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Analytical analysis of stress and displacement for surrounding rock of deep-buried rectangular tunnel in layered strata

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Abstract: This study presents a theoretical solution on stress and displacement of the deep-buried tunnel surrounding rock in layered strata. Based on theory of Покровский equivalent layer method, we convert layered strata into homogeneous ones and then map the rectangular tunnel in the coordinate plane z to the unit circle of complex plane ζ , through using a complex variable method and calculating the analytic functions $\Phi(\zeta)$ and $\Psi(\zeta)$ with Matlab, the study obtains that the stress of the surrounding rock of deep-buried rectangular tunnel in layered strata. Subsequently, we derive the strain and displacement of the surrounding rock. Finally, we study the effects of lateral pressure coefficient and grade of surrounding rock and height-to-width ratio of the tunnel on the stress of tunnel surrounding rock. Based on our analysis, we formulate valuable rules.

1. Introduction

Construction structures, such as tunnels in underground space, often feature cross-layered strata. The cross-layered strata is special trouble in underground engineering, which often leads to uneven subsidence, falling rock, cracking and other diseases in the tunnel structure. Li *et al.* [1] combined field monitoring and numerical simulation of the Gonghe tunnel to discuss failure characteristics of layered rock mass. By theoretical analysis and numerical calculations, Wang *et al.* [2] studied the deformation and failure mechanism and supporting technology of the surrounding rock of cross-layered strata. To date, from the present research results, the studies which focus on stress and displacement of tunnel surrounding rock in layered strata include numerical simulation, field monitoring, and theoretical analysis.

The Покровский equivalent layer method refers to the theory of converting the upper rock and soil mass into an equivalent rock layer with the same elastic modulus as the lower layer rock soil mass. This theory is effectively applied to the study of stress-strain problem of layered strata. Considering the Talahao coal mine of Dongsheng coalfield and theory of equivalent layer method, DAI *et al.* [3] define two kinds of stress paths, namely, the internal and external boundaries. The arch thickness of surrounding rock pressure arch around a straight wall arch and circular section hole under the conditions of 6°, 12°, and 18° were compared and analyzed. AI *et al.* [4] introduced the equivalent layer method, then derived the analytical solutions of stratum displacement, stress and surface settlement of shallow-buried circular tunnel caused by excavation.

Mushhelishvili *et al.* [5] proposed the analytical method of complex variable function. This method provides a new way to solve the displacement and stress on surrounding rock of underground holes.



Kargar *et al.* [6] and Lu *et al.* [7, 8] obtained the stress analytical solution of non-circular tunnel by means of Cauchy solution using complex variable function method. Liu *et al.* [9] By using conformal mapping function, the outer domain of the tunnel is transformed into the outer domain of the unit circle. Cauchy integral and residue theorems are subsequently used to derive the two stress functions and analytical solution of the plane strain problem of stress and displacement of surrounding rock. Starting from Harnack's theorem, Zhu [10] deduced the general expressions of two analytical functions used to solve stress of tunnel surrounding rock with arbitrary excavation sections. Based on the theory of plane elastic complex variable function, Rao *et al.* [11, 12] studied the stress of surrounding rock in an elliptical cave and double ellipsoidal cavity under internal pressure. By means of Cauchy integral method in the solution of complex variable functions, Shi *et al.* [13] solved the analytical expressions of stress and displacement values at any point in an elastic half space for a single-center circular inverted arch horseshoe tunnel.

Complex displacement, stress conditions, and local load conditions at the interface between the upper and lower strata add difficulty to solving the stress and displacement of tunnel surrounding rock in layered strata. In this study, the center of rectangular tunnel is located at the interface between the upper and lower strata. Using the theory of Покровский equivalent layer method, we convert the layered strata into a homogeneous one with the same elastic modulus as the lower stratum. We then use conformal mapping and convert the deep-buried rectangular tunnel in plane z to a unit circle in complex plane ζ . we calculate the analytic functions $\Phi(\zeta)$ and $\Psi(\zeta)$, with Matlab to obtain the stress of surrounding rock of deep-buried rectangular tunnel in layered strata. We then derive the strain and displacement of surrounding rock. Finally, we study the effects of lateral pressure coefficient and grade of surrounding rock and height-to-width ratio of tunnel on the stress of tunnel surrounding rock. The results could provide a theoretical reference for predicting stress and displacement of tunnel surrounding rock in layered strata.

2. Model of layered strata and their transformation

2.1. Simplification of layered strata model

Fig. 1 shows the model of layered strata. Considering that the upper (E_1, μ_1 , and φ_1) and lower strata (E_2, μ_2 , and φ_2) feature different mechanical parameters, the displacement and stress of tunnel surrounding rock in layered strata can be calculated according to the equivalent layer method. According to the equivalent layer method, the upper stratum is equivalent to the lower stratum with thickness of h_1' , elastic modulus of E_2 , Poisson's ratio of μ_2 , and internal friction angle of φ_2 .

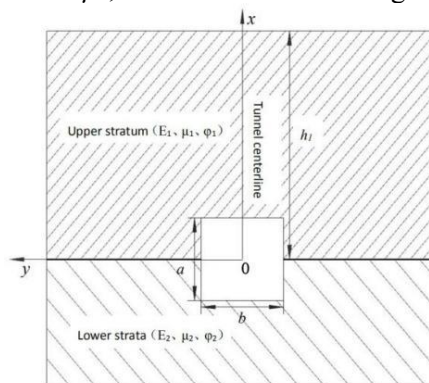


Figure 1. Model of layered strata

$$h_1' = h_1 \sqrt{\frac{E_1}{E_2}} \quad (1)$$

Assuming that the coordinates in any point in the strata are (x, y) , y refers to the horizontal distance from any point to the tunnel centerline, and x denotes the vertical distance from any point to the

centerline of the tunnel, the value of x locating in upper part of the tunnel center is positive. Eq. (2) shows the coordinate transformation relationship. Eq. (2) can simplify the problem of layered strata into that of homogeneous strata with mechanical parameters E_2 , μ_2 , and φ_2 .

$$\begin{cases} y' = y \\ x' = \begin{cases} x\sqrt{\frac{E_1}{E_2}} (x > 0) \\ x (x < 0) \end{cases} \end{cases} \quad (2)$$

2.2. Homogeneous formation and mapping function

From theory of Покровский equivalent layer method, if the tunnel shape differs in the layered strata, the tunnel shape after conversion also usually differs. In this paper, we convert three kinds of rectangular tunnels with different height-to-width ratios in layered strata into corresponding tunnels in the homogeneous strata. We then use Eq. (3) that computes the mapping function and convert the rectangular tunnel on z plane into the circular tunnel on ζ plane. The radius of the circle is $r_0=1$. Table 1 shows the mapping functions of the corresponding tunnel. In Eq. (3), R , C_1 and C_3 all are positive real number, R reflects the tunnel size. C_1 and C_3 reflect the tunnel shape.

$$z = \omega(\zeta) = R\left(\frac{1}{\zeta} + C_1\zeta + C_3\zeta^3\right) \quad (3)$$

Table 1. Mapping functions of different height-to-width ratios of a rectangular tunnel

Layered strata		Homogeneous strata		R	C_1	C_3	Mapping function
Width (m)	Depth (m)	Width (m)	Depth (m)				
4	8	4	6	4.4118	-0.17	-0.15	$\omega(\zeta) = 4.4118\left(\frac{1}{\zeta} - 0.17\zeta - 0.15\zeta^3\right)$
6	8	6	6	3.6145	0	-0.17	$\omega(\zeta) = 3.6145\left(\frac{1}{\zeta} - 0.17\zeta^3\right)$
8	8	8	6	3.0928	0.16	-0.19	$\omega(\zeta) = 3.0928\left(\frac{1}{\zeta} + 0.16\zeta - 0.19\zeta^3\right)$

3. Stress solution of surrounding rock of deep-buried tunnel in layered strata

To solve the stress and displacement of surrounding rock of deep-buried tunnel in layered strata, we first convert the layered strata into a homogeneous ones by using the theory of Покровский equivalent layer method. Then, we use conformal mapping, that is, the outer domain of a tunnel with any shape in plane z is converted into the outer domain of the unit circle of plane ζ . Thus, stress of the tunnel surrounding rock can be solved.

Assuming that the tunnel in homogeneous strata features an infinite length, the problem can be regarded as a plane strain problem. Considering the elastic theory and complex function theory [14], the stress and displacement components of tunnel surrounding rock in homogeneous strata can be determined by two analytic functions $\Phi(\zeta)$ and $\Psi(\zeta)$, in the complex plane. Based on boundary conditions, $\varphi(\zeta)$ and $\psi(\zeta)$ are obtained by Eqs. (4) to (8) [14]. In these equations, f_x and f_y respectively denote the face forces of inner boundary of the entire tunnel in x and y directions. F_x and F_y respectively corresponds to the sum of face forces of inner boundary of the entire tunnel in x and y directions. σ refers to the value of the boundary of the unit circle in plane ζ . Constants B and $B+iC$ are determined by principal stresses at infinity. Eqs. (9) and (10) show the formed relationship, where the principal stresses are σ_1 and σ_2 , respectively. In these equations, β denotes the angle between σ_1 and the y -axis.

$$\varphi(\zeta) = \frac{1}{8\pi(1-\mu)}(\bar{F}_x + i\bar{F}_y)\ln\zeta + B\omega(\zeta) + \varphi_0(\zeta) \quad (4)$$

$$\psi(\zeta) = -\frac{3-4\mu}{8\pi(1-\mu)}(\bar{F}_x + i\bar{F}_y)\ln\zeta + (B' + iC')\omega(\zeta) + \psi_0(\zeta) \quad (5)$$

$$\varphi_0(\zeta) = \frac{1}{2\pi i} \int_{\sigma} \frac{\omega(\sigma)\overline{\varphi_0'(\sigma)}}{\omega'(\sigma)(\sigma-\zeta)} d\sigma - \frac{1}{2\pi i} \int_{\sigma} \frac{f_0}{(\sigma-\zeta)} d\sigma \quad (6)$$

$$\psi_0(\zeta) = \frac{1}{2\pi i} \int_{\sigma} \frac{\omega(\sigma)\overline{\varphi_0'(\sigma)}}{\omega'(\sigma)(\sigma-\zeta)} d\sigma - \frac{1}{2\pi i} \int_{\sigma} \frac{\bar{f}_0}{(\sigma-\zeta)} d\sigma \quad (7)$$

$$f_0 = i\int(\bar{f}_x + i\bar{f}_y)ds - \frac{\bar{F}_x + i\bar{F}_y}{2\pi}\ln\sigma - \frac{1}{8\pi(1-\mu)}(\bar{F}_x + i\bar{F}_y)\frac{\omega(\sigma)}{\omega'(\sigma)} - 2B\omega(\sigma) - (B' - iC')\overline{\omega(\sigma)} \quad (8)$$

$$B = \frac{1}{4}(\sigma_1 + \sigma_2) \quad (9)$$

$$B' + iC' = -\frac{1}{2}(\sigma_1 - \sigma_2)e^{-2i\beta} \quad (10)$$

For the elastic body in the rectangular coordinates in z plane, the relationships between σ_φ and σ_ρ and between the displacement in the x and y directions are shown as follows:

$$\sigma_\rho + \sigma_\varphi = 4\operatorname{Re}\Phi(\zeta) \quad (11)$$

$$\sigma_\varphi - \sigma_\rho + 2i\tau_{\rho\varphi} = 2\frac{\zeta^2}{\rho^2} \left[\frac{\overline{\omega(\zeta)\Phi'(\zeta)}}{\omega'(\zeta)} + \Psi(\zeta) \right] \quad (12)$$

$$\frac{E}{1+\mu}(u + iv) = (3-4\mu)\omega(\zeta) - \frac{\omega(\zeta)}{\omega'(\zeta)}\overline{\varphi'(\zeta)} - \overline{\psi(\zeta)} \quad (13)$$

$$\Phi(\zeta) = \varphi'(z) = \frac{\varphi'(\zeta)}{\omega'(\zeta)} \quad (14)$$

$$\Psi(\zeta) = \psi'(z) = \frac{\psi'(\zeta)}{\omega'(\zeta)} \quad (15)$$

Based on conformal mapping, during conversion of layered strata into a homogeneous one, when tunnel lining is neglected, forces, including $\bar{F}_x$, $\bar{F}_y$, $\bar{f}_x$, and $\bar{f}_y$, on the inner boundary of the tunnel are all zero. We calculate the values of f_0 , $\varphi_0(\zeta)$, and $\psi_0(\zeta)$ and substitute the results in Eqs. (4) and (5). Then, we obtain the values of $\varphi(\zeta)$ and $\psi(\zeta)$ and substitute the results in Eq. (11). Eqs. (12) and (13) can be used to respectively calculate the stress and displacement of tunnel surrounding rock in the layered strata without considering the tunnel lining.

When considering tunnel lining, the calculation of stress and displacement of the tunnel surrounding rock in layered strata is as follows. First, we apply the theory of Покровский equivalent layer method and convert the layered strata into a homogeneous one. Then, we convert the tunnel excavation and lining contours in the layered strata into their corresponding counterparts in homogeneous strata, respectively. We then convert the tunnel surrounding rock, including tunnel excavation and lining contours in plane z into the outer domain of circle in plane ζ , using the formula for stress function of the tunnel surrounding rock; the formula includes the excavation and lining contours in homogeneous strata that were proposed by Kargar et al.[6], Lu et al. [7, 8], and Chen [15]. We then combine the results with the corresponding continuous boundary conditions of stress and displacement. Finally, the stress and displacement of surrounding rock considering lining tunnel can be obtained [16].

4. Comparative analysis of numerical simulations

Based on Midas GTS NX, by comparing the stress of surrounding rock of the rectangular tunnel in layered strata with that of surrounding rock of the corresponding tunnel in homogeneous strata, we discuss the feasibility of an equivalent layered-to-homogeneous formation. When using Midas GTS NX software to perform numerical simulations, we assume the height-by-width dimensions of 8 m $\times$ 6 m

and elastic moduli of $E=8000$ MPa and $E=8000$ MPa for the upper and lower strata, respectively. Uniform pressures of the tunnel vertical and horizontal total $\sigma_1=10.125$ kN/m² and $\sigma_2=76.5$ kN/m², respectively. The results of numerical simulation are shown in Figs. (2) and (3). According to the theory of Покровский equivalent layer method, the elastic modulus of homogeneous strata corresponding to that in layered strata is $E=10000$ MPa, and height multiplied by width yields 6.7804 m x 6 m. The results of numerical simulation are shown in Figs. (4) and (5).

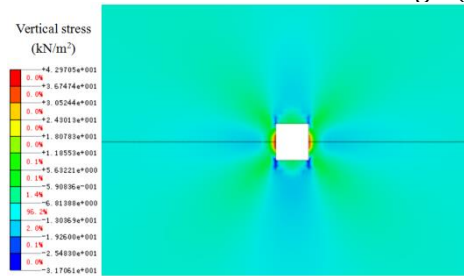


Figure 2. Vertical stress diagram of tunnel in layered strata

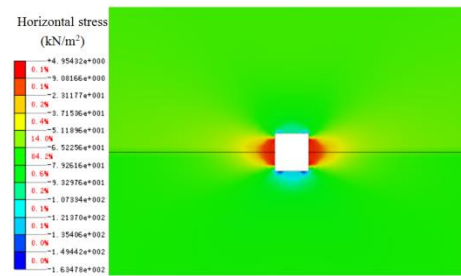


Figure 3. Horizontal stress diagram of tunnel in layered strata

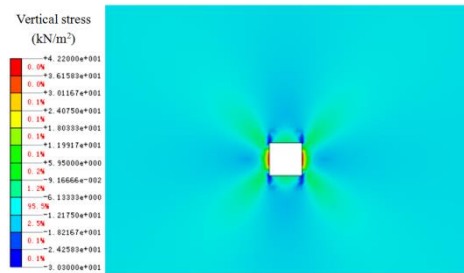


Figure 4. Vertical stress diagram of tunnel in homogeneous strata

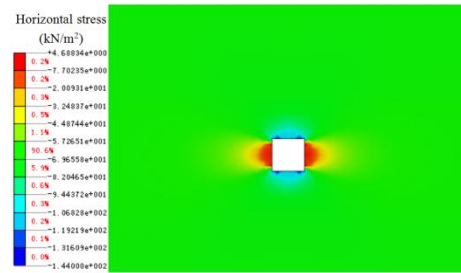


Figure 5. Horizontal stress diagram of tunnel in homogeneous strata

The numerical simulation results show that the point at infinity away from the tunnel is unchanged by stress concentration, and its vertical and horizontal stresses are the same as σ_1 and σ_2 , respectively. Vertical stress σ_y at four corners around a rectangular tunnel shows stress concentration, and its value is significantly greater than that of the other point, as presented in Figs. (2) and (4). The maximum vertical stresses at each corner of the rectangular tunnel in the layered and homogeneous strata reach 4.95 and 4.69 kN/m², respectively. The similarity between these stresses equals 96%. From Figs. (3) and (5), along the x-axis, the horizontal stress values around the rectangular tunnel in layered and homogeneous strata total 4.95 and 4.69 kN/m², respectively, indicating a 95% similarity. Hence, converting the layered strata into a homogeneous one is feasible according to the theory of Покровский equivalent layer method.

5. Sample analysis of deep-buried tunnel in layered strata

Assuming that the tunnel is rectangular and neglecting the influence of lining support, the longitudinal length of the tunnel is larger than its cross-section. This problem is considered a plane strain problem for analysis. The tunnel features a buried depth of $h=100$ m, width of $b=6$ m, and height of $a=8$ m. In the upper stratum, elastic modulus of the tunnel is $E_1=2500$ MPa, Poisson's ratio is $\mu_1=0.3$, and angle of internal friction is $\varphi_1=39^\circ$. Considering the lower stratum, the elastic modulus of the tunnel is $E_2=10000$ MPa, Poisson's ratio is $\mu_2=0.25$, and angle of internal friction is $\varphi_2=45^\circ$ (as shown in Fig. 6). Table 2 provides the physical and mechanical parameters of strata in detail. From the theory of Покровский equivalent layer method, after the layered strata are converted into a homogeneous one, the width of corresponding homogeneous formation is $b=6$ m, and the height is $a=4$ m (shown as Fig. 7). The conversion equation is presented as Eq. (21). The center of the tunnel is used as origin of the x-y coordinate system.

Table 2. Physical and mechanical parameters of strata

Strata	Modulus of elasticity (MPa)	Poisson ratio	Internal friction angle (°)	Density (kg/m ³)
Upper strata	2500	0.3	39	1500
Lower strata	10000	0.25	45	2206

$$\begin{cases} y' = y \\ x' = \begin{cases} \frac{1}{2}x(x > 0) \\ x(x < 0) \end{cases} \end{cases} \quad (16)$$

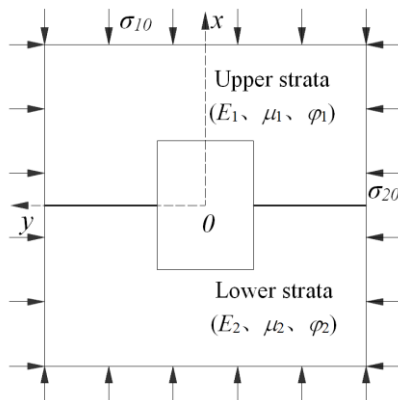


Figure 6. Tunnel shape diagram of layered strata

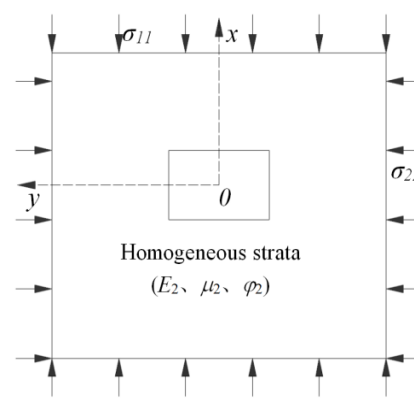


Figure 7. Tunnel shape diagram of homogeneous strata

5.1. Values of $\varphi(\zeta)$ and $\psi(\zeta)$ of rectangular tunnel in layered strata

As the buried depth of the tunnel is larger than the span of the tunnel, the vertical and horizontal stresses (σ_1 and σ_2) of the infinite elastic body, which covers an infinite distance from the tunnel, can be regarded as the vertical σ_{11} and horizontal stresses σ_{22} of the tunnel center, respectively. According to the calculation method used to determine the uniform vertical pressure of deeply buried railway tunnels adopted in China [17], the calculation formula is obtained as shown in Eqs. (17) and (18). We obtain the vertical stress $\sigma_1 = \sigma_{11} = 67.5 \text{ kN/m}^2$ and horizontal stress $\sigma_2 = \sigma_{22} = 10.125 \text{ kN/m}^2$. Substituting the values into Eqs. (9) and (10), we obtain the values of B and $(B' + C'i)$ as 19.4063 and -28.6875 kN/m^3 , respectively. In Eqs. (14) and (15), s refers to the grade of surrounding rock. If the grade of surrounding rock is III, $s=3$. $\omega = 1 + i(B-5)$ denotes the influence coefficient of width; B refers to the tunnel width in meters. Variable i represents the increase and decrease rate of surrounding rock pressure; when $B < 5 \text{ m}$, $i = 0.2$, and when $B > 5 \text{ m}$, $i = 0.1$. λ refers to the lateral pressure coefficient, and $\lambda = 0.15$.

$$\sigma_1 = 0.45 \times 2^{s-1} \times \gamma \omega \quad (17)$$

$$\sigma_2 = \lambda \sigma_1 \quad (18)$$

As the tunnel lining is neglected, the forces, including $\bar{F}_x$, $\bar{F}_y$, $\bar{f}_x$, and $\bar{f}_y$, on the inner boundary of the tunnel are all zero. Eqs. (9) and (10) can be respectively written as follows:

$$\varphi(\zeta) = B\omega(\zeta) + \varphi_0(\zeta) \quad (19)$$

$$\psi(\zeta) = (B' + iC')\omega(\zeta) + \psi_0(\zeta) \quad (20)$$

To solve Eqs. (19) and (20), the mapping function must be solved first. From Eq. (3), the problem of solving the mapping function will eventually become the problem of solving parameters R, C_1 and C_3 , where parameter R can be represented by parameters C_1 and C_3 . Eq. (19) shows the relationship of R, C_1 and C_3 . Compound optimization is adopted to calculate parameters C_1 and C_3 , the objective function is shown in Eq. (20). Matlab is used to calculate the values of C_1 and C_3 . The values of C_1 , C_3 and R are 0, -0.17 and 3.6145, respectively. The mapping function is shown in Eq. (23).

$$R = \frac{r_0}{1 + C_1 + C_3} \quad (21)$$

$$f = \sum_{i=1}^4 \{r_i - R[\cos(\alpha_i - \beta_i) + C_1 \cos(\alpha_i - \beta_i) + C_3 \cos(\alpha_i - 3\beta_i)]\} \quad (22)$$

$$\omega(\zeta) = 3.6145\left(\frac{1}{\zeta} - 0.17\zeta^3\right) \quad (23)$$

$\varphi_0(\zeta)$ is an analytic function of the outer domain of circle, we can obtain the form of Eq. (11) as shown in Eq. (24). From Eq. (24), which calculates $\varphi_0(\zeta)$, Eq. (26) for $\psi_0(\zeta)$ can be obtained. Substituting Eqs. (24) and (25) into Eqs. (19) Eq. (20), respectively, we obtain the values of $\varphi(\zeta)$ and $\psi(\zeta)$ as Eqs. (26) and (27).

$$\varphi_0(\zeta) = -88.6248\zeta + 23.849\zeta^3 \quad (24)$$

$$\psi_0(\zeta) = -15.0662\zeta^{-1} - 70.1441\zeta + 17.6275\zeta^3 + \frac{\zeta^4 - 0.17}{0.51\zeta^4 + 1}(71.547\zeta - 88.6248\zeta^{-1}) \quad (25)$$

$$\varphi(\zeta) = 70.1441\frac{1}{\zeta} - 88.6248\zeta + 11.9245\zeta^3 \quad (26)$$

$$\psi(\zeta) = 88.6248\zeta^{-1} - 140.2881\zeta + \frac{\zeta^4 - 0.17}{0.51\zeta^4 + 1}(71.547\zeta - 88.6248\zeta^{-1}) \quad (27)$$

5.2. Stress analysis of surrounding rock of rectangular tunnel in layered strata

To solve the stress of surrounding rock of tunnel in layered strata, substituting $\varphi(\zeta)$ and $\psi(\zeta)$, which have already been calculated in Eq. (15) and Eq.(16), respectively, we calculate the values of two analytic functions $\Phi(\zeta)$ and $\Psi(\zeta)$, by substituting $\varphi(\zeta)$ and $\psi(\zeta)$ into Eqs. (12) and (13). We can then obtain the relationship of σ_ρ and σ_φ , as shown in Eqs. (28) and (29).

$$\sigma_\varphi + \sigma_\rho = 4\text{Re}\left(19.4063 - \frac{-24.5192 + 19.7944\zeta^2}{\frac{1}{\zeta^2} + 0.51\zeta^2}\right) \quad (28)$$

$$\begin{aligned} \sigma_\varphi - \sigma_\rho + 2i\tau_{\varphi\rho} = & \left[\begin{aligned} & \rho^2(-5.7 \times 10^5 \zeta^{10} - 5.639 \times 10^8 \zeta^8 - 1.375 \times 10^{10} \zeta^6 + 1.689 \times 10^{10} \zeta^4 + 5.818 \times 10^9 \zeta^2 \\ & + 4.436 \times 10^9) - \rho^8(1.675 \times 10^5 \zeta^6 + 7.743 \times 10^8 \zeta^4 + 2.6 \times 10^9 \zeta^2 - 1.723 \times 10^9) \\ & - 9.072 \times 10^9 \zeta^4 + 1.368 \times 10^{10} \zeta^6 + 4.076 \times 10^9 \zeta^8 + 8.815 \times 10^5 \zeta^{10} + \rho^4(1.41 \times 10^5 \zeta^8 \\ & + 6.521 \times 10^8 \zeta^6 + 2.189 \times 10^9 \zeta^4 - 1.451 \times 10^9 \zeta^2) \end{aligned} \right] \times \\ & \left[\begin{aligned} & \rho^2(2.864 \times 10^7 \zeta^{12} - 2.412 \times 10^7 \zeta^{10} + 1.575 \times 10^8 \zeta^8 - 8.525 \times 10^7 \zeta^6 \\ & + 2.763 \times 10^8 \zeta^4 - 7.423 \times 10^7 \zeta^2 + 1.546 \times 10^8) \end{aligned} \right]^{-1} \end{aligned} \quad (29)$$

Along the x-axis, $\varphi=0$, and $\zeta=\rho$. Substituting their values in Eqs. (29) and (30), we calculate the results of theoretical stresses and numerical simulation; the results are shown in Figs. (9) and (10). Fig. (9) shows that the theoretical calculation results of tangential stress σ_φ agree with those of numerical simulation. When the point of tunnel surrounding rock is from near to far, tangential stress σ_φ initially increases and then decreases. Eventually, this stress forms a smooth line whose value is equal to the horizontal stress applied on the boundary. Fig. (10) shows that the theoretical calculation results of radial stress σ_ρ show good agreement with those of numerical simulation. When the point of surrounding rock of tunnel is from near to far, radial stress σ_ρ increases first and then decreases. Eventually, this stress forms a smooth line whose value is equal to the radial stress applied on the boundary.

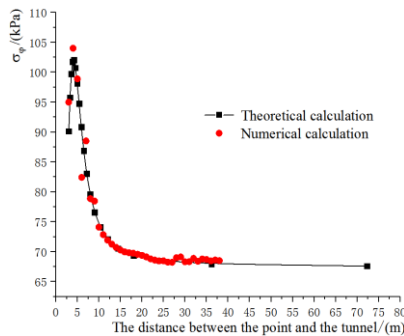


Figure 8. Contrast diagram of tangential stress

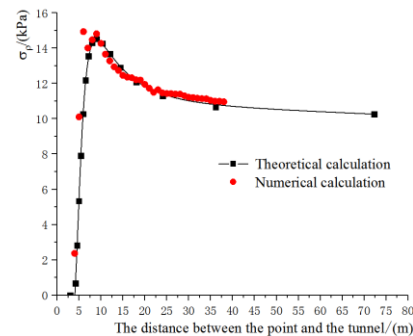


Figure 9. Contrast diagram of radial stress

6. Analysis of influencing factors

The force of surrounding rock of underground engineering is extremely complex, and various factors affect the stress of the tunnel surrounding rock. For rectangular tunnel, using Matlab, Eqs. (16) and (17) are calculated to study the influence of lateral pressure coefficient of surrounding rock, grade of surrounding rock of tunnel, and height-to-width ratio of the tunnel on stress of surrounding rock of the rectangular tunnel. The method can provide theoretical basis for similar projects.

6.1. Influence of the coefficient of lateral pressure on surrounding rock

Fig. (10) shows the tangential stress value of the rectangular tunnel at lateral pressure coefficients (λ) of 0.15, 0.30, 0.5, and 1. The perimeter of the rectangular tunnel corresponds to a circle with radius $\rho=1$ on the complex plane ζ . Fig. (10) indicates that the tangential stress at the four corners of the rectangular tunnel is significantly greater than that at other locations around the tunnel. When the lateral pressure coefficient λ is 0.3, the tangential stress at the four corners of the rectangular tunnel is lower than that when the lateral pressure coefficient λ values are 0.15, 0.5, and 1. The values of tangential stress in increasing order are $\sigma_{\varphi}(\lambda=0.3) < \sigma_{\varphi}(\lambda=0.15) < \sigma_{\varphi}(\lambda=0.5) < \sigma_{\varphi}(\lambda=1)$.

Considering the lateral pressure coefficient λ values of 0.15, 0.30, 0.5, and 1, Fig. (11) shows that when the point of tunnel surrounding rock on the x-axis is from near to far, the tangential stress σ_{φ} increases first and then decreases. Finally, the radial stress stabilizes, and the value is equal to that of radial stress on the model boundary. Fig. (12) shows that when the point of tunnel surrounding rock on the x-axis is from near to far, the radial stress σ_r increases first and then plateaus. The value is equal to that of horizontal stress of the model boundary. Then, we can determine that a greater lateral pressure coefficient indicates a greater tangential stress at the same point.

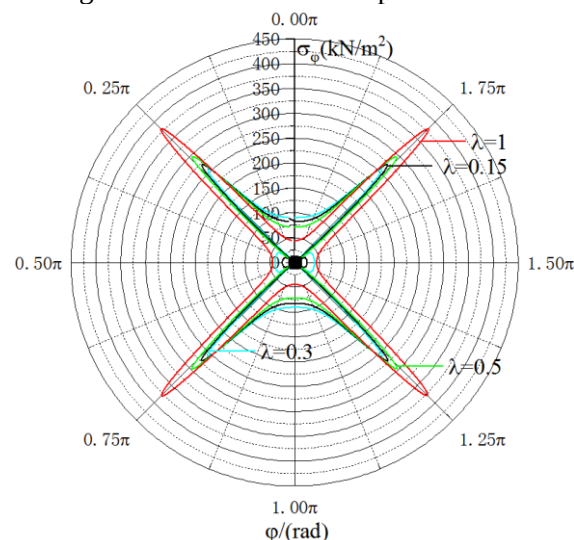


Figure 10. Tangential stress value around the tunnel

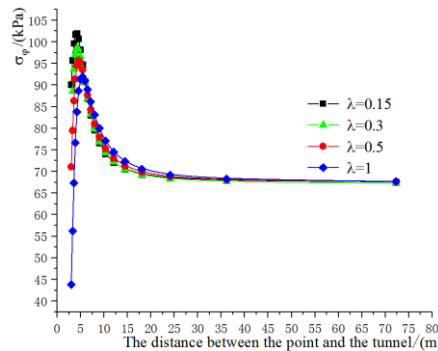


Figure 11. Tangential stress diagram of tunnel surrounding rock

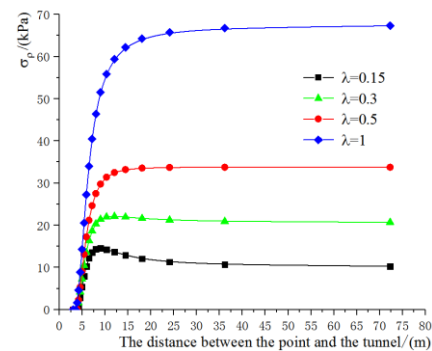


Figure 12. Radial stress diagram of tunnel surrounding rock

6.2. Influence of width-to-height ratio of tunnel on surrounding rock

We study the influence of different height-to-width ratios on the stress of surrounding rock of rectangular tunnel in layered strata; Table 1 shows the width and height of the rectangular tunnel. Fig. (13) shows that with decreasing height-to-width ratio of the tunnel, the tangential stress values at the four corners around the tunnel increase. Figs. (14) and (15) show that when the point of tunnel surrounding rock on the x-axis is from near to far, the radial stress σ_r and tangential stress σ_ϕ increase first and then decrease. The radial stress σ_r and tangential stress σ_ϕ stabilize, and their values are equal to those of stresses on the model boundary.

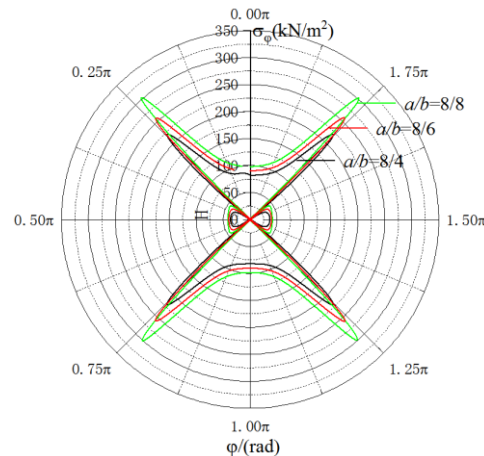


Figure 13. Tangential stress value around the tunnel

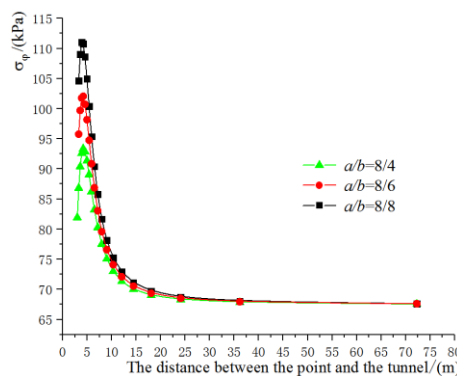


Figure 14. Tangential stress diagram of tunnel surrounding rock

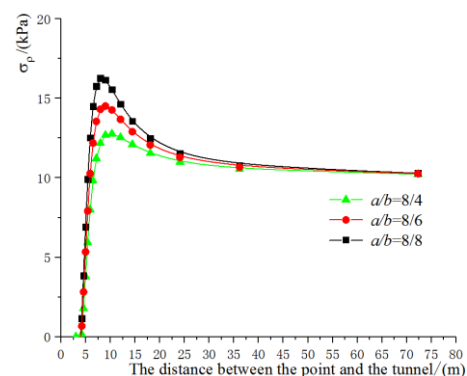


Figure 15. Radial stress diagram of tunnel surrounding rock

6.3. Influence of the grade of surrounding rock on surrounding rock

According to Eq. (17), which calculates the vertical uniform pressure of deep-buried railway tunnel adopted in China [17], a proportional relationship exists between the surrounding rock pressure of deep-buried tunnels and the grade of tunnel surrounding rock. In layered strata, the height and width of the tunnel are 8 and 6 m, respectively. The weight of the surrounding rock of corresponding tunnel in homogeneous strata is $\gamma=2206 \text{ kg/m}^3$, and the coefficient of lateral pressure of surrounding rock is $\lambda=0.15$. We compare and analyze the stresses of three grades of tunnel surrounding rock, denoted as III, IV, and V. Table 3 presents the stress of surrounding rock corresponding to the grade of tunnel surrounding rock. Fig. (16) shows that the tangential stress at the four corners of the rectangular tunnel is significantly greater than that at other locations around the tunnel. A higher grade of surrounding rock implies a larger tangential stress. Figs. (17) and (18) show that when the point of tunnel surrounding rock on the x-axis is from near to far, the radial stress σ_r and tangential stress σ_θ increase first and then decrease. The radial stress σ_r and tangential stress σ_θ reach a plateau, and the values are equal to those of the model boundary.

Table 3. Stress corresponding to classification of surrounding rock

Grade of surrounding rock	Homogeneous strata tunnel		Density (kg/m ³)	Lateral pressure coefficient	σ_1 (kN/m ³)	σ_2 (kN/m ³)
	Depth (m)	Width (m)				
III	6	6	2206	0.15	67.5	10.125
IV	6	6	2206	0.15	135.0072	20.2511
V	6	6	2206	0.15	270.0144	40.5022

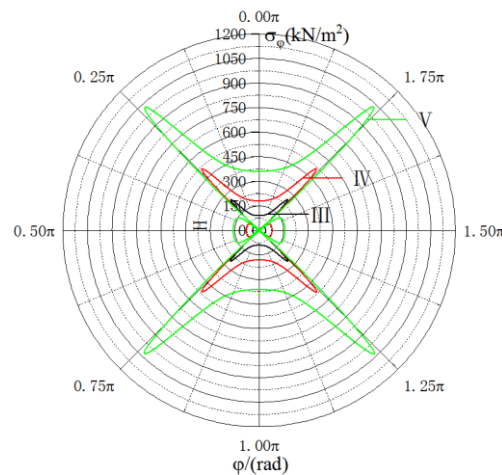


Figure 16. Tangential stress values around the tunnel

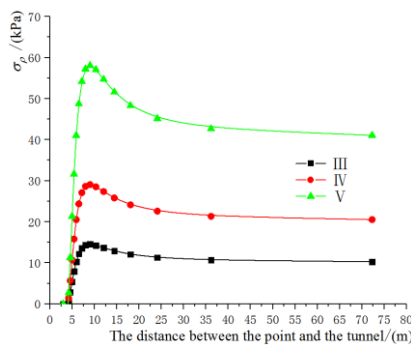


Figure 17. Tangential stress diagram of tunnel surrounding rock

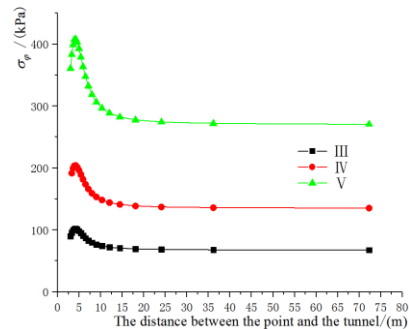


Figure 18. Radial stress diagram of tunnel surrounding rock

7. Conclusion

(1) Starting from the theory of Покровский equivalent layer method, we convert the layered strata into a homogeneous one and then use conformal mapping to convert the rectangular tunnel in plane z to the unit circle in plane ζ . The stress of tunnel surrounding rock in the layered strata is obtained by solving the analytic functions $\Phi(\zeta)$ and $\Psi(\zeta)$.

(2) Based on Midas GTS NX, by comparing the stress of surrounding rock of the rectangular tunnel in layered strata with that of surrounding rock of the corresponding tunnel in homogeneous strata, we know that converting the layered strata into a homogeneous one is feasible according to the theory of Покровский equivalent layer method.

(3) We study the effect of lateral pressure coefficient on deep-buried rectangular tunnel. Studies have shown that when the lateral pressure coefficient λ is 0.3, tangential stress at the four corners of the rectangular tunnel is lower than when lateral pressure coefficients λ are 0.15, 0.5, and 1. The values of tangential stress in increasing order are $\sigma_{\varphi(\lambda=0.3)} < \sigma_{\varphi(\lambda=0.15)} < \sigma_{\varphi(\lambda=0.5)} < \sigma_{\varphi(\lambda=1)}$.

(4) For deep-buried rectangular tunnels, we study the influence of four height-to-width ratios on the stress of tunnel surrounding rock. With decreasing height-to-width ratios, the tangential stress at the four corners of the tunnel increases.

(5) We select three grades of the surrounding rock and study the effect of grade on deep-buried rectangular tunnel. It can be concluded that the higher the surrounding rock grade of the tunnel is, the greater the tangential stress at the four corners of the tunnel is.

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