $k_3 = R - b - bs_{\vartheta}$. Equation (2) presents eight assembly modes possessed by PRoM-120. It is due to the arrangement of PRoM-120 shown in figure 1 can be selected the positive value of λ_i .

2.3. Velocity equation of PRoM-120

A velocity of PRoM-120 can be sought by taking the derivative of equation (2) concerning time once. Thus, it can be presented in the simple equation as

$$\mathbf{J}_q \dot{\mathbf{q}} = \mathbf{J}_x \dot{\mathbf{x}} \text{ or } \dot{\mathbf{q}} = \mathbf{J} \dot{\mathbf{x}}, \quad (3)$$

with \mathbf{J}_q and \mathbf{J}_x denote the Jacobian of inverse kinematics and the Jacobian of the direct kinematic

$$\mathbf{J}_q = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \text{ and } \mathbf{J}_x = \begin{bmatrix} \lambda_1 & -bs_{\vartheta}s_{\phi}k_1 & ac_{\phi}\lambda_1 + h_1k_1 \\ \lambda_2 & -bs_{\vartheta}s_{\phi}k_2 & -ac_{\phi}\lambda_2 - h_2k_2 \\ \lambda_3 & b(s_{\vartheta}c_{\phi}\lambda_3 + c_{\vartheta}k_3) & bc_{\vartheta}s_{\phi}\lambda_3 \end{bmatrix}; \quad (4)$$

With $h_1 = as_{\phi} + bc_{\vartheta}c_{\phi}$ and $h_2 = as_{\phi} - bc_{\vartheta}c_{\phi}$; respectively. $\mathbf{J} = \mathbf{J}_q^{-1} \mathbf{J}_x$ Denotes the Jacobian matrix PRoM-120 and can be computed if and only if \mathbf{J}_q is not a singular matrix. Also, $\dot{\mathbf{q}}$ and $\dot{\mathbf{x}}$ are the

velocity of the actuator joint space, $(\dot{d}_1, \dot{d}_2, \dot{d}_3)^T$, and velocity of the platform space, $(\dot{z}, \dot{\theta}, \dot{\phi})^T$, respectively.

3. The screw system of PRoM-120

A screw theory is a useful tool to address several issues in PMs such as DOF determination of over-constrained PMs, the singularity of PMs, and structural synthesis of PMs. A screw, $\$$, is defined by a dual vector representation which consists of a primal part and a dual part. The primal part is a direction vector where the screw is pointing to in the space. Whereas, the dual part describes the moment of the primal part about a point in the space. Relating to its physical sense, a screw can be known as a twist or a wrench. The twist represents a velocity – angular velocity as its primal part and a vector linear velocity as the dual part. The wrench is composed by a force vector and a moment vector as its primal and dual part, respectively. For ease of use in building up the screw, it is usually represented in its unit which referred as the unit screw, $\hat{\$}$.

The unit screw is commonly used to develop a screw system – a twisted system or a wrench system – for each limb of PMs. With regards to the principle of virtual works, the twist and the wrench – also for the twist system and the wrench system – can be related into a formulation which later known as a reciprocal screw product. It is defined mathematically as

$$\$_i \circ \$_j^r = 0; i = 1, 2, \dots, n; \text{ and } j = 1, 2, \dots, 6 - n. \quad (5)$$

Equation (5) can be known that for any screw $\$_i$ that forms n -system of twists will produce a reciprocal screw $\$_j^r$ of $(6 - n)$ -system of wrenches, and vice versa.

The initial step is to find all twist systems or motion screw systems of PMs. This is a simple task because every joint on each limb possess its allowable motion whether translation or rotation in one, two, or three directions that depending on the type of joints. Thus, the twist system on each limb is referred to as a limb twist system or LTS in short. The LTS for the first two limbs of PRoM-120 can be constructed by using unit screws as assigned in figure 1. For each limb, it consists of the unit screw of one active joint and three passive joints,

$$\text{LTS} = \begin{cases} \text{LTS}_1 = [\hat{\$}_{11}^{\text{LTS}} & \hat{\$}_{12}^{\text{LTS}} & \hat{\$}_{13}^{\text{LTS}} & \hat{\$}_{14}^{\text{LTS}}]^T \\ \text{LTS}_2 = [\hat{\$}_{21}^{\text{LTS}} & \hat{\$}_{22}^{\text{LTS}} & \hat{\$}_{23}^{\text{LTS}} & \hat{\$}_{24}^{\text{LTS}}]^T \\ \text{LTS}_3 = [\hat{\$}_{31}^{\text{LTS}} & \hat{\$}_{32}^{\text{LTS}} & \hat{\$}_{33}^{\text{LTS}} & \hat{\$}_{34}^{\text{LTS}} & \hat{\$}_{35}^{\text{LTS}}]^T \end{cases} \quad (6)$$

with $\hat{\$}_{11}^{\text{LTS}} = (0 \ 0 \ 0; \ 0 \ 0 \ -1)$, $\hat{\$}_{31}^{\text{LTS}} = \hat{\$}_{21}^{\text{LTS}} = \hat{\$}_{11}^{\text{LTS}}$, $\hat{\$}_{12}^{\text{LTS}} = (0 \ 1 \ 0; \ p_{12} \ 0 \ r_{12})$, $\hat{\$}_{22}^{\text{LTS}} = (0 \ 1 \ 0; \ p_{22} \ 0 \ r_{22})$, $\hat{\$}_{32}^{\text{LTS}} = (1 \ 0 \ 0; \ 0 \ -d_3 \ R - b)$, $\hat{\$}_{13}^{\text{LTS}} = (0 \ 1 \ 0; \ p_{13} \ 0 \ r_{13})$, $\hat{\$}_{23}^{\text{LTS}} = (0 \ 1 \ 0; \ p_{23} \ 0 \ r_{23})$, $\hat{\$}_{33}^{\text{LTS}} = (1 \ 0 \ 0; \ 0 \ q_{33} \ bs_g)$, $\hat{\$}_{14}^{\text{LTS}} = (c_\phi \ 0 \ -s_\phi; \ 0 \ q_{14} \ 0)$, $\hat{\$}_{24}^{\text{LTS}} = (c_\phi \ 0 \ -s_\phi; \ 0 \ q_{24} \ 0)$, $\hat{\$}_{34}^{\text{LTS}} = (0 \ c_g \ s_g; \ -c_g q_{33} - bs_g^2 \ 0 \ 0)$, $\hat{\$}_{35}^{\text{LTS}} = (c_g s_\phi \ -s_g \ c_g c_\phi; \ -s_g z \ q_{33} c_g s_\phi \ bs_g c_g s_\phi)$, $p_{12} = b + d_1$, $r_{12} = R$, $p_{13} = as_\phi + z$, $r_{13} = ac_\phi - bc_g s_\phi$, $q_{14} = -bc_g s_\phi^2 - c_\phi z$, $p_{22} = b + d_2$, $r_{22} = -R$, $p_{23} = -as_\phi + z$, $r_{23} = -ac_\phi - bc_g s_\phi$, $q_{24} = q_{14}$, and $q_{33} = bc_g c_\phi - z$.

Since the LTS for each limb of PRoM-120 has been found, it can be determined wrench system on each limb. This wrench system is force/moment constraining the motion of each limb and is referred to a limb wrench system or LWS. The LWS for each limb of PRoM-120 is calculated by applying a reciprocal screw product to each unit screw of LTS as listed in equation (6). Thus, it has

$$\text{LWS} = \begin{cases} \text{LWS}_1 = [\hat{\$}_{11}^{\text{LWS}} & \hat{\$}_{12}^{\text{LWS}}]^T \\ \text{LWS}_2 = [\hat{\$}_{21}^{\text{LWS}} & \hat{\$}_{22}^{\text{LWS}}]^T \\ \text{LWS}_3 = \hat{\$}_{31}^{\text{LWS}} \end{cases}, \quad (7)$$

with $\hat{\$}_{11}^{\text{LWS}} = \hat{\$}_{21}^{\text{LWS}} = (0 \ 0 \ 0; \ s_\phi \ 0 \ c_\phi)$, $\hat{\$}_{12}^{\text{LWS}} = \hat{\$}_{22}^{\text{LWS}} = (0 \ c_\phi \ 0; \ 0 \ -q_{14} \ 0)$, and $\hat{\$}_{31}^{\text{LWS}} = (1 \ 0 \ 0; \ 0 \ q_{35} \ bs_g)$.

Then, a platform wrench system or PWS of PRoM-120 can be determined by using the information given by the LWS for each limb. This PWS is determined by taking a union of all LWSs and then find the basis of them. It yields

$$\text{PWS} = [\hat{\$}_1^{\text{PWS}} \ \hat{\$}_2^{\text{PWS}} \ \hat{\$}_3^{\text{PWS}}]^T, \quad (8)$$

With $\hat{\$}_1^{\text{PWS}} = \hat{\$}_{11}^{\text{LWS}}$, $\hat{\$}_2^{\text{PWS}} = \hat{\$}_{12}^{\text{LWS}}$, and $\hat{\$}_3^{\text{PWS}} = \hat{\$}_{31}^{\text{LWS}}$, which represents the constraint forces and moments acting on the platform. Afterward, applying the reciprocal product to the PWS in equation (8) will produce a twisted system on the platform which is known as the platform twist system or the PTS. The PTS provides direct information about the number and the type of DOF owned by PRoM-120.

A screw system which represents the motion/force transmission system can be found in the PMs having combined motion. They are an input twist system (ITS), a transmission wrench system (TWS), and an output twist system (OTS). The ITS for each limb can be constructed simply by taking the twist of the active joint on each limb, which gives for PRoM-120 as

$$\text{ITS} = [\hat{\$}_1^{\text{ITS}} \ \hat{\$}_2^{\text{ITS}} \ \hat{\$}_3^{\text{ITS}}]^T \quad (9)$$

with $\hat{\$}_i^{\text{ITS}} = \hat{\$}_{i1}^{\text{LTS}}$, for $i = 1, 2, 3$. Next, another wrench system of each limb can be obtained by locking its respective active joint. It means that the reciprocal screw product is applied to each LTS which only contains the passive twists. To simplify, this wrench system is named with the passive LWS. Hence, the TWS of each limb can be determined by seeking a unique screw in the passive LWS which does not belong to the PWS [10]. It can be written mathematically as

$$\text{TWS} = [\hat{\$}_1^{\text{TWS}} \ \hat{\$}_2^{\text{TWS}} \ \hat{\$}_3^{\text{TWS}}]^T. \quad (10)$$

With $\hat{\$}_1^{\text{TWS}} = (-k_1 \ 0 \ -\lambda_1; \ 0 \ p_{13}k_1 + r_{13}\lambda_1 \ 0)$, $\hat{\$}_2^{\text{TWS}} = (k_2 \ 0 \ \lambda_2; \ 0 \ -p_{23}k_2 - r_{23}\lambda_2 \ 0)$, and $\hat{\$}_3^{\text{TWS}} = (0 \ k_3 \ \lambda_3; \ -q_{33}k_3 - \lambda_3bs_g \ 0)$. Equation (10) shows that the TWS of PRoM-120 for each limb is a transmission force along the link B_iC_i which transmits the force from the actuator to the platform.

The OTS of each limb is a twist produced on the platform when one limb is actuated while the others are locked [10]. It can be achieved by finding a reciprocal screw of a new wrench system in PRoM-120. The new wrench system for the i^{th} limb is determined by taking a union of the PWS and the TWS other than the respective limb. In other words, the new wrench system of the first limb is the

union of the PWS, TWS₂, and TWS₃. Thus, applying the reciprocal product to the new wrench system of the first limb will lead to the OTS of the first limb, and the same procedure can be performed to obtain the other two. The OTS of each limb is given in the simplified form as

$$\text{OTS} = [\hat{\$}_1^{\text{OTS}} \quad \hat{\$}_2^{\text{OTS}} \quad \hat{\$}_3^{\text{OTS}}]^T, \quad (11)$$

with $\hat{\$}_i^{\text{OTS}} = (\alpha_i \quad \beta_i \quad \gamma_i; \quad \eta_i \quad \mu_i \quad \xi_i)$; for $i = 1, 2, 3$, $\alpha_1 = c_\phi \lambda_3 \chi_1$, $\alpha_2 = -c_\phi \lambda_3 \chi_2$, $\alpha_3 = -c_\phi \lambda_3$, $\beta_1 = b[s_g(c_\phi \lambda_2 + s_\phi k_2)\lambda_3 + c_g \lambda_2 k_3]$, $\beta_2 = -b[s_g(c_\phi \lambda_1 + s_\phi k_1)\lambda_3 + c_g \lambda_1 k_3]$, $\beta_3 = bs_\phi s_g(\lambda_1 k_2 - \lambda_2 k_1)$, $\gamma_1 = -s_\phi \lambda_3 \chi_1$, $\gamma_2 = s_\phi \lambda_3 \chi_2$, $\gamma_3 = s_\phi \lambda_3$, $\eta_1 = b[(k_3 q_{33} - bs_g \lambda_3)c_g \lambda_2 + (c_\phi \lambda_2 - s_\phi k_2)s_g \lambda_3 p_{23}]$, $\eta_2 = b[(k_3 q_{33} + bs_g \lambda_3)c_g \lambda_1 - (c_\phi \lambda_1 + s_\phi k_1)s_g \lambda_3 p_{13}]$, $\eta_3 = bs_g s_\phi[\lambda_1 k_2 p_{23} - \lambda_2 k_1 p_{13} - 2\lambda_1 \lambda_2 ac_\phi]$, $\mu_1 = q_{14} \lambda_3 \chi_1$, $\mu_2 = -q_{14} \lambda_3 \chi_2$, $\mu_3 = -q_{14} \lambda_3$, $\xi_1 = b(s_g c_\phi \lambda_3 + c_g k_3)\chi_1$, $\xi_2 = -b(s_g c_\phi \lambda_3 + c_g k_3)\chi_2$, $\xi_3 = bs_g s_\phi \lambda_3$, $\chi_1 = \lambda_2 r_{23} - k_2 h_2$, $\chi_2 = \lambda_1 r_{13} + k_1 h_1$, and $\chi_3 = \lambda_1 k_2 h_2 + \lambda_2 k_1 h_1 + 2\lambda_1 \lambda_2 ac_\phi$.

4. Workspace of PRoM-120

In this research, the discretization method is used because it provides a simple way to calculate the workspace of PMs. This method is applied by discretizing the domain – which is the platform space – into some points. To determine the workspace of PRoM-120, it is utilized two sets of numerical values for each PRoM-120 with three and four kinematic constants as given in table 1.

Table 1. Numerical values for three cases of PRoM-120 that applied to determine its workspace and evaluate its performance.

PRoM-120	L [mm]	R [mm]	a [mm]	b [mm]
Case 1: PRoM-120-4	300	200	80	120
Case 2: PRoM-120-3	300	200	80	80
Case 3: PRoM-120-4	300	200	80	40

There are several steps applied to determine the workspace of PRoM-120. The first step is conducted by discretizing each variable of the platform space for a given value of the lower bound and the upper bound. In this research, it is employed a bound of $z = [-700 \text{ mm}, -250 \text{ mm}]$, $\vartheta = [-180^\circ, 20^\circ]$, and $\phi = [-60^\circ, 60^\circ]$ for the listed three cases. For the discretization purpose, it is used 46 points, 1001 points, and 121 points for the given bound of z , ϑ , and ϕ , respectively. Hence, it gives 5571566 points in total.

Afterward, every point that have been discretized are fed into a solution of the IKP given in equation (3), indeed, for the positive value of λ_i . This calculation produces the value of the actuated joint spaces for knowing the discretized value of the platform space. All prismatic joints installed on PRoM-120 have bound from -200 mm to 0 mm. Applying the condition for each prismatic joint reduces the number of discretizing points. Then, the points within a solution of the inverse kinematic problems are checked whether they are in singular condition or not. The singularity condition which taken into account is the inverse kinematic singularity, $|\mathbf{J}_q| = 0$, and the direct kinematic singularity, $|\mathbf{J}_x| = 0$. Finally, it can be obtained the points that are the workspace of PRoM-120 itself.

All numerical calculations for the workspace determination – also for performance evaluation – are conducted by using scientific python stacks (Numpy, Scipy, and Matplotlib). The workspace of PRoM-120 for the three cases are plotted in figure 2. In brief, the workspace of PRoM-120 for the three cases can be known from the number of points within their workspace. The number of points within the computed workspace is 991243, 1094921, and 1096524 for each case, respectively.

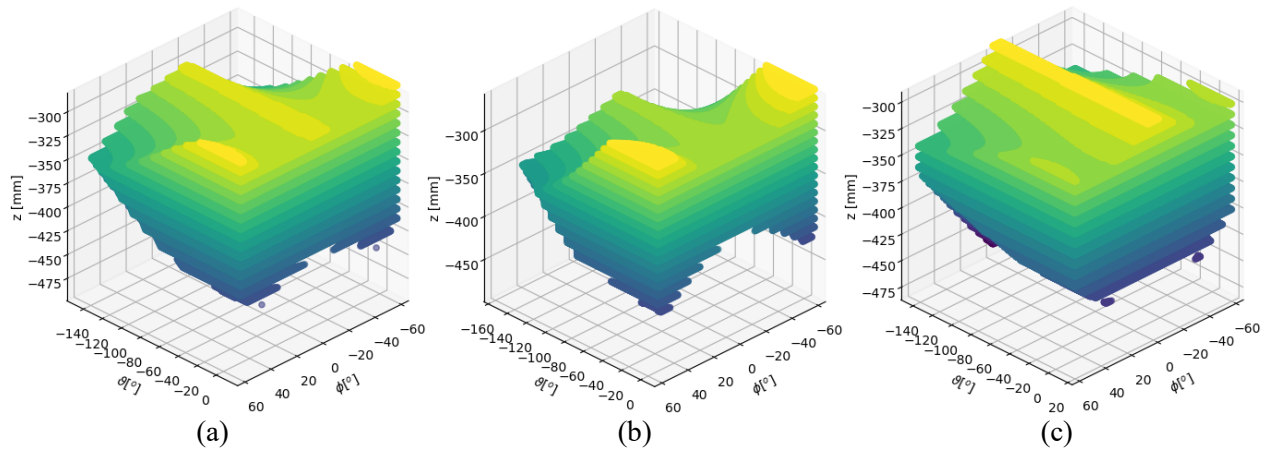


Figure 2. The workspace of PRoM-120 for (a) Case 1, (b) Case 2, and (c) Case (3).

5. Performance evaluation of PRoM-120

Both workspace determination and performance evaluation for any PMs are the important subjects for optimal design of PMs. The performance evaluation is usually applied to measure the kinematic performance of PMs that stated in several indices. For PRoM-120 presented in this paper, it is used four indices namely conditioning index (local and global), input transmission index, output transmission index and transmission index (local and global). These indices are applied to measure the performance of PRoM-120 for those three cases that already mentioned in table 1.

The local conditioning index (LCI) can be computed from the inverse of the condition number of the Jacobian matrix. It is defined mathematically as $LCI = 1/\kappa$, with κ denotes the condition number of the Jacobian matrix, $\kappa = \|\mathbf{J}^{-1}\| \cdot \|\mathbf{J}\|$ [9]. The LCI is evaluated for each point within the workspace which computed previously. Meanwhile, the global condition index (GCI) is computed over entire workspace as stated mathematically as

$$GCI = \left[\int dW \right]^{-1} \cdot \int (1/\kappa) dW, \quad (12)$$

with W denotes the workspace of the manipulator.

The LCI of the PRoM-120 for the three cases within it workspace is depicted in figure 3. It can be inferred from those cases that the highest value of LCI is lower than 0.04. The GCI of PRoM-120 for case 1 to 3 is 0.015, 0.016, and 0.016, respectively. Hence, the LCI of these cases is near 0 which indicates that the index cannot be used to assess the performance of PMs having combined DOF [9, 10].

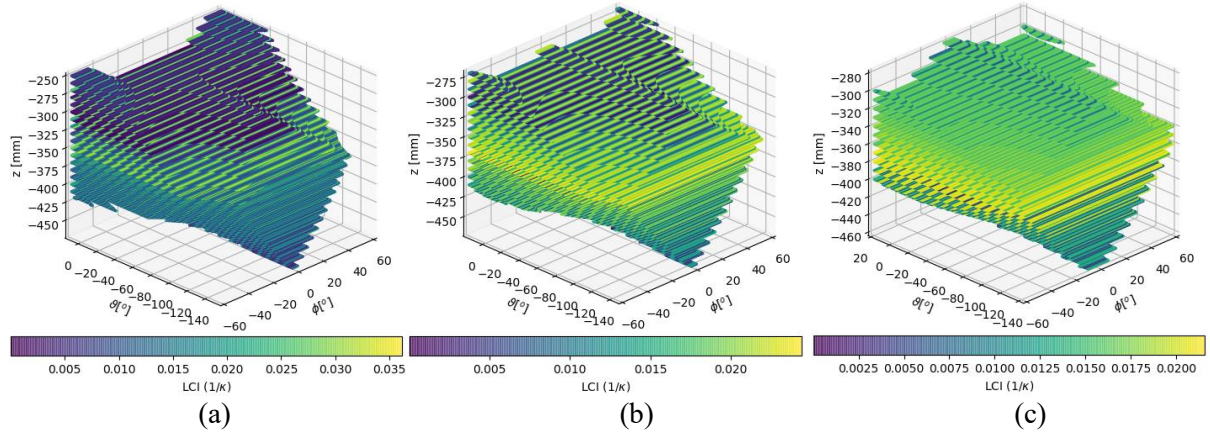


Figure 3. The LCI of PRoM-120 for (a) Case 1, (b) Case 2, and (c) Case (3).

There are four indices used in transmission based performance evaluation for PRoM-120, i.e., the input transmission index (ITI), the output transmission index (OTI), the local transmission index (LTI), and the global transmission index (GTI). There is one index need to be included to calculate its performance, which is the constraint transmission index (CTI). Since PRoM-120 does not possess constraint singularity which can be known from the linearly independent check of PWS, then the CTI can be excluded for calculation of the LTI.

The ITI and the OTI of each limb can be computed by taking the product between the TWS and the ITS, and the product between the TWS and the OTS, respectively. They are computed for every point within the workspace. It can be represented mathematically as [10]

$$ITI_i = \frac{|\$ _i^{TWS} \circ \$ _i^{ITS}|}{|\$ _i^{TWS} \circ \$ _i^{ITS}|_{\max}}, \text{ and } OTI_i = \frac{|\$ _i^{TWS} \circ \$ _i^{ITS}|}{|\$ _i^{TWS} \circ \$ _i^{ITS}|_{\max}}; \text{ for } i = 1, 2, 3. \quad (13)$$

The ITI and the OTI calculated in equation (14) are aimed at each limb of PRoM-120. The ITI of PRoM-120 can be found by discovering the minimum value of the ITI from each limb $\gamma_1 = \min(ITI_1, ITI_2, ITI_3)$. This condition also holds for finding the OTI of PRoM-120, $\gamma_o = \min(OTI_1, OTI_2, OTI_3)$. Both ITI and OTI of PRoM-120 are plotted in figure 4 and 5, respectively. It is only for the third case of PRoM-120 which possesses the ITI over 0.8 within the workspace. Meanwhile, the OTI shows a wide range from 0 to 1 within the obtained workspace.

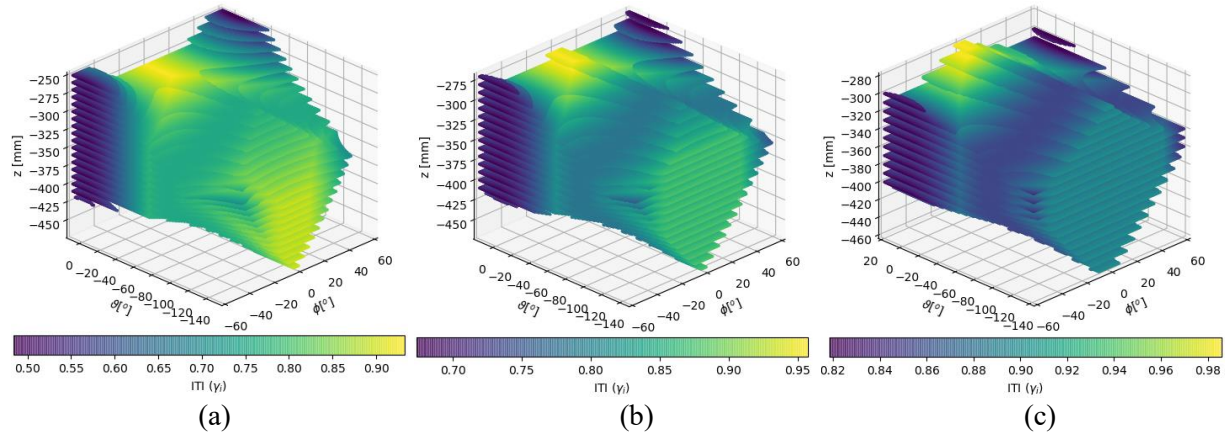


Figure 4. The ITI of PRoM-120 for (a) Case 1, (b) Case 2, and (c) Case (3).

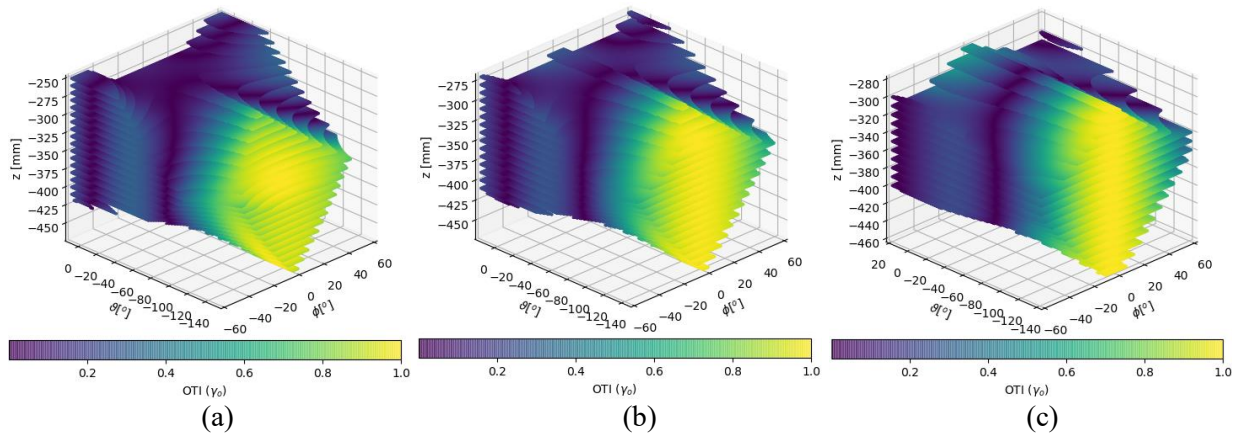


Figure 5. The OTI of PRoM-120 for (a) Case 1, (b) Case 2, and (c) Case (3).

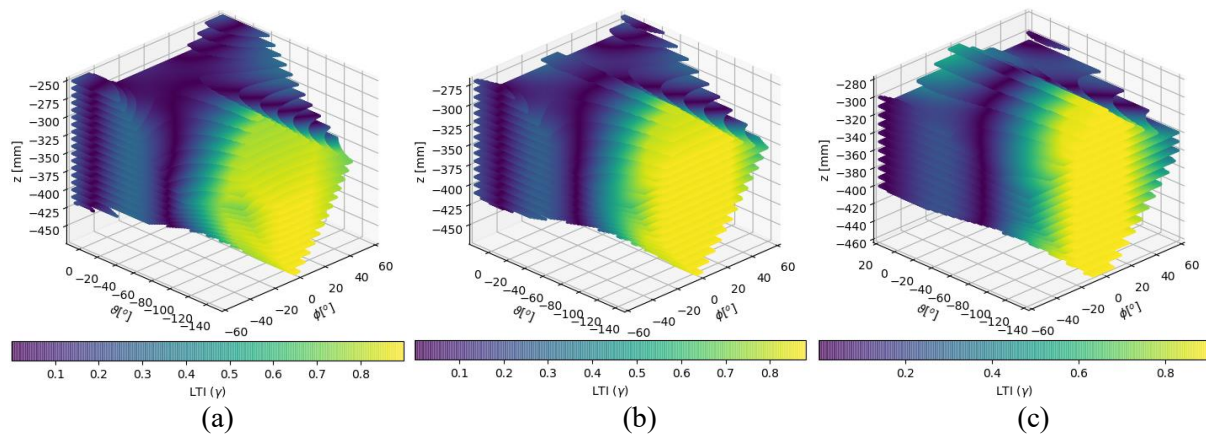


Figure 6. The LTI of PRoM-120 for (a) Case 1, (b) Case 2, and (c) Case (3).

The LTI (γ) of PRoM-120 is computed by taking the minimum value of both ITI and OTI, $\gamma = \min(\gamma_I, \gamma_O)$, obtained previously. From the obtained LTI can be perceived a region within workspace which fulfills the good motion/force transmissibility. To know such region, a threshold value is required which is set for the value of $\text{LTI} \geq 0.7$ [10]. The region that possesses the $\text{LTI} \geq 0.7$ is characterized as the good transmission workspace (GTW). Applying such condition to each case gives the percentage of GTW over the computed workspace as 19.69%, 19.43%, 18.18% for case 1 to 3, respectively. The LTI within the workspace of PRoM-120 for the three cases is displayed in figure 6. Meanwhile, the GTI can be computed in the same way as the GCI is calculated, indeed, by replacing the integrand with the LTI, γ . It produces the value of GTI for case 1 to 3 is 0.308, 0.317, and 0.364, respectively.

6. Conclusions

The workspace determination and performance evaluation for PRoM-120 with three and four kinematic constants have been conducted by examining three cases. These cases are stated that based on the numerical value of kinematic constants possessed by PRoM-120. The Jacobian matrix of PRoM-120 was obtained from the derivation of the IKP solution. Also, the screw system was established to obtain the motion/force transmission system which utilized for the performance evaluation of the PRoM-120.

The workspace of PRoM-120 was determined by applying the discretization method, that discretized the platform space into some points. The IKP solution, singularity condition, and actuator

limitation were utilized to compute the workspace of PRoM-120. Every point within the obtained workspace were employed to evaluate the performance of PRoM-120 using conditioning index and transmission based index. It is shown that conditioning index was poor for assessing the performance of PRoM-120. Meanwhile, the transmission based index revealed a small region within the workspace that can be characterized the GTW. It showed only less than 20% of the entire calculated workspace felt into the GTW. To obtain larger GTW within the workspace, an optimization method must be employed for the future research.

7. Acknowledgments

The authors wish to say our gratitude to the General Directorate of Research and Development – The Ministry of Research, Technology and Higher Education that funded this research under the scheme “Penelitian Dosen Pemula” contract no. 052/K10/KM/KONTRAK-PENELITIAN/2018.

References

- [1] Xu L, Li Q, Tong J, et al. 2018 Tex3: An 2R1T Parallel Manipulator with Minimum DOF of Joints and Fixed Linear Actuators. *Int J Precis Eng Manuf* **19** 227–238.
- [2] Herrero S, Pinto C, Altuzarra O, et al. 2014 Analysis and Design of the 2PRU-1PRS Manipulator for Vibration Testing. In: *Proceedings of the ASME 2014 International Mechanical Engineering Congress and Exposition* pp. 1–8.
- [3] Carretera J A, Nahon M, Gosselin C M, et al. 1999 Kinematic analysis of a three-of parallel mechanism for telescope application. In: *Proceeding of ASME Design Automation Conference*. 1999, pp. 17–24.
- [4] Yasir A, Kiper G. 2018 *Mech Mach Sci* **52**. Epub ahead of print 2018. DOI: 10.1007/978-3-319-60702-3.
- [5] Pandilov Z and Dukovski V 2014 *Fascicule* **1** 2067–3809.
- [6] Hunt K H 1983 *ASME J Mech Transm Autom* **105** 705–712.
- [7] Li Q, Chen Z, Chen Q, et al. 2011 *Robot Comput Integr Manuf* **27** 389–396.
- [8] Herrero S, Pinto C, Altuzarra O, et al. 2015 Workspace Study of the 2PRU-1PRS Parallel Manipulator. *14th IFToMM World Congr*. Epub ahead of print 2015. DOI: 10.6567/IFTToMM.14TH.WC.OS13.063.
- [9] Jacobian M J and Manipulability 2005 *J Mech* **128** 199–206.
- [10] Liu X-J, Wu C, Wang J. 2012 *J Mech Robot* **4** 1-10.