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Research on Moment of Inertia Measurement Method

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Abstract. A method for measuring the moment of inertia has been proposed. The dynamic model of stator current and load moment of inertia during the motor's frequency speed control has been built. Research show that load moment of inertia and the max of stator current are submitting linear relationship, which means that moment of inertia can been calculated after the stator current during the motor's frequency speed control has been measured. Finally, the linear relationship between the moment of inertia and the stator current has been proved with a series of experiment, therefore, the feasibility of the method has been validated. The method has a better repeatability while the repeatability error is 3%.However, by recalculating the experiment data with the math model, the result indicate that the maximum error is 39%.The main factors of the error are math model building error and the instrument uncertainty. In order to improve the precision of the moment of inertia measurement, research should be taken to build a more exact math model.

1. Introduction

Moment of inertia is a index of rigid body inertia while turning. It is a very important physical meaning, usually represented by letter I or J. Moment of inertia is obtained by calculation [1] and experimental method. Calculation is always using for solving the inertia of rigid body with regular shape and uniform mass distribution. However, moment of inertia of most objects cannot been calculated, experimental means are generally using in practice.

The theoretical basis of the experimental method for moment of inertia measurement is the Newton's second law of the rotor, torque is equal to the product of moment of inertia and angular acceleration. Therefore, the measurement of the moment of inertia depends on measuring of the torque and the angular acceleration of the rotor. The traditional methods of offline measurement are basically the direct detection of torque and angular acceleration what have proposed a high requirement for equipment and operators [2]. For this purpose, it is necessary to start the study of indirect measurement of torque and angular acceleration in order to achieve the measurement of moment of inertia.

2. Dynamic model of moment of inertia measurement

2.1. Hysteresis model of asynchronous motor

The basic model of asynchronous motors is a transformer. Energy is transmitted by means of a magnetic field between stator and rotor. Hysteresis is existed inevitably, therefore, dynamic hysteresis model should be established. The dynamic hysteresis model introduced in literature [3] is

$$B(s) = \frac{1}{\sigma_e s + 1} H(s) \tag{1}$$

In the formula, H represents external magnetic field strength, B represents magnetic induction strength, σ_e represents eddy coefficient.

Power output by the stator is the remaining power excluding the proceeds winding resistance consumption, which is represented by P_ω. The equivalent electromagnetic torque output by the stator is $T_\omega = P_\omega/\omega$. The power output by the stator is transmitted to the rotor through the magnetic field between stator and rotor. The power obtained by the rotor is electromagnetic power, represented by P_{em}. Then the electromagnetic torque of the motor is $T_{em} = P_{em}/\omega$ [4]. In fact, when ignoring the leakage of magnetic flux, the values of P_ω and P_{em} are equal. However, due to the existence of hysteresis, the electromagnetic torque T_{em} and the stator output torque T_ω has the following relationship

$$T_{em}(s) = \frac{1}{\sigma_e s + 1} T_\omega(s) \tag{2}$$

Frequency output is generally linear during the frequency control process. It makes the motor operating point slowly changing in the mechanical properties curve. And in order to achieve constant torque, E/f must be a constant [5]. When the initial speed n_0, target speed n_1 and acceleration time Δt are all known, the theoretical angular acceleration of the motor is $\beta = (n_1 - n_0)/\Delta t$.

The torque output by the motor stator changes momentarily after the acceleration command is issued. A step signal is produce, amplitude of the step signal is $\Delta T_\omega = J\beta$. The Laplace transformation of the dynamic change of the stator torque output is $\Delta T_\omega(s) = J\beta/s$.

Then the Laplace transform of dynamic change of the electromagnetic torque is

$$\Delta T_{em}(s) = \frac{1}{\sigma_e s + 1} \Delta T_\omega(s) = \frac{1}{\sigma_e s + 1} \frac{1}{s} J\beta \tag{3}$$

Inverse Laplace transform of Eq.(3) is

$$\Delta T_{em}(t) = 2\pi J \frac{f_1 - f_0}{\Delta t}(1 - e^{-\frac{t}{\sigma_e}}) \tag{4}$$

2.2. Dynamic model of moment of inertia measurement
Asynchronous motor must overcome friction and other mechanical loss during the stable operation, this torque is called motor no-load torque, represented by T_0. The load of the motor, represented by T_2, is constant under normal circumstances [6]. For the acceleration of the motor, another part of the torque is required, which is dynamic torque ΔT_{em}. Then the dynamic electromagnetic torque of the motor can be expressed as

$$T_{em} = \Delta T_{em} + T_2 + T_0 = \Delta T_{em} + T_0' \tag{5}$$

The expression of dynamic electromagnetic torque during frequency conversion acceleration can be obtained from Eq.(4) and Eq.(5)

$$T_{em} = 2\pi J \frac{f_1 - f_0}{\Delta t}(1 - e^{-\frac{t}{\sigma_e}}) + T_0' \tag{6}$$

In addition, we can know form the characteristic curve that the asynchronous motor is always running in areas with low slip. The electromagnetic torque [7~8] of the motor can be approximately expressed as

$$T_{em} = \frac{3p}{2\pi f} I_2'^2 \frac{R_2'}{s} \approx \frac{3p}{2\pi} \frac{E}{f} I_2' \tag{7}$$

The flux balance equation [9] shows that the relationship between the stator current and the equivalent rotor current is

$$I_1 = I_2' + I_{10} \tag{8}$$

In the formula, I_1 represents stator current, I_2' represents equivalent rotor current, I_{10} represents initial stator current.

The dynamic relationship between moment of inertia and asynchronous motor stator current is obtained from Eq.(6) ,Eq.(7) and Eq.(8)

$$I_1 = \frac{2\pi}{3p} \frac{f}{E} [J \frac{f_1 - f_0}{\Delta t} (1 - e^{-\frac{t}{\sigma_e}}) + T_0'] + I_{10} \tag{9}$$

3. Simulation analysis of measurement model

Eq.(12) is the dynamic model of stator current, frequency control parameters and moment of inertia of asynchronous motor. Parameters set in the Matlab simulation of the asynchronous motor shown in Table 1

Table 1. Parameter values set in simulaion

σ_e	E/f	p	f_0（Hz）	f_1（Hz）	I_{10}(A)
0.1	2	4	15	40	0.6

Fig.1 and Fig.2 are the result graph of the stator current of the asynchronous motor under the condition of different moment of inertia J and eddy current coefficient σ_e when the frequency control parameters are the same.
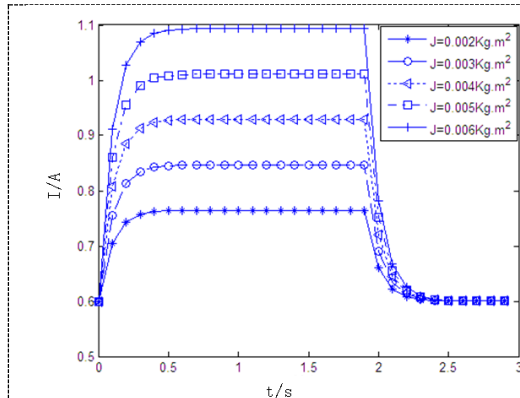

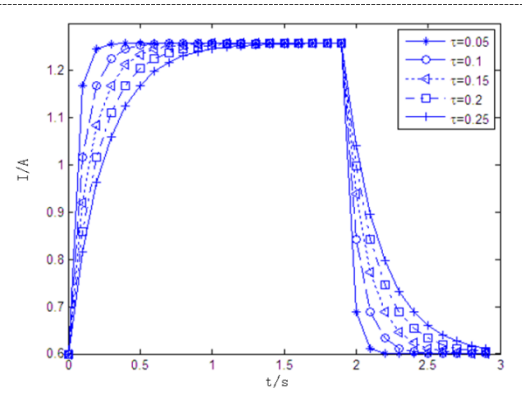| Figure 1. Stator current of different moment of inertia. | Figure 2. Stator current of different eddy current coefficient. |

Obviously from Figure 1, when other the parameters are constant, the stator current always has a stable value (ie, the maximum value), which can be obtained from Eq. (9).

$$I_{1max} = \frac{2\pi}{3p} \frac{f}{E} (J \frac{f_1 - f_0}{\Delta t} + T_0') + I_{10} \tag{10}$$

For a given induction motor, $\dfrac{f}{E}$, $\dfrac{f_1 - f_0}{\Delta t}$, T_0' and I_{10} are all constant when the frequency control

parameters are defined. Then the Eq.(10) can be expressed as linear relationship $I_{1\max} = kJ + b$.

Therefore, for a specific asynchronous motor and a known frequency control conditions, the steady-state value (maximum value) of the stator current of the asynchronous motor has a linear relationship with the moment of inertia of the object to be tested. $I_{1\max} = kJ + b$ is the measurement model of moment of inertia and the two unknown parameters k , b can be calculated by experiments using two standard rotors.

4. Experiments and analysis
We have designed four standard rotors whose moment of inertia can be calculated by the size in order to verify the model. Size parameters of the standard rotors are shown in Table 2.

Table 2. Size parameters of standard rotors.

Inner diameter /mm	Outter diameter /mm	Length /mm	Weight /Kg	Moment of inertia /kgm^2
31	72	80	2.0696	0.0063588
31	86	80	3.1537	0.0131777
31	96	80	4.0457	0.0205865
31	108	80	5.2454	0.0331116

A series of experiments have been conducted with the four standard rotors, the four standard rotors and experimental device are shown in Fig.3.

Figure 3. Standard rotors and experimental device.
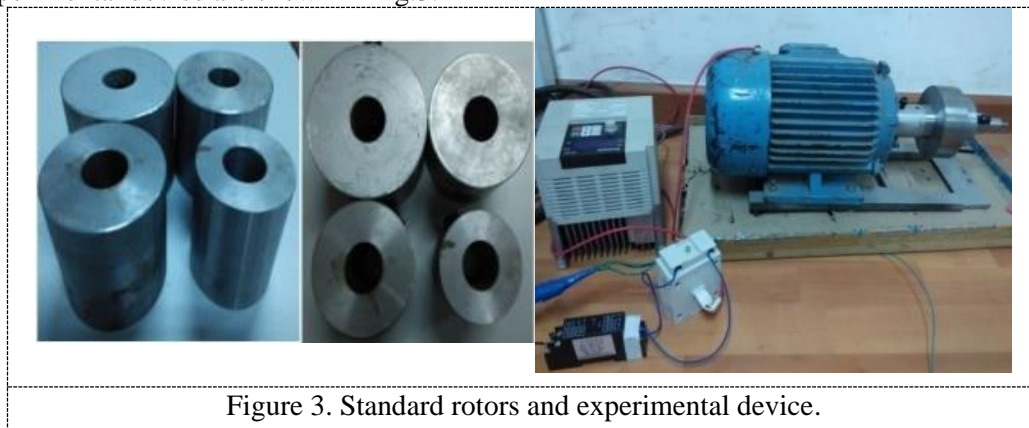
The asynchronous motor used in the experiment is Y90L-2 three-phase motor and is setted to accelerate from 900r / min to 2400r / min in 1 second. Fig.4 shows the actual detected stator current with the four standard rotors under a same conditions.
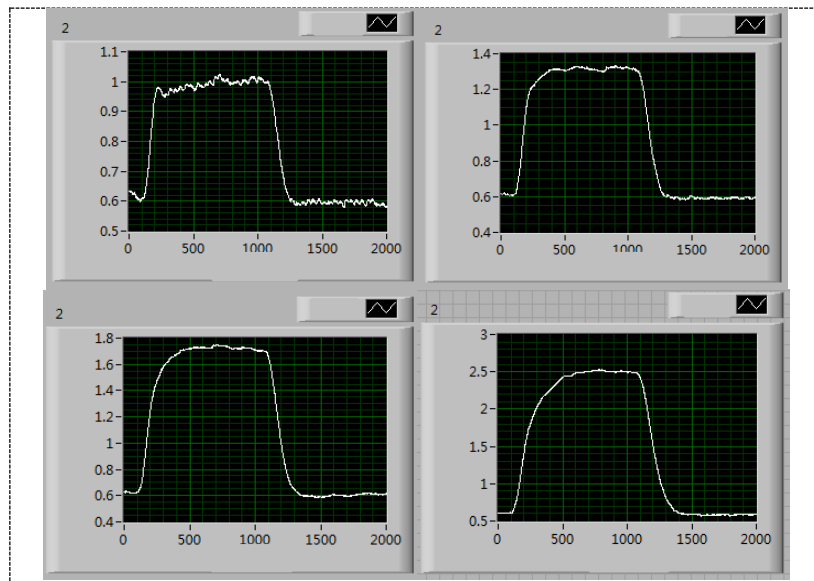
Figure 4. Measured stator current with the four rotors.

Five experiments have been conducted to assess the stability of the measurement method. Table 3 shows the measured stator current of asynchronous motor.

Table 3.Measured stator current of asynchronous motor

1st	2nd	3rd	4th	5th	Mean
1.0182	1.0150	1.0222	1.0182	1.0094	1.0170
1.3400	1.3368	1.3455	1.3300	1.3440	1.3393
1.7386	1.7471	1.7272	1.7467	1.7431	1.7415
2.5240	2.5311	2.5317	2.5427	2.5345	2.5328

The parameters k and b of the measurement model in Eq.(15)can be fitted using the data in Table 3. The fitted model is $I_{1\max} = 57.0222 \times J + 0.6137$. Fig.5 shows the relationship between maximum stator current and moment of inertia.
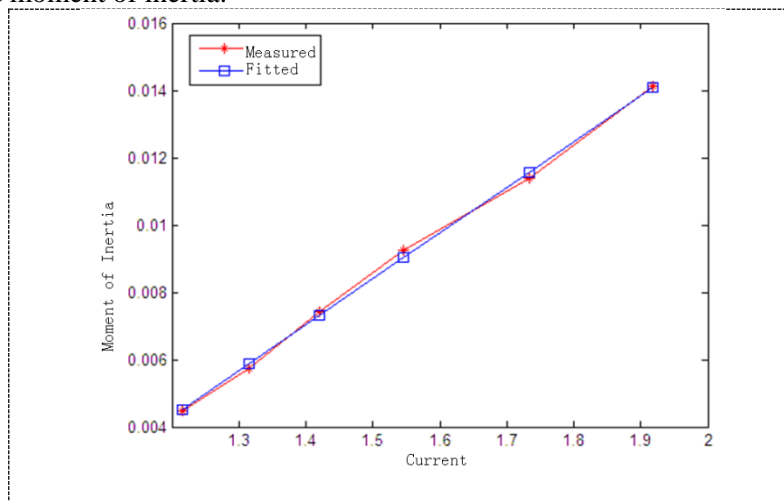

Figure 5. Relationship between maximum stator current and moment of inertia.

There is a linear relationship between the maximum stator current and the moment of inertia of the standard rotors to be tested. It is proved that it is feasible to measure moment of inertia by measuring stator current. At the same time, it can be seen from Table 2 that when the standard rotor is determined,

the maximum standard deviation of the detected stator current is -0.014A and the relative error is 3%. Therefore, repeatability of the measurement method has been proved.

5. Conclusion

This paper presents a method of measuring moment of inertia and establishes a dynamic mathematical model between the stator current and the moment of inertia of the load during the process of frequency control. The research shows that the maximum value of the stator current and the moment of inertia of the load have a linear relationship when the frequency control condition is certain. Therefore, when the experimental conditions are the same, the moment of inertia of the measured object can be calculated by measuring the stator current of the asynchronous motor. Finally, the feasibility and repeatability of the method have been verified through a series of experiments.

The method of moment of inertia measurement proposed in this paper is more convenient and faster than the traditional methods, and has a wide range of applications. A more accurate mathematical model should be established through further research to improve the accuracy of measurement of moment of inertia, in order to put into practical application.

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