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Numerical study of a container ship model for the uncoupled parametric rolling

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Abstract. Parametric roll on ships is a resonance phenomenon whose onset causes heavy roll oscillations leading to dangerous situations for the ship, the cargo and the crew. It affects both small vessels with marginal stability and large container ships if four conditions are met simultaneously: the wave length and ship length are approximately equal, the frequency of encounter wave is twice the roll natural frequency, the ship's roll damping is low enough and the wave height exceeds a limit value characteristic to each vessel. For a container ship, the first two conditions result in a time-varying geometry of the submerged hull and thus to a periodic variation of the transverse stability. In the last two decades, several mathematical models for parametric rolling have been proposed by scientists. In the paper, we cancelled the couplings terms between heave, pitch and roll in a 3 DOF model proposed by Neves and Rodriguez to obtain an uncoupled 1 DOF model for the parametric roll. Even if it seems too simplistic, such a model based on Mathieu type equation is a good start to capture the basic features of the analysed phenomenon. There are no less than seven parameters that can influence the ship stability and the appearance of large roll amplitudes. By modifying two of these parameters one by one and keeping the others constant, a series of colour plots have been drown that capture the role played by the ship's parameters in obtaining dangerous roll amplitudes or too short times to take countermeasures. Analysing the instability regions suggested by these colour plots, one can imagine control measures to reduce the roll amplitudes, such as changing the forward velocity or the heading angle. The data used in the paper have been obtained from experiments with a container ship model in a towing tank followed by expansion to a full scale ship.

1. Introduction

Parametric roll resonance is a nonlinear phenomenon that results in heavy roll motions, with amplitudes of 30 – 40 degrees for ships sailing in following and head seas, which may put the ship in a very dangerous situation that can even capsize it. Fishing vessels, container carriers and passenger vessels are known to be prone to parametric roll in both regular and irregular waves. Several incidents have been reported with significant damage to cargo and ship for millions of euro's, including APL China and Maersk Carolina, and then carefully analysed [1, 2].

Parametric resonance is related to the periodic changes of those coefficients in the roll equation which describe the restoring arm variation in longitudinal waves (connected to the ship stability). These changes are most accentuated in waves of length close to that of the ship and involve an increase in stability when a wave trough is amidships or a decrease in stability when a wave crest is in the same position. Other conditions for parametric roll may also be considered such as the wave

encounter period is approximately half the ship roll natural period, the ship damping in roll is insufficient to avoid the onset of the resonance condition and the wave height exceeds a ship-dependent threshold value [3, 4].

For the modern containerships, factors as the exaggerated bow flare and the pronounced overhang stern make the vessel more susceptible to parametric roll [5, 6].

Apart from the varying restoring arm, other causes for parametric resonance are the coupling between roll, heave and pitch motions. In the last decades, several mathematical models including these couplings have been proposed by scientists [7–10].

In the paper, we intended to find out in a simpler and more enlightening manner the role played by different ship parameters in the appearance and development of parametric roll. To this aim, we settled the coupling term to zero in a three degrees of freedom model for heave, pitch and roll proposed by Neves and Rodriguez [7] to obtain an uncoupled equation for parametric roll. Even if it can be criticized for simplicity, this model proves to be able to capture the basic features of the parametric roll resonance.

2. The parametric roll equation

The equation (1) used throughout the paper for describing the parametric roll motion is written as:

$$\left(I_x + \delta I_x\right)\ddot{\phi} + d_1\dot{\phi} + d_2\left|\dot{\phi}\right|\dot{\phi} + \rho\,g\,\nabla\left(GM_m + GM_a\cos\omega_e t\right)\phi + k_3\phi^3 = 0 \qquad (1)$$

with I_x and δI_x denoting the ship and added mass inertia in roll, d_1 and d_2 the linear and quadratic roll damping coefficients, ρ the water density, g the gravitational acceleration, ∇ the water displacement, k_3 the cubic coefficient of the roll restoring moment, GM_m and GM_a the constant and variable parts of the metacentric height, and ω_e the wave encounter frequency. The last parameter results from, equation (2):

$$\omega_e = \omega_0 - \frac{\omega_0^2}{g}U\cos\beta \qquad (2)$$

where ω_0 stands for the wave frequency, U for the ship forward velocity and β for the ship heading angle. The parameters' values have been taken from an experimental research with a container ship model in a towing tank followed by an expansion to a full scale ship [11, 12]. They are presented in table 1.

Table 1: The ship parameters' values requested in equation (1).

Parameter	Value	Unit
I_x	$1.4014\cdot10^{10}$	$kg\cdot m^2$
δI_x	$2.17\cdot10^9$	$kg\cdot m^2$
d_1	$3.5393\cdot10^8$	$kg\cdot m^2/s$
d_2	$1.8826\cdot10^8$	$kg\cdot m^2/rad$
k_3	$2.974\cdot10^9$	$kg\cdot m^2/s^2\,rad^2$
ρ	1000	kg/m^3
∇	76,468	m^3
GM_m	1.91	m
GM_a	0.84	m

ω_0	0.464	rad/s
β	180	$degrees$
g	9.81	m/s^2

The roll natural frequency, ω_ϕ, is provided by equation (3):

$$\omega_\phi = \sqrt{\frac{\rho g \nabla GM_m}{I_x + \delta I_x}} \tag{3}$$

For the values given in table 1, one obtains $\omega_\phi = 0.2975\, rad/s$ and $\omega_e = 0.6265 rad/s$, such as $\omega_e / \omega_\phi = 2.1055$.

3. Numerical simulations

For avoiding, reducing or controlling the parametric roll resonance effect, it is of great importance to know the influence each ship parameter has on the vessel's stability. In this section, we performed an analysis of the ship response not only for the values included in table 1 but for wide intervals in their closeness. There are no less than seven parameters that can influence the ship stability and the appearance of large roll amplitudes, namely d_1, d_2, GM_m, GM_a, k_3, U and β. We will change some of the parameters one by one while we will fix the others and we will report a series of colour plots which clearly suggest the contribution of each parameter in the vessel's response.

It is well known that in achieving the resonance conditions, a mechanical system greatly amplifies any initial perturbation, regardless of its size. The motion described by equation (1) is not an exception. Thus, figure 1 shows four theoretical similar evolutions of the analysed ship for the GM_m and d_1 values included in table 1 or increased with 20%. It is noticeable that the initial perturbation of only 0.1 degrees is amplified in 300 – 500 seconds to values that raises serious problems for, at least, the transported goods. Of course, an increase in ship stability by increasing the metacentric height or damping leads to diminution of the steady state roll amplitudes.
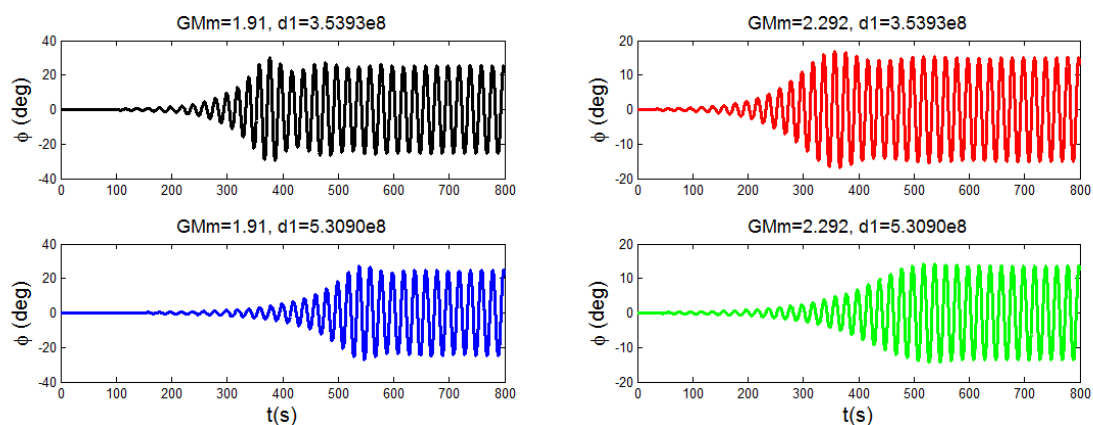

Figure 1. Examples of parametric roll resonance for four combinations of parameters GM_m and d_1.

From the figure above, it is apparent that the stationary state is reached in a few hundred seconds. The maximum amplitude is associated either with the stationary state or with a pre-run time interval. In the following we chose as parametric roll amplitude the average value of the last ten periods of oscillation for a study time of 800 seconds.

Figure 2 presents the steady state parametric roll amplitudes for $\omega_e/\omega_\phi \in [0.2, 2.2]$, $GM_m \in [0.1, 2.6]$ and different combinations (GM_a, d_1). The black point in figure 2(a) stands for the data included in table 1. Doubling or halving the damping coefficient (figures 2(b) and (c)) produces a considerably smaller effect than the constant metacentric height modification (here doubling, see figures 2(d) and (e)).
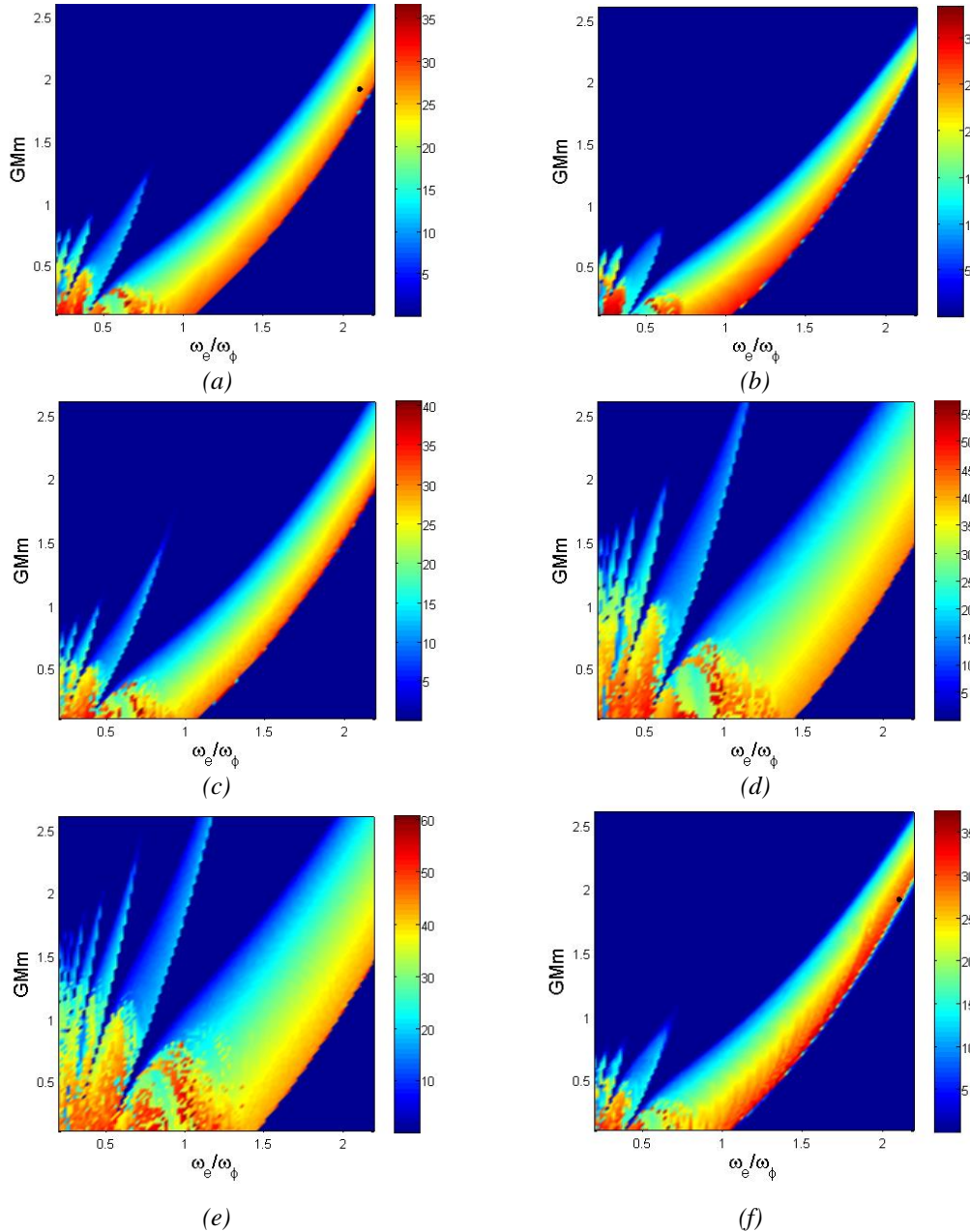

Figure 2. Steady state roll angle for different parameters ω_e/ω_ϕ and GM_m :

(a) $GM_a = 0.84$, $d_1 = 3.5393 \cdot 10^8$, $\Delta t = 800 s$;(b) $GM_a = 0.84$, $d_1 = 7.0786 \cdot 10^8$, $\Delta t = 800 s$

(c) $GM_a = 0.84$, $d_1 = 1.7696 \cdot 10^8$, $\Delta t = 800 s$;(d) $GM_a = 1.68$, $d_1 = 3.5393 \cdot 10^8$, $\Delta t = 800 s$

(e) $GM_a = 1.68$, $d_1 = 1.7696 \cdot 10^8$, $\Delta t = 800 s$;(f) $GM_a = 0.84$, $d_1 = 3.5393 \cdot 10^8$, $\Delta t = 400 s$.

By reducing the running time to 400 seconds, the roll oscillations have not reached significant amplitudes or the steady state has not been obtained and the reported values belong to the transitory period, as illustrated in figure 2(e).

For the analysed case, the quadratic damping $d_2 \dot{\phi}\left|\dot{\phi}\right|$ has an insignificant effect on the reduction of dangerous amplitudes. The colour plots obtained in the $\left(\omega_e/\omega_\phi, GM_m\right)$ plane for $d_2 = 0$ and for four times the original value of d_2 look almost identical, as shown in figure 3.

Figure 3. Steady state roll angle for different parameters ω_e/ω_ϕ and GM_m and:

(a) $d_2 = 0$; (b) $d_2 = 7.5304 \cdot 10^8$.

By maintaining the other parameters at the value in table 1, the damping effect is more readily identifiable in figure 4. For a fixed encounter frequency ω_e, it is virtually impossible to reduce the parametric roll amplitude only by increasing the quadratic damping coefficient d_2. Instead, this desideratum can be achieved by increasing the linear damping coefficient d_1 by more than 200%. Of course, getting out from the dangerous zone can be realized by changing the encounter frequency.
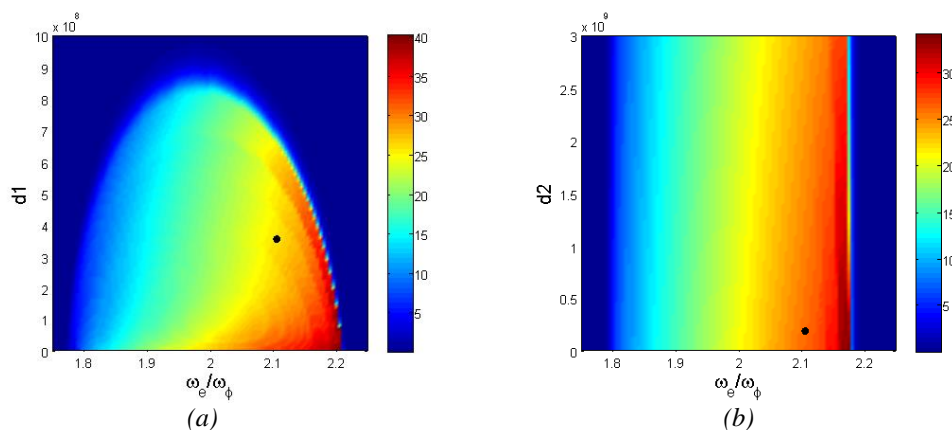

Figure 4. The damping effect on the steady state amplitudes:
(a) linear damping; (b) quadratic damping.

A parameter with much more pronounced influence on the solution is the cubic coefficient of the restoring moment, k_3. If $k_3 = 0$, the equation (1) transforms into a damped Mathieu equation having in the unstable region unbounded solutions, as presented in figure 5(a). Here, the roll amplitudes have been capped at 90^0. With the increase of k_3 there is an obvious diminution of the steady state amplitudes (see figure 5(b)).
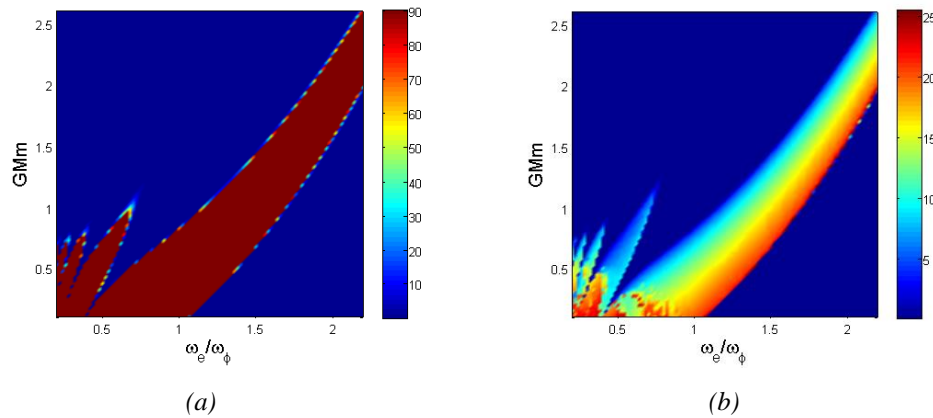

(a)   *(b)*

Figure 5. The cubic coefficient of the restoring moment's effect on the steady state amplitudes:
(a) $k_3 = 0$; (b) $k_3 = 5.948 \cdot 10^9$.

As for the variable part of metacentric height, GM_a, there is a relationship of direct proportionality between its magnitude and the steady state roll amplitude. Over a certain threshold the ship is in a real danger of capsizing. From figure 6, it can be noticed that, besides the principal resonance $\omega_e / \omega_\phi = 2$ there exist a subharmonic resonance, $\omega_e / \omega_\phi = 1$.
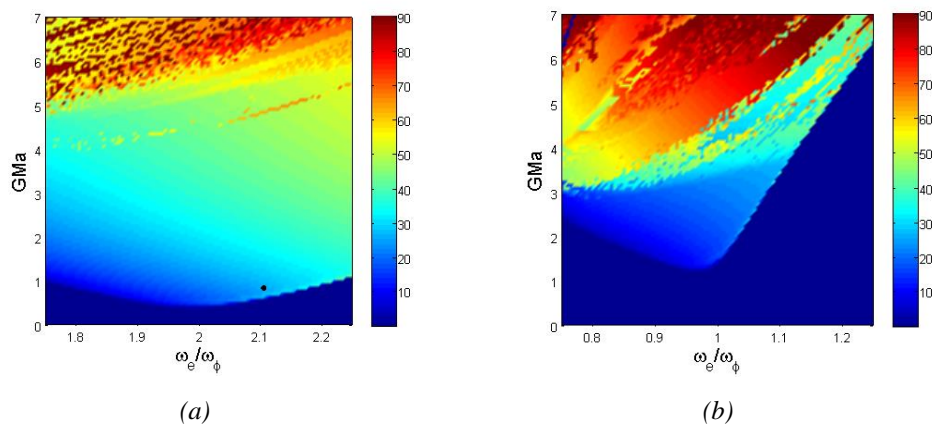

(a)   *(b)*

Figure 6. The variable part of the metacentric height's effect on the steady state amplitudes.

All the above figures indicate that a relatively small variation in ω_e (about 10%) moves the black dot in a safe region. According to equation (2), ω_e depends on the ship forward speed U and the ship heading angle β. We deduct that a (possible) change in these parameters, in a short time after parametric roll detection, is desirable for the ship and cargo safety. The effect is even more visible as other parameters are improved, as shown in figure 7.
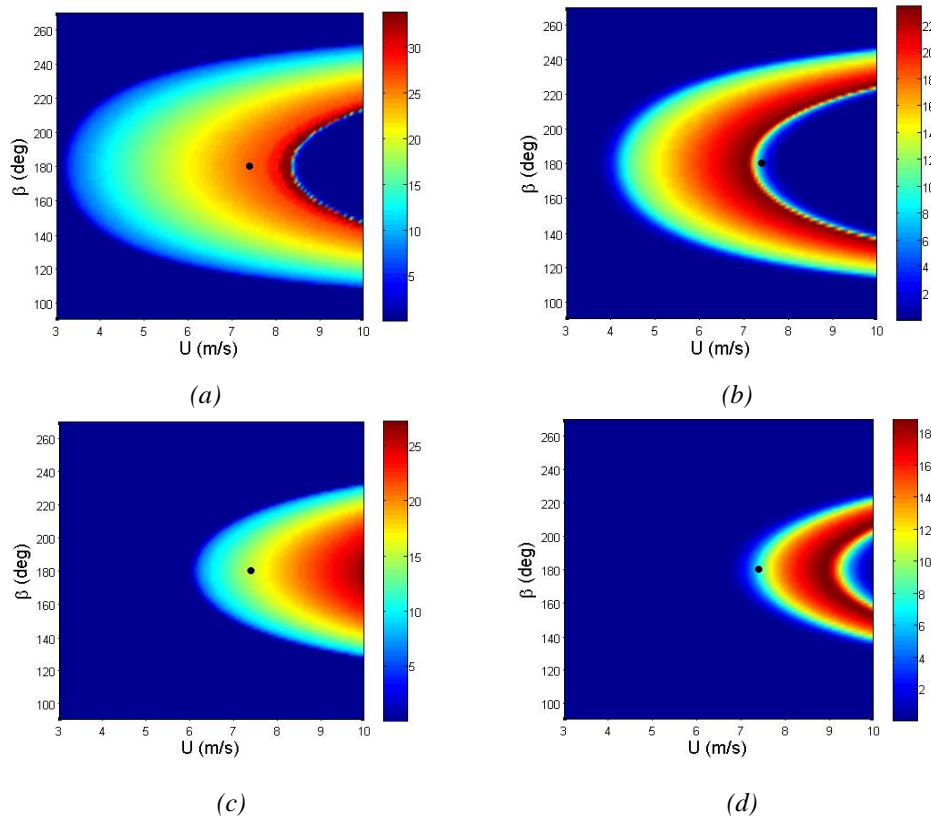
Figure 7. The influence of ship forward speed U and ship heading angle β on the steady state amplitudes: (a) $d_1 = 3.5393 \cdot 10^8, GM_m = 1.91$; (b) $d_1 = 7.0786 \cdot 10^8, GM_m = 1.91$; (c) $d_1 = 3.5393 \cdot 10^8, GM_m = 2.292$; (d) $d_1 = 7.0786 \cdot 10^8, GM_m = 2.292$.
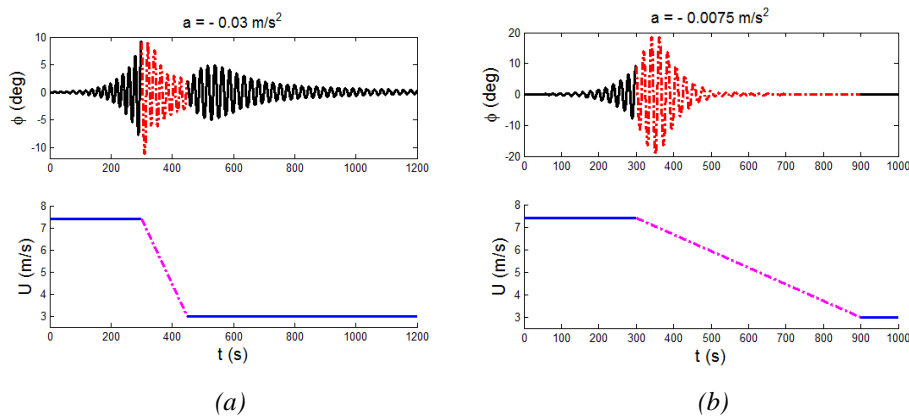

Figure 8. Roll angle and forward speed in time for the negative acceleration:(a) $a = -0.03\, m/s^2$; (b) $a = -0.0075\, m/s^2$. The speed started to decrease after 300s.

Suppose now that the ship parameters are those displayed in table 1. At 300s after a small perturbation (of only 0.1^0) affected the ship this increased to 15^0 and, after 450s, the stationary state was reached with the ship oscillating at 25^0 (see figure 1). This situation corresponds to the black point in figure 7(a). Keeping the value $\beta = 180^0$ unmodified, the ship will return to moderate rolling angles if the forward

speed will be decreased to 3m/s or increased to 9m/s. The ship's behaviour is somewhat different depending on the time chosen for changing the speed and how long this transformation takes.
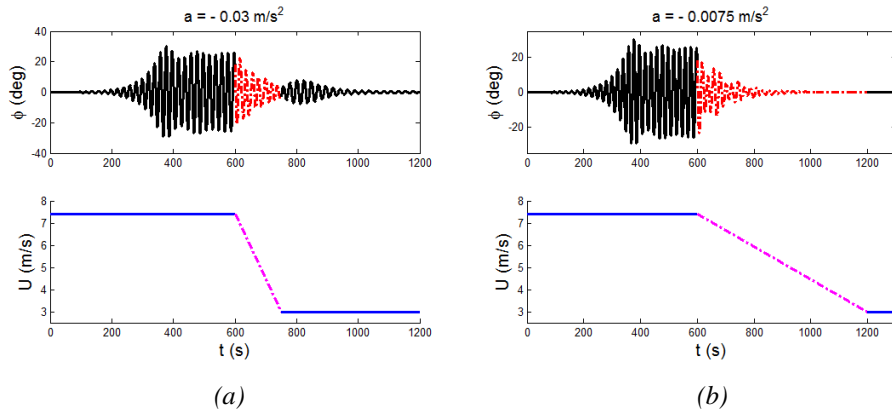

Figure 9. Roll angle and forward speed in time for the negative acceleration: (a) $a = -0.03\,m/s^2$; (b) $a = -0.0075\,m/s^2$. The speed started to decrease after 600 s.

Figures 8 and 9 present the ship response when the forward speed is decreased to 3m/s with two different accelerations. This action began after 300s and 600s respectively. In all four cases the roll amplitude decays towards/ at zero after 1200s from the initial excitation. It should be noted, however, the different way in which the ship behaved after the (safe) speed of 3m/s was reached.

On the other hand, figures 10 and 11 show the same type of information but for a forward speed that rises from 7.399m/s to 9m/s. If this happens in 90s (acceleration $a = 0.018\,m/s^2$) then, regardless of the time chosen for the speeding, the roll angle decays to zero. For a slower speed increase (in 270s), the oscillation amplitudes continue to grow and, after reaching the forward speed U = 9m/s, they stabilize at $30 - 40^0$.
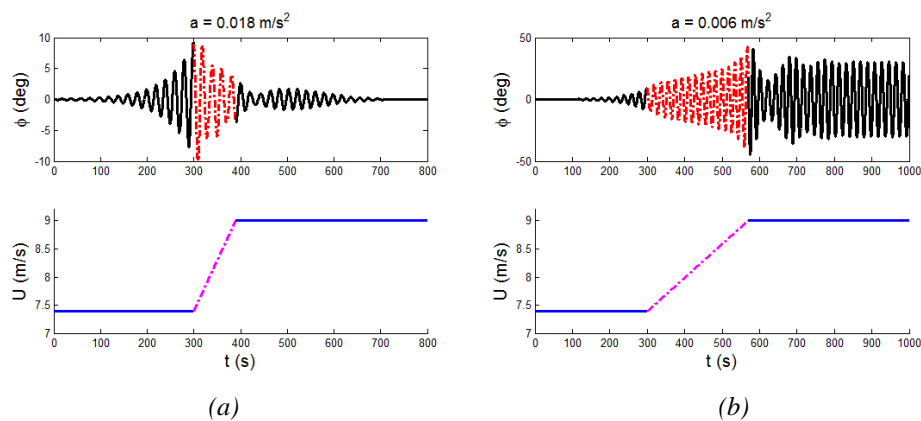

Figure 10. Roll angle and forward speed in time for the positive acceleration: (a) $a = 0.018\,m/s^2$; (b) $a = 0.006\,m/s^2$. The speed started to increase after 300s.

A possible explanation can be given if we look more closely at figure 7(a). When the forward speed is gradually decreased, the black dot crosses less and less dangerous areas (with smaller stationary amplitudes). Once it gets into the safe area, the oscillations are vanishing in time. If the same speed is increased slowly over time, the black dot is moving into more and more dangerous regions and has enough time to "store" energy that will be difficult to dissipate by damping at U = 9m/s.
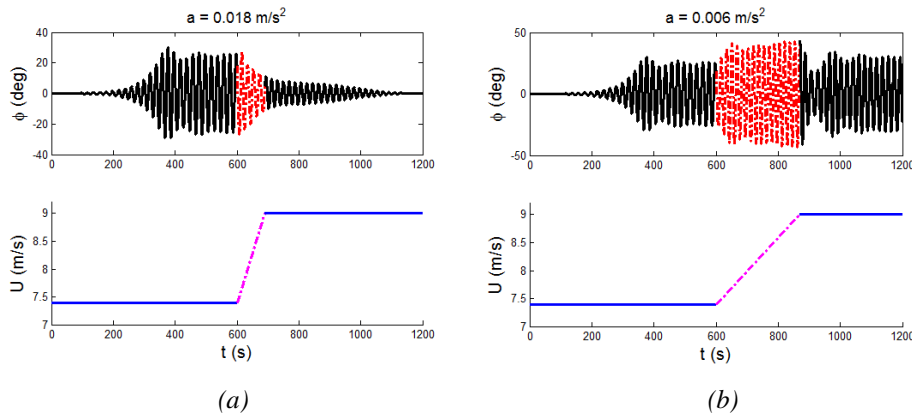
Figure 11. Roll angle and forward speed in time for the positive acceleration: (a) $a = 0.018\,m/s^2$; (b) $a = 0.006\,m/s^2$. The speed started to increase after 600s.

If the acceleration $a = 0.006\,m/s^2$ is maintained, a solution would be the increase of forward speed at 10m/s, as soon as possible after the parametric roll's detection to avoid the extremely dangerous amplitudes of $40 - 50^0$, as shown in figure 12.
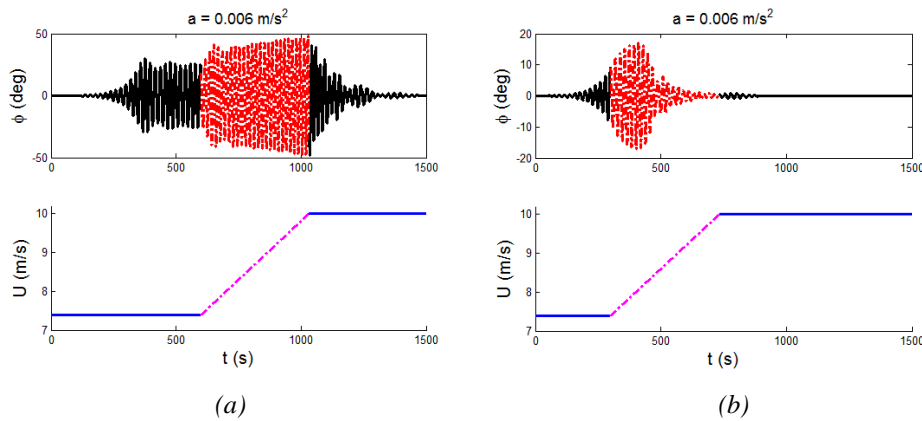

Figure 12. Roll angle and forward speed in time for the positive acceleration $a = 0.006\,m/s^2$. The speed started to increase after 600s (a) or 300s (b).
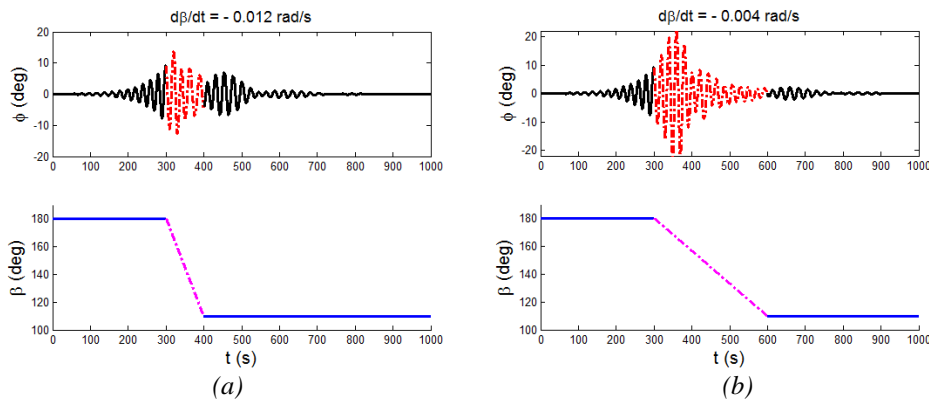

Figure 13. Roll and heading angles in time for the angular velocity $d\beta/dt = -0.012\,rad/s$ (a) or $d\beta/dt = -0.004\,rad/s$ (b). The heading angle started to decrease after 300s.

Finally, the alternative to changing the forward speed is modifying the heading angle. Figure 7(a) suggests for U = 7.399 m/s an angle β in the safe area of 110^0. The roll angle decays to zero, no matter the angular velocity $d\beta/dt$ is used, as proven by figure 13.

4. Conclusions

In the paper we analyzed the influence of certain ship parameters on the stability of a given containership with respect to parametric roll resonance. The model we used had as a starting point a three-degrees-of-freedom model in which the degrees of freedom were heave, pitch and roll. Neglecting the coupling terms, we obtained a Mathieu type equation with quadratic damping and cubic restoring terms. The simplified model succeeded to capture the basic features of the parametric roll. The paper contains a set of color plots that emphasizes the role played by each of the seven parameters involved in the parametric roll equation in obtaining a safe or a dangerous behavior of the ship model. Analyzing the stability regions revealed by the color plots, we suggested some possibilities to get out from the dangerous areas, such as modifying the ship forward speed or heading angle. Further research should concentrate on the influence of couplings on roll amplitude in a higher degrees of freedom model, including at least pitch and heave motions.

5. References

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