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A scalable fuzzy division model of topological space on GIS

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Abstract. It is difficult to describe topological relationship of complicated geographical phenomena and entity with fuzziness and uncertainty in the real world. Aiming at the above problem, this paper has summarized to the topological relationship of between spatial entities with 4-intersection model and 9-intersection model firstly. On the basis of point-set topology, the formal description model of the topological relationship between fuzzy geographical entities is analysed and gives a detailed and accurate formalization-description and strict math formula. Lastly, this paper proposes a method, which improves and expands topological division to between fuzzy geographical entities. It shows that analysing result, the model can analyse and divide the entity fuzzily from simple entity to complicated entity.

1. Introduction

Geographic entity can be divided into two categories approximately in the real world [1], one is exact spatial range of geographic entity: such as buildings and artificial models, the other is inexact spatial range of geographic entity: such as plough, forest cover and a distribution of the polluted zone act. Those attributes of geographic entities are continuous distributed in space, and it is fuzziness because of spatial variations and the definition and classification of property. Those spatial entities are described by the method of target or fields, and the corresponding is the vector data and grid data. Point, Line and area are the basic elements of vector data, and the grid data is represented by pixel. The topological relationship of spatial entities is the most important relations, and it is also the basis of spatial query, analysis and reasoning [2].

Previous studies have focused on location data error and include random uncertainties [3]. Many scholars have been carry out intensive study for the fuzziness of spatial entities. One of them is proposed the method of describing fuzzy region by fuzzy theory, analyses uncertainty of the space and gives morphological description of boundary, inside and outside [4]. The geometric structures of fuzzy region is discussed, and the shortcomings of existing researches are analyzed and pointed out. Finally, a generalized model of describing fuzzy region is set up, and new methods of formalization for fuzzy region in thematic map is presented [5]. This paper deals with building a special fuzzy topological space for fuzzy sets. Based on it, a formal definition of simple fuzzy regions is given. A 4*4-intersecion approach is proposed for the formalization of topological relations [6]. This paper presents a method of fuzzy representation of regional boundary based on fuzzy set theory, and obtained the conclusion that regional boundary is a fuzzy band. On the basis of discussing appropriateness of geographically cartographic boundary from degree of changes between regional attribute values, presents a new method of handling meaningless polygons on overlaid map derived from vector GIS [7]. The describing methods of uncertain linear objects in the present are summarized, and the 9-intersection model for describing topological relations between crisp linear objects is applied. Then the topological relations between uncertain linear objects are described and determined by a qualitative analyzing method [8]. Rough set theory is used based on a 4-intersection model, which can describe the spatial topological relations of certainty and uncertainty region has been put forward, and the topological relations of fuzzy objects are discussed [9].

2. The summary of 4-Intersection model and 9-Intersection model

Research on spatial topological relations of geographical entities, Egenhofer has defined 4-intersection model and 9-intersection model on the basis of point-set topology early [10]. Topological relationship between two objects is presented by determining whether the intersection of internal set and boundary set of object is empty or not in 4-Intersection model, that is $T(A,B)=\begin{bmatrix} A^0 \cap B^0 & A^0 \cap \partial B \\ \partial A \cap B^0 & \partial A \cap \partial B \end{bmatrix}$, there are A^0, B^0, ∂A,∂B, which are presented internal of *A*, internal of *B*, boundary of *A* and boundary of *B* separately; There is two cases, one is intersection using 1 representation, the other is non-intersection using 0 representation, and the model can be distinguishable 16 relations in theory.

Topological relationship between two objects is presented by determining whether the intersection of internal set, boundary set and external set of object is empty or not in 9-intersection model, that is $T(A,B)=\begin{bmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^0 & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$, there are A^0, A^- and ∂A, which represent internal, external and boundary of A respectively. The model can be distinguishable 512 relations in theory, but most of relations is meaningless. It is include disjoint, meet, overlap, covers, contains, covered-by, inside and equal. There are meaningful topological relations, such as topological relation of between area and area, topological relation of between line and area, topological relation of between line and line [11]. There is disadvantage, which is unable to deal with uncertain topological relation. It is impossible to build model a case with a certain width of boundary, and can't describe the topological relation of between fuzzy objects.

3. Fuzzy division of point and area for GIS vector data

3.1. The fuzzy dividing of point

In fuzzy geography information space, point-object is divided into fuzzy interior and fuzzy exterior as follow respectively. There is overlap about fuzzy interior and fuzzy exterior, just membership is different.

The fuzzy interior of point entity refers to origin *point(x,y)* as circle α as radius fuzzy circle constitute set, the interior fuzzy membership reduce to 0 from fuzzy circle to boundary. The fuzzy exterior of point entity refers to origin *point(x,y)* as circle α as radius fuzzy circle constitute set, the exterior fuzzy membership increase from fuzzy circle to boundary, and the exterior membership in fuzzy circle is 1.Parameter α satisfy $\alpha>0$, α is radius of fuzzy circle; Which *d(x,y)* represents the distance of *point(x,y)* to fuzzy circle. The fuzzy membership function of point is shown in figure 1.

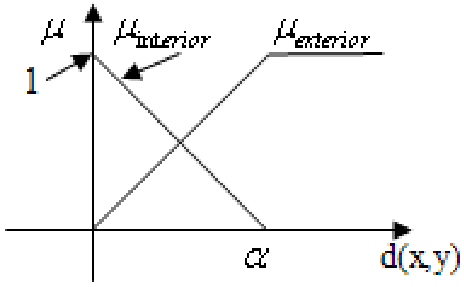

Figure 1. The fuzzy membership function of point

The interior fuzzy membership of point is shown in formula 1, and the exterior fuzzy membership of point is shown in formula 2 [11].

$$\mu_{in}(x,y)=\begin{cases}1-\dfrac{d(x,y)}{\alpha} & 0\le d(x,y)\le\alpha \\ 0 & others\end{cases} \tag{1}$$

$$\mu_{ex}(x,y)=\begin{cases}\dfrac{d(x,y)}{\alpha} & 0\le d(x,y)<\alpha \\ 1 & others\end{cases} \tag{2}$$

3.2. The fuzzy dividing of area

Area is common element of GIS, and the topological relation is related to the distance between of entities. When the distance is far enough, topological relations is weak, that is only disjoint; the distance is near, there are many topological relations and measure information. The ambiguity of topological relations caused to the ambiguity of boundary of various states. The distribution of spatial entity is non-uniform, but the boundary of region is certain, the distribution character's centroid as center, the extent of character display changes are non-uniform to exterior dispersion, and the centroid inside the region. For fuzzy area of spatial objects attribute is distributed evenly [11], fuzzy space partition of area entity is divided into fuzzy boundary, fuzzy interior and fuzzy exterior. The definitions as follow respectively.

Definition 1: The fuzzy boundary of area entity refers to each point on the boundary of area object as circle α>0 as radius constitute fuzzy circle. In the original borders, the fuzzy membership is 1. Farther away from the border, the boundary fuzzy membership degree is smaller. When it is not inside fuzzy brand, the boundaries fuzzy membership degree is 0.

Definition 2: The fuzzy exterior of area entity refers to original boundary' exterior of area object constitute sets, the exterior fuzzy membership degree is bigger and bigger. When it is on the fuzzy exterior boundary, the fuzzy membership degree is 1.

Definition 3: The fuzzy interior of area entity refers to original boundary' interior of area object constitute sets, farther away from the original border, the interior fuzzy membership degree is bigger, When it is on the fuzzy interior boundary, the fuzzy membership degree is 1.

Let's assume that boundary dividing function $D(x, y)$, $d_{\min}(x,y)\geqslant 0$ represents any $p(x, y)$ to original boundary of area entity's distance is minimum. A^-, ∂A, A^0 represents the interior, boundary and exterior of certain entity A. The fuzzy membership function of area is shown in figure 2. The function of boundary division $D(x, y)$ is shown in formula 3.

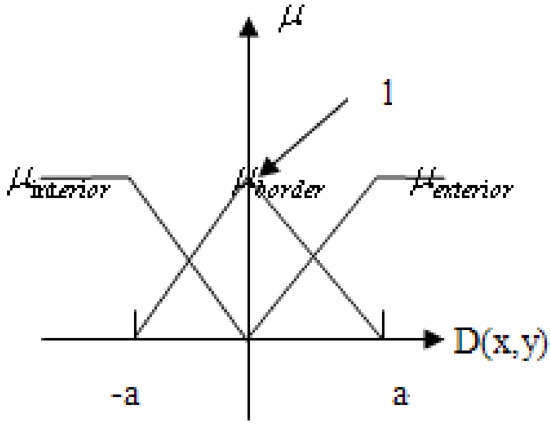

Figure 2. The fuzzy membership function of area

$$D(x,y)=\begin{cases} d(x,y) & (x,y)\in A^{-} \\ -d(x,y) & (x,y)\in A^{0} \\ 0 & (x,y)\in \partial A \end{cases} \tag{3}$$

The fuzzy boundary membership degree of area entity is shown in formula 4 [11].

$$\mu_{border}(x,y)=\begin{cases} 0 & D(x,y)\le -\alpha \\ \dfrac{\alpha+D(x,y)}{\alpha} & -\alpha<D(x,y)<0 \\ 1 & D(x,y)=0 \\ \dfrac{\alpha-D(x,y)}{\alpha} & 0<D(x,y)<\alpha \\ 0 & D(x,y)\ge \alpha \end{cases} \tag{4}$$

The interior fuzzy membership of area is shown in formula 5, and the exterior fuzzy membership of area is shown in formula 6 [11].

$$\mu_{in}(x,y)=\begin{cases} 1 & D(x,y)\le -\alpha \\ \dfrac{D(x,y)}{-\alpha} & -\alpha<D(x,y)<0 \\ 0 & others \end{cases} \tag{5}$$

$$\mu_{ex}(x,y)=\begin{cases} 1 & D(x,y)\ge \alpha \\ \dfrac{D(x,y)}{\alpha} & 0<D(x,y)<\alpha \\ 0 & others \end{cases} \tag{6}$$

4. Scalable fuzzy topological division of uncertain area

It is difficult to confirm metric parameter of space relation because of being affected by surroundings and other factors in geographic information space [12]. The distributions of entity is non-uniform, and the original boundary is uncertain, each of distribution feature is the centroid as center, the extent of characteristic display is non-uniform proliferation, and the centroid is inside region.

It is certain centroid by taking average value of system position of the mass of each point in point-system as the weight value. Let's assume that it take the same volume of n points in the area evenly, those mass of points are $m_1, m_2, \dots, m_n$. The coordinates of each point are (x_1,y_1), (x_2,y_2),…,(x_n,y_n) separately, M is the summation of mass as $M=\sum_i m_i$; Let's assume that (x_c,y_c) is coordinates of centroid, and it is certain that the coordinates of centroid by $x_c=\frac{\sum_i m_i x_i}{M}$ and $y_c=\frac{\sum_i m_i y_i}{M}$.

Let's assume that whole regional space is limited set, and it calculates the nearest Euclidean distance from all points to the original boundary in confined the original boundary of area beyond as $\beta_{\min}(x,y)$, finds the maximum distance from one of *point* (x_f,y_f) to the original boundary as $\beta_{\max}=\max\{\beta_{\min}(x,y)\}$.

Definition 4: The fuzzy boundary of uncertain area object refers to each point on the boundary of uncertain area object as a center of circle, ($0 < \alpha < \beta_{max}$)as radius constitute fuzzy circle, and then structures fuzzy band. When it is on the original borders, the fuzzy membership of border is 1. Farther away from the border, the fuzzy membership degree of border is smaller. When it is not inside fuzzy band, the fuzzy membership degree of border is 0.

Definition 5: The fuzzy exterior of uncertain area object refers to the limited space original boundary' exterior of uncertain area object constitute sets, farther away from the border, the exterior fuzzy membership degree is bigger and bigger. When it is on the most further point, the exterior fuzzy membership degree is 1.

Definition 6: The fuzzy interior of uncertain area object refers to the limited space within the original boundary of uncertain area object constitute sets, near from the centroid, the interior fuzzy membership degree is bigger and bigger. When it is on the centroid, the interior fuzzy membership degree is 1.

$D(x,y)$ is a partition function, which is formula 7, and $d(x,y) \geqslant 0$ represents minimum distance from any point to original boundary of area; $d_{in\max}(x,y)$ represents the distance from extension line of start with the centroid passing through any point $P_{any}(x, y)$ to the intersection $P_{int}(x, y)$ with the boundary; $d_{in\min}(x,y)$ represents the distance from the centroid within the original boundary of area object to any point $P_{any}(x, y)$, which is shown in formula 8 and formula 9.

$$D(x,y) = \begin{cases} d(x,y) & (x,y) \in A^- \\ -d(x,y) & (x,y) \in A^0 \\ 0 & (x,y) \in \partial A \end{cases} \tag{7}$$

$$d_{in\max}(x,y) = dis_{\overline{C-Pany}} + dis_{\overline{Pany-Pint}} \quad (P_{any} \in A^0) \tag{8}$$

$$d_{in\min}(x,y) = dis_{\overline{C-Pany}} \quad (P_{any} \in A^0) \tag{9}$$

A^-, ∂A and A^0 represent internal, boundary and external of A respectively.

The fuzzy boundary membership degree of uncertain area is shown in formula 10.

$$\mu_{border}(x,y) = \begin{cases} 0 & \alpha \le |D(x,y)| \\ \dfrac{\alpha - |D(x,y)|}{\alpha} & 0 < |D(x,y)| < \alpha \\ 1 & D(x,y) = 0 \end{cases} \tag{10}$$

The fuzzy interior membership degree of uncertain area is shown in formula 11.

$$\mu_{in}(x,y) = \begin{cases} 1 & d_{in\min}(x,y) = 0 \\ \dfrac{d_{in\min}(x,y)}{d_{in\max}(x,y)} & D(x,y) < 0 \\ 0 & others \end{cases} \tag{11}$$

The fuzzy exterior membership degree of uncertain area is shown in formula 12.

$$\mu_{ex}(x,y)=\begin{cases}1 & D(x,y)=\beta_{max}\\ \dfrac{\beta_{min}(x,y)}{\beta_{max}} & 0<D(x,y)\\ 0 & others\end{cases} \tag{12}$$

The above research is improvement the topological partition model for fuzzy area in complex space on the basis of the literature [11], and proposed corresponding fuzzy membership function. There are detailed definitions and formal description with a rigorous mathematical method in theory. This model is applicable to fuzzy geographic entities of different levels of complexity and helps readers have a clearer understanding of the division of spatial fuzzy entities.

5. Conclusion

This paper analyzes and summarizes in detail on the basis of existing fuzzy formal description of GIS simple elements. Meanwhile, aiming at the deficiency on description of the topological partition of fuzzy points and fuzzy areas, this paper gives a more detailed and accurate description of the formal description and the expression of mathematical formula, and it has achieved topology partitioning from simple entity to complex fuzzy geographic entity finally. The author will further research more complex geographic phenomena of existing many different geographic features in an area and partition fuzzy space by experiment.

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