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Appraisal of MC2010 shear resistance approaches coupled with a residual flexural strength prediction model

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Abstract. In the present work the predictive performance of the two approaches proposed by Model Code 2010 for the evaluation of the shear capacity of fiber reinforced concrete (FRC) elements flexurally reinforced with conventional steel bars is assessed considering a database (DBs) constituted by 80 FRC beams do not including conventional transverse reinforcements. The accuracy of these shear models is evaluated by statistical analysis of the prediction ratio between the experimental and estimated shear capacity of the beams of the DBs, and applying the Demerit Points Classification approach for further information about the reliability of the two approaches in design context. Due to the absence of the post-cracking experimental characterization of the FRC used in several beams considered in the DBs, an approach was developed for estimating the residual flexural strength parameters from the most relevant known variables of steel fiber reinforcement mechanisms for concrete, namely the fiber volume and aspect ratio, and the concrete compressive and tensile strength. The residual flexural strength prediction model is assessed and its influence on the performance of the shear resistance models is evaluated.

1. Introduction

Regarding the use of discrete fibers in concrete reinforcement, previous research has already pointed that it increases concrete shear resistance and deformation capacity, decreases crack width and spacing, and can change the type of fracture mode for members without transverse conventional reinforcements, from brittle to a ductile one [1–5]. In addition, the fiber reinforcement can be adopted for partial or total replacement of steel stirrups in structural elements preserving the required shear capacity to the member [2,6,7].

Based on the extensive research already carried out regarding the experimental assessment of the shear behavior of fiber reinforced concrete (FRC) members, a database (DB_s) that collects the shear tests data of beams flexurally reinforced with conventional steel bars and made by steel fiber reinforced concrete (SFRC) has been developed [8]. These beams are herein abbreviated by the acronym R/SFRC. The DB_s has already been used for the appraisal of R/SFRC shear resistance prediction models [9–11]. Currently, the DB_s is being extended, and some difficulties have been encountered in using the data of some shear tests due to the lack of some key information, namely the residual flexural strength of the SFRC, which is used in both approaches proposed by CEB *fib* Model Code 2010 (MC2010) [12] for the evaluation of the shear capacity of FRC elements flexurally reinforced with conventional steel bars.

In fact, the final version of MC2010 [12], published in 2013, covers the design guidelines and recommendations for the dimensioning process of FRC structural members, including two shear resistance prediction models. Considering the expansion of the FRC shear tests DB_s, a new opportunity has been found to appraise two of the most widely used FRC shear resistance prediction models, coupled with a residual flexural strength prediction model. In the context of a model for predicting the punching capacity of SFRC slabs, Moraes-Neto *et al.* [13] have mounted a database (DB_f) formed by results from three point notched beam bending tests (3PNBBT) with specimens of concrete reinforced with hooked ends steel fibers (SFRC), executed according to the RILEM TC 16-TDF [14] and MC2010 recommendations. Treating statistically the results from the DB_f, the authors derived simple equations for determining the residual flexural tensile strength parameters ($f_{Ri},(i=1,2,3,4)$) from the volume content (V_f) and aspect ratio (l_f/d_f) of the hooked ends steel fibers. The authors have recognized that one of the debilities of the proposed equations is their independence of the quality of the matrix, since the fiber reinforcement mechanisms, and consequently the post-cracking tensile capacity of a FRC, depend on this property. In the present work the DB_f is extended in order to have enough information for including the concrete strength as a variable making part of a new generation of more reliable and consistent equations for determining f_{Ri}. These equations are used to estimate the contribution of the fiber reinforcement for the SFRC shear capacity according to the two approaches of the MC2010 whenever f_{Ri} are not known experimentally.

2. MC2010 FRC shear resistance prediction models

The Model Code 2010 presents two models to predict the shear resistance of FRC structural members based on: (i) the concept of the residual flexural strength of FRC (f_{Ri}); (ii) the simplified modified compression field theory (SMCFT) [15] coupled with the Variable Engagement Model (VEM) [16].

The resistance of FRC members using the model based on the concept of the residual flexural strength, denoted in this paper as MC2010_EEN, is determined by the expression [12]:

$$V_{Rd,F}=\left\{\frac{0.18}{\gamma_c}\cdot k\cdot\left[100\cdot\rho_{sl}\left(1+7.5\cdot\frac{f_{Ftuk}}{f_{ctk}}\right)\cdot f_{ck}\right]^{1/3}+0.15\cdot\sigma_{cp}\right\}b_w\cdot d \;\;;\;\; \sigma_{cp}<0.2\cdot f_{cd} \tag{1}$$

where γ_c is the partial safety factor for concrete without fibers; $k=1+\sqrt{200/d}\le 2.0$ is a factor that takes into account the beam's size effect; d is the effective depth of the cross section [mm]; $\rho_{sl}=A_{sl}/(b_w\cdot d)$ is the longitudinal reinforcement ratio; A_{sl} is the cross-sectional area of the longitudinal reinforcement [mm^2]; f_{Ftuk} is the characteristic value of the ultimate residual tensile strength of the FRC [MPa]; f_{ctk} is the characteristic tensile strength for the concrete without fibers [MPa]; f_{ck} is the concrete characteristic compressive strength [MPa]; σ_{cp} is the average axial stress acting in the cross-section [MPa], f_{cd} is the design value of the concrete compressive strength; and b_w is the smallest width of the tensile zone of the cross-section [mm].

The shear resistance, $V_{Rd,F}$, is assumed to be not smaller than the minimum value, $V_{Rd,F,\min}$, obtained from [12]:

$$V_{Rd,F,\min}=\left(\nu_{\min}+0.15\cdot\sigma_{cp}\right)\cdot b_w\cdot d \;\;;\;\; \nu_{\min}=0.035\cdot k^{3/2}\cdot f_{ck}^{1/2} \tag{2}$$

For evaluation of the shear resistance, the ultimate residual tensile strength is computed from equation (3) considering the residual flexural strength parameters of FRC, $f_{Ri}\,(i=1,3)$ and the ultimate crack width equal to $w_u=1.5mm$. For the design situations of FRC members, in equation (3) should be considered the characteristic values of the residual flexural strength of the FRC.

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3\,(=2.5mm)} \cdot (\, f_{Fts} - 0.5 \cdot f_{R3} + 0.2 \cdot f_{R1}) \geq 0 \;\; ; \;\; f_{Fts} = 0.45 \cdot f_{R1} \tag{3}$$

The MC2010_EEN model is based on the shear contribution of plain concrete members without transverse reinforcements, as proposed in Eurocode 2 [17]. The increased post-cracking toughness and crack-opening restriction provided by the fiber reinforcement to the concrete is empirically considered by multiplying the longitudinal reinforcement ratio by the factor $(1+7.5\, f_{Ftuk} / f_{ctk})$. Thus, the fiber reinforcement contribution is considered as an additional flexural reinforcement [18].

According to the model based on the SMCFT/VEM theory, hereafter denoted as MC2010_MCFT, the general expression that predicts the shear resistance of FRC member is equal to:

$$V_{Rd} = V_{Rd,F} + V_{Rd,s} \tag{4}$$

where $V_{Rd,F}$ is the FRC shear resistance contribution, while $V_{Rd,s}$ is the added shear resistance provided by the transverse conventional reinforcement.

The FRC shear resistance, $V_{Rd,F}$, is the shear resistance contribution of the concrete matrix, $V_{Rd,c}$, plus of the contribution of the fibers bridging the shear failure crack, $V_{Rd,f}$, being determined by the following expression[12]:

$$V_{Rd,F} = V_{Rd,c} + V_{Rd,f} = \frac{1}{\gamma_F} \cdot \left[k_v \cdot \sqrt{f_{ck}} + k_{fd} \cdot f_{Tk}\,(w_u) \cdot \cot\theta \right] \cdot z \cdot b_w \;\; , \;\; \sqrt{f_{ck}} \leq 8MPa \tag{5}$$

where k_v is the parameter that determines the contribution of the aggregate interlock mechanism for the shear strength of the cross section ((6)); k_{fd} is a fiber dispersion reduction factor [19], assuming the value of $k_{fd} = 0.8$; $f_{Tk}\,(w_u)$ is the characteristic value of the post-cracking tensile capacity of FRC, determined from direct tensile tests, evaluated at the ultimate crack width of w_u; θ is the inclination of the compressive stress field, corresponding to the inclination of the critical diagonal crack; z is the internal lever arm that can be estimated as $z = 0.9 \cdot d$ [mm] [12].

The k_v is function of the parameter that considers the aggregate size influence, k_{dg} (equation (7)), of the longitudinal strain at the mid depth of the effective shear area, ε_x, and of the transverse reinforcement ratio of the cross-section, ρ_w. In equation (7) the term d_g is the maximum aggregate dimension in the concrete matrix [mm].

$$k_v = \begin{cases} \dfrac{0.4}{1+1500 \cdot \varepsilon_x} \cdot \dfrac{1300}{1000 + z \cdot k_{dg}} & \text{for} \quad \rho_w < 0.08 \cdot \sqrt{f_{ck} / f_{yk}} \\ \dfrac{0.4}{1+1500 \cdot \varepsilon_x} & \text{for} \quad \rho_w \geq 0.08 \cdot \sqrt{f_{ck} / f_{yk}} \end{cases} > 0.0 \tag{6}$$

$$k_{dg} = \begin{cases} \dfrac{32}{16 + d_g} \geq 0.75 & \text{for} \quad f_{ck} \leq 70MPa \\ 2.0 & \text{for} \quad f_{ck} > 70MPa \text{ and for ligthweight concrete} \end{cases} \tag{7}$$

The strain, ε_x, at mid-depth of the effective shear area is defined as half the tension chord strain, $\varepsilon_x = \varepsilon_s / 2$, namely [12,20]:

$$\varepsilon_x = 0 \le \frac{1}{2 \cdot E_s \cdot A_{sl}} \left[\frac{M_{Ed}}{z} + \frac{V_{Ed}}{2} \cot\theta + N_{Ed} \left(\frac{1}{2} \pm \frac{\Delta e}{z} \right) \right] \le 0.003 \quad (8)$$

where Δe is the eccentricity of the beam axis with respect to its mid-depth of the effective shear area; M_{Ed} and V_{Ed} correspond to the acting bending moment and shear force at the cross-section (assumed positive values) and N_{Ed} is the axial force at the cross-section (positive for tension and negative for compression).

According to MC2010, the value of θ can be freely chosen in the interval of $\theta_{\min} \le \theta \le 45^{\circ}$, while the value of $\theta_{\min}$ is related with the longitudinal strain level in the mid-depth of the cross-section, ε_x, [12]:

$$\theta_{\min} = \left(29^{\circ} + 7000 \cdot \varepsilon_x \right) \quad (9)$$

Based on the work of [21], where a $\sigma - w$ relationship of FRC was derived from inverse analysis using the results of prism bending tests, the direct post-cracking tensile capacity of FRC can be estimated from the expression [9]:

$$f_{Tk}(w) = k_G \min\left(0.4 f_{R2k} + 1.2 \left(f_{R4k} - f_{R2k} \right) \xi(w),\ f_{ctk,\min} \right) ;\ \xi(w) = \alpha w - 0.25 \quad (10)$$

The factor k_G takes into account fibre alignment due to casting bias and wall influences that occur in the prism bending test. In Table 1 is presented the value of k_G considering the different prism bending test standards. The value for the factor α also depends on the prism bending test configuration and is presented in Table 1.

Table 1. Value of k_G and α depending on prism bending test standard [9].

Prism bending test standard	k_G	α
ASTM C1609 [22]	0.70	1/3
EN 14651 [23]	0.60	5/12
RILEM TC 162-TDF [14]	0.60	5/12
UNI 11039 [24]	0.60	43/84

The ultimate crack width, w_u, orthogonal to the critical diagonal crack is determined according to equation (11) [12].

$$w_u = \left(0.2 + 1000 \cdot \varepsilon_x \right) \ge 0.125\ mm \quad (11)$$

The shear reinforcement resistance, $V_{Rd,s}$, is determined according to the following equation [12]:

$$V_{Rd,s} = A_{sw} / s_w \cdot z \cdot f_{ywd} \cdot \left(\cot\theta + \cot\alpha \right) \cdot \sin\alpha \quad (12)$$

where A_{sw} is the shear reinforcement area [mm^2], f_{ywd} is the design value of the yield stress of the shear reinforcement, and α is the inclination of the transverse reinforcement.

The design shear resistance cannot exceed the crushing capacity of concrete in the web, determined as [12]:

$$V_{Rd,\max} = k_c \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot z \cdot \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta} \quad (13)$$

where k_c is a strength reduction factor defined by $k_c = k_\varepsilon \cdot \eta_{fc}$; k_ε is a factor that takes into account the strain in the web of the structural element, being determined according to the level of approximation defined in MC2010 (level of approximation I: $k_\varepsilon = 0.55$; level of approximation II and III: equation (14)); η_{fc} is a factor to consider the influence of the concrete strength class on the level of the shear failure brittleness , being determined from equation (15).

$$k_\varepsilon = \left(1.2 + 55 \cdot \varepsilon_1\right)^{-1} \le 0.65 \;;\; \varepsilon_1 = \varepsilon_x + \left(\varepsilon_x + 0.002\right)\cot^2\theta \tag{14}$$

$$\eta_{fc} = \left(30/f_{ck}\right)^{1/3} \le 1.0 \quad \left(f_{ck} \text{ in MPa}\right) \tag{15}$$

Both models can only be adopted if conventional steel longitudinal reinforcement is present in the cross-section of the FRC member.

3. FRC beams shear tests database (DB_s)

An extended version of a database (DB_s) comprising shear tests of FRC beams [8] is considered for the assessment of the MC2010 shear prediction models.

The DB_s includes the data of 113 steel fiber reinforced concrete (R/SFRC) beams submitted to shear, collected from the following sources [1,5,21,25–38]. In terms of cross section configuration of the beams considered in the DB_s, 99 beams are of rectangular shape, while 14 are of T-shape. Within the 113 R/SFRC beams, 99 are of deep type cross section $\left(d/b_w > 1.0\right)$ and 14 of current and shallow type cross section $\left(d/b_w \le 1.0\right)$. The effective shear span ratio, a/d, ranges from 2.0 to 4.0, therefore it is assured the applied load is not directly conducted to the closest support of the beam in any of the R/SFRC of the DB_s.

All the R/SFRC of the DB_s are flexurally reinforced with passive steel bars (prestress effect is not treated in the present work). The flexural reinforcement ratio varies between 1.0 to 3.1%, and the effective depth of the cross-section, *d*, is within the interval 150 to 1440 mm. None of the R/SFRC beams is shear reinforced with conventional reinforcements (like stirrups). In the 113 R/SFRC beams of the DB_s only hooked end type of fibers were used, with a fiber aspect ratio, l_f/d_f (l_f and d_f are the length and diameter of the fiber, respectively) varying from 48 to 80. The fiber volume, V_f, ranges from 0.25% to 1.5%. The concrete maximum aggregate size, d_g, varies between 10 and 25 mm, while the average compressive strength of the SFRC ranges between 19.2 to 64.6 MPa. Regarding the SFRC residual flexural tensile strength, the average values of f_{R1} and f_{R3} are not reported for 39 samples, while for f_{R2} and f_{R4} are not provided for 59 samples. In the DB_s three different prism bending test standards were used, namely the EN 14651 [23], RILEM TC 162-TDF [14], the ASTM C1609 [22] and the UNI 11039[24]. Considering the main differences for the evaluation of the residual flexural strength using the referenced standards, the EN 14651 and RILEM TC 162-TDF adopt a 3-point bending test configuration with notched FRC beams, while the ASTM C1609 and UNI 11039 adopt a 4-point bending test, the former one with un-notched beams, and the later with notched beams.

Due to the use of un-notched SFRC samples on the prisms bending tests performed according to the ASTM C1609, the crack appears at the weakest point between the two loading points, and the measurement of the crack mouth opening displacement (CMOD) is not possible due to the unknown location of the crack. Consequently, in order to calculate the residual flexural strength of the SFRC samples of the database, the CMODs were obtained by correlating the CMOD with the measured mid-span deflection of the SFRC prisms using the methodology proposed in [39] that considers a rigid body kinematic mechanism similar to the one adopted in 3-point bending tests of notched beams [40]. Due to this simplification, the residual flexural strength values that were determined according to the ASTM C1609 can present some error.

Due to the significant number of R/SFRC beams of the DB_s were the residual flexural strength is not reported, there was the need to adopt a model to predict these mechanical properties, which is discussed in the next section.

After implementation of this prediction model, the extreme values of the residual flexural strength of the R/SFRC beams studied in the DB_s are: $f_{R1}=[1.5-8.5]$ MPa, $f_{R2}=[1.8-8.9]$ MPa, $f_{R3}=[1.5-8.9]$ MPa and $f_{R4}=[1.1-7.9]$ MPa.

A first analysis of the DB_s has consisted in the exclusion of test results where a flexural-shear failure mechanism could be feasible. For this purpose, the flexural capacity of each R/SFRC beam was assessed following the guidelines of MC2010. For the samples where the ultimate resisting bending moment was lower than 95% of the acting bending moment (due to shear capacity registered experimentally), the shear test results were not considered in this study. After this analysis, a total of 80 R/SFRC beams were considered for the assessment of the shear resistance prediction models, within which the complete data of the post-cracking residual flexural strength (f_{Ri}, $i=1,2,3,4$) was available for 42 beams, and partial information (f_{Ri}, $i=1,3$) was provided for 59 beams. Since some of the beams in the DB_s are casted with the same SFRC mix, the results of the experimental characterization of the post-cracking residual strength for these beams is the same. The 59 beams with experimental data of f_{R1} and f_{R3} were casted from 44 different SFRC mixes, while the 42 beams with f_{R2} and f_{R4} resulted from 35 different SFRC mixes. In Table 2 is presented the different standards considered in the evaluation of the available residual flexural strength values, considering the different SFRC mixes present in the DB_s.

Table 2. Prism bending test standards adopted for the evaluation of the residual flexural strength, f_{Ri}, considering the different SFRC mixes present in the DB_s.

Number of samples	Prism bending standards				
	EN 14651	RILEM TC 16	ASTM C1609	UNI 11039	Total
f_{R1} and f_{R3}	9	9	25	1	44
f_{R2} and f_{R4}	9	0	25	1	35

4. Prediction model of residual flexural strength of SFRC

Based on the analysis of a database (DB_f) formed by 89 samples of hooked end steel fiber reinforced concrete (SFRC) notched beams submitted to 3-point bending according to EN 14651 [23], in [41] is described in detail a strategy for establishing a relationship between the residual flexural strength of SFRC, $f_{Ri}, (i=1,2,3,4)$, and other material properties capable of being easily obtained in design context.

For the DB_f under consideration, a very good agreement was obtained for the relation between the f_{R1}, the reinforcement index, IR, and the concrete average tensile strength, f_{ctm}. The reinforcement index is defined as:

$$IR = V_f \cdot l_f / d_f \tag{16}$$

The derived relationship between f_{R1}, IR and f_{ctm} is:

$$f_{R1} = k_{1,R1} \cdot f_{ctm} \cdot (IR)^{k_{2,R1}} \tag{17}$$

where $k_{1,R1}$ and $k_{2,R1}$ are model coefficients.

Due to the inherit complexity to perform direct and indirect tensile tests, the tensile strength of concrete can be estimated from the concrete compressive strength by the expression presented in MC2010:

$$f_{ctm} = \begin{cases} 0.3 \cdot (f_{ck})^{2/3} & , f_{ck} \le 50MPa \\ 2.12 \cdot \ln[1+0.1 \cdot (f_{ck}+8)] & , f_{ck} > 50MPa \end{cases} \tag{18}$$

Strong correlation between $f_{Ri}(i=2,3,4)$ and f_{R1} was also verified in [41], namely:

$$f_{Ri} = k_{1,Ri} \cdot \left(f_{R1}\right)^{k_{2,Ri}} \quad i = 2,3,4 \tag{19}$$

Based on the results of the DB_f, three intervals of average compressive strength were adopted for best correlate the coefficients $k_{1,Ri}$ and $k_{2,Ri},(i=1,2,3,4)$ with the $f_{cm}[MPa]$, having been obtained the following equations:

$$k_{1,R1} = \begin{cases} 3.2 & , f_{cm} < 25 \\ \dfrac{1.1}{1000} \cdot f_{cm}^{\,2} - \dfrac{f_{cm}}{10} + 5 & , 25 \le f_{cm} \le 65 \\ 3.2 & , f_{cm} > 65 \end{cases}; \quad k_{2,R1} = \begin{cases} 0.5 & , f_{cm} < 25 \\ \dfrac{8}{1000} \cdot f_{cm} + 0.32 & , 25 \le f_{cm} \le 65 \\ 0.85 & , f_{cm} > 65 \end{cases} \tag{20}$$

$$k_{1,R2} = \begin{cases} 0.9 & , f_{cm} < 25 \\ \dfrac{-3.8 \cdot f_{cm}^{\,2}}{10000} + \dfrac{3.2 \cdot f_{cm}}{100} + 0.35 & , 25 \le f_{cm} \le 65 \\ 0.8 & , f_{cm} > 65 \end{cases}; \quad k_{2,R2} = \begin{cases} 1.1 & , f_{cm} < 25 \\ \dfrac{3 \cdot f_{cm}^{\,2}}{10000} - \dfrac{2.8}{100} \cdot f_{cm} + 1.6 & , 25 \le f_{cm} \le 65 \\ 1.1 & , f_{cm} > 65 \end{cases} \tag{21}$$

$$k_{1,R3} = \begin{cases} 0.88 & , f_{cm} < 25 \\ \dfrac{-8 \cdot f_{cm}^{\,2}}{10000} + \dfrac{6.05 \cdot f_{cm}}{100} - 0.14 & , 25 \le f_{cm} \le 65 \\ 0.4 & , f_{cm} > 65 \end{cases}; \quad k_{2,R3} = \begin{cases} 1.12 & , f_{cm} < 25 \\ \dfrac{8 \cdot f_{cm}^{\,2}}{10000} - \dfrac{6.78 \cdot f_{cm}}{100} + 2.31 & , 25 \le f_{cm} \le 65 \\ 1.3 & , f_{cm} > 65 \end{cases} \tag{22}$$

$$k_{1,R4} = \begin{cases} 0.8 & , f_{cm} < 25 \\ \dfrac{-1.06 \cdot f_{cm}^{\,2}}{1000} + \dfrac{8.4 \cdot f_{cm}}{100} - 0.66 & , 25 \le f_{cm} \le 65 \\ 0.3 & , f_{cm} > 65 \end{cases}; \quad k_{2,R4} = \begin{cases} 1.12 & , f_{cm} < 25 \\ \dfrac{1.1 \cdot f_{cm}^{\,2}}{1000} - \dfrac{9.26 \cdot f_{cm}}{100} + 2.75 & , 25 \le f_{cm} \le 65 \\ 1.38 & , f_{cm} > 65 \end{cases} \tag{23}$$

The use of equations (17), (19)-(23) is restricted for $f_{cm} \le 70MPa$, as the formulation tends to overestimate the residual flexural strength of SFRC for $f_{cm} > 70MPa$ (few data exist in this interval).

The prediction model of the residual flexural strength presented a very good agreement with the DB_f studied in [41], as the average ratio between the experimental, $f_{Ri,\exp}$, and predicted, $f_{Ri,\mathrm{mod}}$, values of the residual flexural strength ranged between 1.02 to 1.09, while the coefficient of variation varied between 17.8% to 28.94% (Table 3). It is verified that the coefficient of variation has increased with the crack opening at which the f_{Ri} is evaluated, which is a consequence of the increase of the dispersion of the results from f_{R1} to f_{R4}.

The prediction model was also applied to the set of SFRC mixes used in the beams of the DB_s where the residual flexural strength was experimentally characterized (Table 2). In Table 4 is presented the average and coefficient of variation (COV) of the ratio between experimental, $f_{Ri,\exp}$, and estimated values, $f_{Ri,est}$, of the residual flexural strength, considering the prism bending test standard adopted in the evaluation of the experimental values. It is possible to verify that when comparing the estimated and experimental f_{Ri} values determined from the test configuration that adopts notched beams (EN 14651/RILEM TC 162-TDF/UNI 11039), the model exhibits an higher dispersion and average values of the ratio $f_{Ri,\exp}/f_{Ri,est}$ than the observed in [41]. For the samples tested according to the ASTM C1609, the model presents in average a slight underestimation of the SFRC residual flexural strength, which can be related with lower values of $f_{Ri,\exp}$ due to cracking on the weakest section between the loading

points of the un-notched beams, and to the simplification adopted in the adopted correlation between the mid-span deflection and CMOD.

Table 3. Statistical results of the residual flexural strength prediction model [41].

Residual flexural strength	$f_{Ri,\exp}/f_{Ri,\mathrm{mod}}$		
	Average	SD[a]	COV[b] (%)
f_{R1}	1.02	0.18	17.78
f_{R2}	1.08	0.22	20.61
f_{R3}	1.09	0.26	23.68
f_{R4}	1.05	0.30	28.94

[a] Standard deviation; [b] Coefficient of variation.

Table 4. Statistical analysis of the ratio between the experimental and estimated values of the residual flexural strength, f_{Ri}.

Residual flexural strength	$f_{Ri,\exp}/f_{Ri,est}$					
	Global		EN 14651/RILEM TC 162-TDF/UNI 11039		ASTM C1609	
	Average	COV[a] (%)	Average	COV[a] (%)	Average	COV[a] (%)
f_{R1}	1.06	26.9	1.23	24.4	0.92	19.4
f_{R2}	1.14	28.9	1.49	21.0	0.99	20.2
f_{R3}	1.14	31.9	1.35	27.6	0.99	27.8
f_{R4}	1.09	37.9	1.49	28.3	0.93	30.4

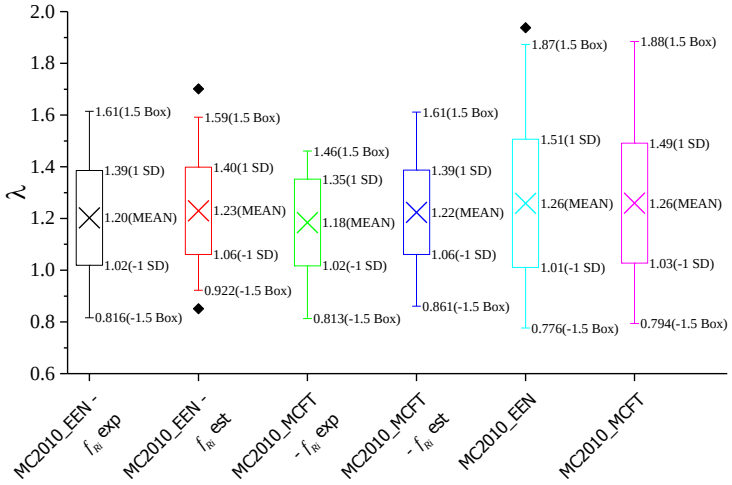
[a] Coefficient of variation.

5. Appraisal of MC2010 shear resistance and residual flexural strength prediction models

The predictive performance of shear models MC2010_EEN and MC2010_MCFT is appraised by using the information collected in the R/SFRC beams shear tests database (DB_s). The shear resistance obtained by both models, $V_{u,\mathrm{model}}$, is compared with the experimentally obtained shear resistance of each R/SFRC beam, $V_{u,\exp}$, and the prediction ratio $\lambda = V_{u,\exp}/V_{u,\mathrm{model}}$ is obtained. For the evaluation of the strain at mid-depth of the effective shear area, ε_x, in the MC2010_MCFT model (equation (8)), the internal forces in the member are determined for a control section at the distance $a/2$ from the beam's support, being a the shear span, where shear failure has occurred, considering that the reaction at the support is equal to the beam's shear resistance. During the analysis, the partial safety factors adopted in the theoretical models are set to 1.0, and average values for the SFRC material properties are adopted, in order to properly compare the experimental and theoretical results, being the predictive performance of the model considered as better as closer to 1.0 is $\lambda = V_{u,\exp}/V_{u,\mathrm{model}}$. In the case of the samples were the residual flexural strength data was not experimentally assessed, it was adopted the prediction model presented in the previous section to estimate the values of this material properties. The characteristic value of the SFRC compressive strength was evaluated from average value present in the DB_s, considering the expression $f_{ck} = f_{cm} - 8MPa$ proposed by MC2010.

In Figure 1 is presented a statistical analysis of the prediction ratio, λ, for each shear resistance model considering the source of the residual flexural strength values (experimental/estimated). When assessing the influence of the use of experimental or estimated values of the residual flexural strength of SFRC for the MC2010_EEN and MC2010_MCFT, it is possible to denote that the average values of the prediction ratio, λ, is slightly higher when using estimated values of f_{Ri}, which is related to the slight underestimation of the estimated values of f_{Ri}, as shown in the column "Global" of Table 4. However, when considering the source of the residual flexural strength values the average values of the prediction ratio λ for the analyzed scenarios have very small differences, including the coefficients of

variation. Due to this, it is possible to conclude that the residual flexural strength prediction model presented in the previous section is suitable to be used in the evaluation of the shear resistance of R/SFRC beams, when the values of f_{Ri} are absent in the database. When analysing the global performance of the shear prediction models, both models present a satisfactory approximation to the experimental results (average value of λ equal to 1.26), with acceptable values of dispersion (coefficient of variation lower than 20%). It should be noticed the existence of only one outlier in 80 samples ($\approx$1%), which corresponds to a sample of the MC2010_EEN model.



Prediction ratio λ	No. Samples	Average	COV[a] (%)
MC2010_EEN f_{Ri} experimental	59	1.202	15.2
MC2010_EEN f_{Ri} estimated	59	1.230	13.7
MC2010_MCFT f_{Ri} experimental	42	1.184	14.1
MC2010_MCFT f_{Ri} estimated	42	1.224	13.3
MC2010_EEN	80	1.258	19.7
MC2010_MCFT	80	1.259	18.4

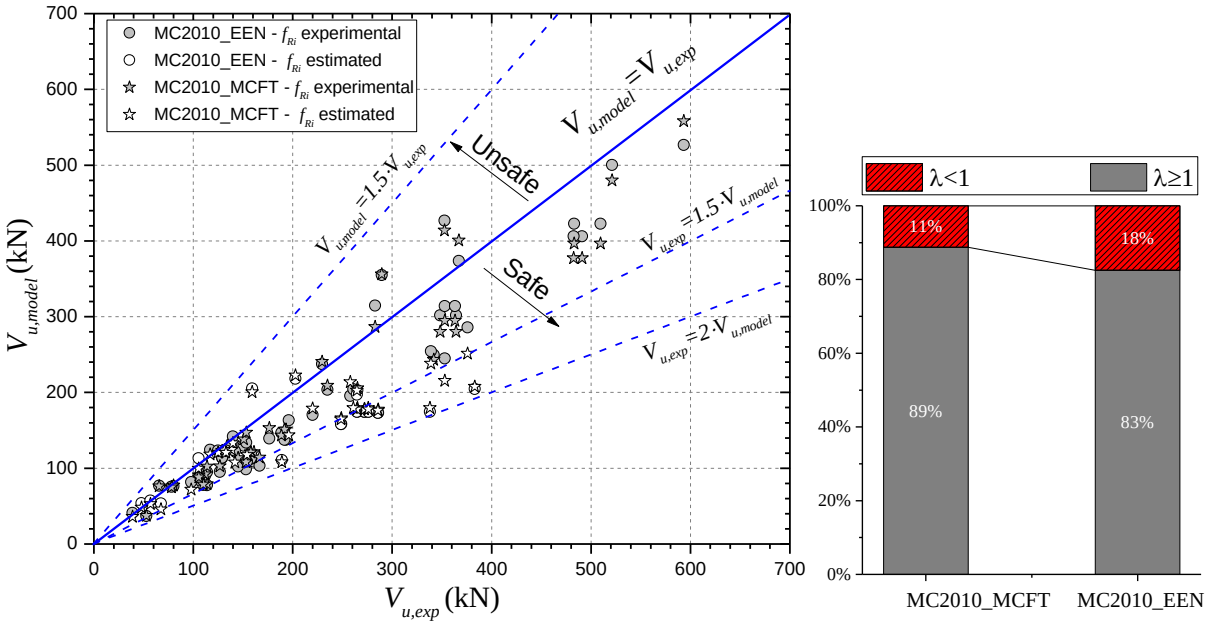
[a] Coefficient of variation

Figure 1. Statistical analysis of λ for both MC2010 shear resistance models considering the use of experimental and estimated values of f_{Ri}.

In Figure 2a is presented the comparison between the experimental and prediction values of the shear resistance for each beam. The results are also divided in categories considering the source of the residual flexural strength values considered in the shear resistance model. As can be seen, the predictive performance of the shear models is very similar and a satisfactory agreement with the experimental values of the DB_s is attained. Considering the influence of the source of the residual flexural strength values, it is possible to observe that the use of the f_{Ri} prediction model appears to be related with a more pronounced underestimation of the shear resistance (a relatively high percentage of results is in the interval of $V_{u,\exp} = 1.5 \cdot V_{u,\text{model}}$ to $V_{u,\exp} = 2.0 \cdot V_{u,\text{model}}$) for the R/SFRC beams under analysis. In Figure 2b is presented the overall analysis of both models, regarding that the prediction values are safe for $\lambda \geq 1$ and unsafe for $\lambda < 1$. From these results is possible to verify that the MC2010_MCFT model presents a slightly higher percentage of safe predictions when compared to the MC2010_EEN model. In addition, the comparison between the performance of both MC2010 models was assessed according to an adapted version of the *Demerit Points Classification* proposed in [42]. This classification is based on the determination of the cumulative number of penalties for each value of λ. The penalty points scale is defined in Table 5, and the lower the number of penalties is, the safer is the performance of the model. For this case, both models present a very similar number of total penalties points, with a slight better performance of the MC2010_MCFT model presenting 62 penalty points while the MC2010_EEN model has 68 penalty points.

Table 5. Adapted version of the *Demerit Points Classification*.

$\lambda = V_{u,\exp} / V_{u,\text{model}}$	Classification	Penalty
< 0.50	Extremely Dangerous	10
[0.50-0.85[	Dangerous	5
[0.85-1.15[	Appropriate Safety	0
[1.15-2.00[	Conservative	1
$\geq$ 2.00	Extremely Conservative	2



(a) (b)

Figure 2. a) Comparison of experimental and shear strength determined from both MC2010 prediction models. b) Predictive performance of theoretical model, safe ($\lambda \geq 1$) and unsafe ($\lambda < 1$).

6. Conclusions

In the present work was assessed the performance of the two shear resistance models available in CEB *fib* Model Code 2010 (MC2010) by comparison of its predictive capability in a database that includes the results of 80 samples of steel fiber reinforced concrete (R/SFRC) beams submitted to shear.

Due to the absence of the characterization of the residual flexural strength of the SFRC used in several tests, a model that estimates these mechanical parameters based on the reinforcement index and concrete strength class was proposed and coupled with the shear resistance models.

For the assessment of the suitability of the residual flexural strength prediction model a comparison of the prediction ratio, λ, was performed, which was computed by considering estimated and experimentally obtained values of the residual flexural strength of the SFRC. The obtained results demonstrated a very small difference between the average and dispersion of the prediction ratio, λ, which validates the use of the residual flexural strength prediction model in the assessment of the shear resistance of FRC elements.

For evaluating the overall performance of the shear models a statistical analysis of the prediction ratio, λ, was conducted, which revealed that the performance of the MC2010_MCFT and MC2010_EEN models is very similar, with the MC2010_MCFT presenting a very slightly minor coefficient of variation for the λ than the MC2010_EEN model (18.4%-19.7%). For the majority of the cases, both MC2010 shear models predict safe values for the shear resistance of the R/SFRC beams, and when the partial safety factor is introduced, the models always return safe predictions.

An additional comparison of the MC2010 shear models was done considering the Demerit Points Classification, having both approaches presented similar performance in this regard (the difference was only 6 penalty points between both models).

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