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The frictional contact of coated bodies. Part II – The slip-stick contact

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Abstract. Fretting is a special wear mode that occurs in a contact subjected to minute relative motion by vibration or other perturbations. The solution of the slip-stick elastic contact is required to address the problem of tractions and stresses arising in a fretting contact. This solution may be particularly difficult to obtain for coated bodies, considering that, unlike the case of homogenous materials, the response of layered materials to unit load has not yet been expressed in closed form. However, explicit expressions, also known as the frequency response functions, have been derived in the Fourier transform domain. Considering that displacement and stress calculation in an elastic contact process yields convolution products arising from superposition of effects, the existing frequency response functions can be used to calculate the response of layered materials directly in the spectral domain, and subsequently transferred to the space domain. Once a method for elastic displacement calculation in layered materials is made available, the slip–stick elastic contact problem can be solved in the same manner as for homogenous materials. The contact problems in the normal and in the tangential direction are coupled, as opposed to the sliding contact model. The numerical solution allows for identification of the slip and stick regions, and of the corresponding contact tractions. The latter tractions can be subsequently used to assess the stress state in the coated body, and thus to formulate competent design decisions.

1. Introduction

Fretting wear and fretting fatigue play a chief role in decreasing the service life of machine elements subjected to oscillating tangential displacements arising in concentrated contacts. The theoretical framework of this tribological scenario involves the solution of the Cattaneo-Mindlin problem [1, 2], i.e. the problem of the mechanical contact under partial slip, in which the tangential force is not large enough to induce the macro sliding of the contacting bodies. While the contacting bodies are globally sticking, the established contact area comprises regions of stick, where corresponding particles on the two contacting boundaries undergo the same displacements, and regions of micro slip, where there exists relative displacement between the initially matching surface particles. The closed-form solution of the latter problem [1, 2] was obtained in the frame of Linear Theory of Elasticity under limiting assumptions: (I) the contact pressure is independent of the shear tractions (which is true for contacting bodies with similarly elastic properties), and (II) the contacting bodies are homogenous, isotropic and linear elastic. These assumptions are relaxed in the numerical study performed in this paper, aiming to advance a simulation technique for the partial slip contact involving coated bodies. To our best knowledge, an analytical solution to the latter contact scenario has not been achieved, and its derivation may be difficult as: (I) the contact area and its evolution with the load level, as well as the contact tractions (pressure and shear), are a priori unknown; (II) the boundaries between the regions of stick and those of slip are also unknown, (III) the consideration of a coated body in contact implies that the contacting bodies are dissimilarly elastic, and consequently pressure is not independent of the shear tractions, and (IV) the elastic response of multi-layered materials to unit load (i.e., the Green's functions) is only known in the frequency domain as the frequency response functions (FRFs).

The simulation technique employed in this paper is known in the literature as a semi-analytical method (SAM) [3]. SAM implies the discretization of a surface region of the contacting body, as opposed to the finite element method that requires the meshing of the bulk. The SAM computational efficiency also derives from the use of spectral techniques [4, 5] to rapidly compute the convolution products yielding the displacement response of the contacting bodies.

Gallego, Nélias and Jacq [6] first applied SAM in the study of the slip-stick contact problem. They solved repeatedly the contact problem in the normal direction while considering the change in contact conformity due to wear. A numerical technique for the partial slip contact considering tangential tractions was advanced by Chen and Wang [7]. Alternate numerical solutions were obtained by Gallego, Nélias and Deyber [8], and by Spinu and Glovnea [9]. Spinu and Frunza [10] investigated the hysteretic effects in the fretting contact between dissimilarly elastic materials, and proved that, due to the irreversibility of friction as a dissipative process, the load must be applied in small increments to correctly reproduce the path of the contact process. More recent, Spinu and Cerlinca [11] advanced a SAM-based numerical solution to the Cattaneo-Mindlin problem for viscoelastic materials.

The mathematical description of displacements and stresses arising in layered materials pioneered with the work of Burmister [12]. The FRFs are the counterpart of the Green's functions in the frequency domain, and allow for computation of the elastic response of multi-layered bodies. O'Süllivian and King [13] obtained the FRFs for the contact of bi-layered materials under normal and shear loads. The fast Fourier transform (FFT) was first applied to contact mechanics by Ju and Farris [14]. The contact of rough surfaces with coatings was modelled by Nogi and Kato [15] with the aid of the FFT assisted by the FRFs derived by O'Süllivian and King [13]. Liu and Wang [5] applied FFT enhanced techniques in the study of contact stress fields caused by surface tractions, and advanced alternate FRFs that are more suitable to numerical implementations. Wang et al. [16] investigated the partial slip contact of coated bodies. Recently, Yu et al. [17] obtained the analytical FRFs of multi-layered materials in a recurrence format and studied the elastic contact of layered bodies with various configurations.

The SAM advanced in [9-11] is extended in this paper to account for the contact of coated bodies. The use of state-of-the-art numerical tools based on the fast Fourier transform and the convolution theorem allows fine spatial meshes and small loading increments, leading to rapidly converging numerical solutions.

2. The contact model

The contact problem domain discretization coupled with the superposition of effects in the frame of linear elasticity allows for a trial-and-error search of the contact area and of the slip-stick boundary. Displacement computation is the essential step in solving efficiently the contact problem. Influence coefficients derived from the Green's functions may be used from homogenous materials, whereas a special technique [18] based on the FRFs and the discrete cyclic convolution theorem is needed for multi-layered materials. Once the displacement response of the contacting bodies can be expressed for arbitrary loading, the iterative search of the contact area and of the slip and stick regions can be performed in the frame of the contact model briefly discussed in this section.

The contact problem is conveniently reported to a Cartesian coordinate systems having its x and y axes contained in the common plane of contact, allowing for the description of initial contact geometry $hi(x, y)$. The latter axes are referred to as the tangential directions, whereas the z-axis is the

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normal direction. Forces and moments may be transmitted through the contact: the normal force W, the tangential force $\mathbf{T}(T_x, T_y)$, the bending (or flexing) moments M_x, M_y, and the torsional moment M_z. As the contacting bodies subjected to load undergo elastic deformation, the initial point of contact evolves into a contact region A, on which contact tractions arise to balance the applied load: the pressure p in the normal direction and the shear traction $\mathbf{q}(q_x, q_y)$ in the tangential direction. The particles of the contacting bodies exhibit elastic displacements u_i, whereas the bodies as a whole move as rigid bodies with the translations ω_i and the rotations ϕ_i, with $i = x, y, z$. In the framework of the Cattaneo-Mindlin problem [1, 2], although the contacting bodies are globally sticking, slip occurs at the periphery of the contact area, to release the otherwise infinite tractions that would result in a fully sticking contact. Consequently, the contact area is divided into a stick area S and a complementary slip region $A - S$. The frictional coefficient μ is assumed uniform on the contact area and load-independent. The additional parameter referred to as the relative slip distances $\mathbf{s}(s_x, s_y)$ defined in [19], allows for the iterative identification of the slip/stick boundary.

The principles of model discretization are detailed in the companion paper. The SAM assumes continuous distributions as piecewise constant over a rectangular mesh P laying in the common plane of contact, enfolding the contact area at any load increment. The elementary cell area is denoted by Δ. Discrete model parameters have at most three arguments: the first two are for spatial localization, whereas the third one indicates the number of the loading increment, e.g. $p(i, j, k)$ is the pressure in the elementary cell (i, j), attained after k loading increments. Parameters having two arguments depend only on spatial localization, e.g. the mesh nodes coordinates, whereas those with one argument depend only on the loading increment, e.g. the rigid-body movements.

As suggested in previous studies concerning the contact of homogenous materials [9-11], the contact model can be divided in two submodels possessing the same structure,: (I) the contact in the normal direction described by equations (1), (3) and (5), and (II) the tangential effects governed by equations (2), (4) and (6). The solution of each submodel may be consequently achieved with the same algorithm. Three types of equations may be identified for each submodel:

1. The static force and moment equilibrium:

$$W(k) = \Delta \sum_{(i,j)\in A(k)} p(i,j,k); \quad M_x(k) = \Delta \sum_{(i,j)\in A(k)} p(i,j,k) y(i,j); \quad M_y(k) = \Delta \sum_{(i,j)\in A(k)} p(i,j,k) x(i,j); \tag{1}$$

$$T_n(k) = \Delta \sum_{(i,j)\in A(k)} q_n(i,j,k), n = x, y; \quad M_z(k) = \Delta \sum_{(i,j)\in A(k)} \left[q_y(i,j,k) x(i,j) - q_x(i,j,k) y(i,j) \right]. \tag{2}$$

2. The geometrical condition of deformation:

$$h(i,j,k) = hi(i,j) + u_z(i,j,k) - \omega_z(k) - \phi_x(k) y(i,j) - \phi_y(k) x(i,j), (i,j) \in P; \tag{3}$$

$$\begin{gathered} \begin{bmatrix} s_x(i,j,k) - s_x(i,j,k-1) \\ s_y(i,j,k) - s_y(i,j,k-1) \end{bmatrix} = \begin{bmatrix} u_x(i,j,k) - u_x(i,j,k-1) \\ u_y(i,j,k) - u_y(i,j,k-1) \end{bmatrix} - \begin{bmatrix} \omega_x(k) - \omega_x(k-1) \\ \omega_y(k) - \omega_y(k-1) \end{bmatrix} - \\ \left(\phi_z(k) - \phi_z(k-1) \right) \begin{bmatrix} y(i,j) \\ x(i,j) \end{bmatrix}, \quad (i,j) \in A(k). \end{gathered} \tag{4}$$

3. The contact complementarity conditions:

$$\begin{cases} p(i,j,k) > 0 \quad \text{and} \quad h(i,j,k) = 0, \quad (i,j) \in A(k); \\ p(i,j,k) = 0 \quad \text{and} \quad h(i,j,k) > 0, \quad (i,j) \in P - A(k); \end{cases} \tag{5}$$

$$\begin{cases} \|\mathbf{q}(i,j,k)\| \le \mu p(i,j,k) \quad \text{and} \quad \|\mathbf{s}(i,j,k)-\mathbf{s}(i,j,k-1)\| = 0, (i,j)\in S(k); \\ \|\mathbf{q}(i,j,k)\| = \mu p(i,j,k) \quad \text{and} \quad \|\mathbf{s}(i,j,k)-\mathbf{s}(i,j,k-1)\| > 0, (i,j)\in A(k)-S(k). \end{cases} \tag{6}$$

The two submodels cannot be solved independently becouse either the normal or the tangential displacements comprise contributions from both normal and shear tractions, as shown in the following section. A detailed description of the scheme to solve the contact model (1) - (6) can be found elsewhere [9-11].

3. Computation of elastic displacement in multi-layered materials

In the framework of the Linear Theory of Elasticity, stresses and displacements arising in a half-space due to general loadings are obtained by superposition of solutions of point forces, also referred to as the Green's function method. The Green’s function describes the half-space response in the space domain, and its counterpart in the frequency domain, is referred to as the FRF. This duality is of particular interest in case of coated bodies, where only the FRFs were derived in closed-form [5,13,15].

As shown in a previous work by the same authors [18], the FRFs can be used to calculate displacement in a coated system with the aid of the discrete convolution theorem [4]. The convolution product is computed directly in the frequency domain, where it is converted to an element-wise product between the Fourier transforms of the convolution members. Special care must be taken with the handling of the periodicity error, i.e. the error associated with the application of the FFT to discrete series. The transfer to frequency domain via FFT effectively transforms a non-periodic problem into a periodic one, with a period equal to the discretization window. An efficient way for the reduction of this error was decribed in [4, 5], and implies an extension of the problem physical domain, resulting in an increase of the resolution in the frequency domain. The same method is applied here to derive the normal and tangential displacements of the contact surface subjected to contact tractions. The displacements required in the contact model (1) - (6) are obtained by inverse FFT (IFFT), as shown in the following equation:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \mathrm{IFFT}\left(\begin{bmatrix} \tilde{f}_{xx} & \tilde{f}_{xy} & \tilde{f}_{xz} \\ \tilde{f}_{yx} & \tilde{f}_{yy} & \tilde{f}_{yz} \\ \tilde{f}_{zx} & \tilde{f}_{zy} & \tilde{f}_{zz} \end{bmatrix} \cdot \begin{bmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{p} \end{bmatrix} \right), \tag{7}$$

in which the symbol $(\tilde{\ })$ denotes the double Fourier transform with respect to the x and y-axes, and $\tilde{f}_{ij}$, $i, j = x, y, z$, is the FRF expressing the displacement in the direction of $\vec{i}$ induced by the contact traction acting in the direction of $\vec{j}$. The closed-form expressions of the needed FRFs are given below, with m and n the coordinates in the frequency domain corresponding to x and y, respectively:

$$\tilde{f}_{zz}(m,n) = -(1-\nu_1)(1+4\alpha h\kappa\theta - \lambda\kappa\theta^2)\alpha R/G_1 ; \tag{8}$$

$$\tilde{f}_{xz}(m,n) = \sqrt{-1}m(A^{(1)} + \bar{A}^{(1)})/(2G_1) ; \tag{9}$$

$$\tilde{f}_{yz}(m,n) = \sqrt{-1}n(A^{(1)} + \bar{A}^{(1)})/(2G_1) ; \tag{10}$$

$$\tilde{f}_{xx}(m,n) = (2G_1)^{-1}\left[\sqrt{-1}m(D^{(1)} + \bar{D}^{(1)}) - 4(1-\nu_1)(B^{(1)} + \bar{B}^{(1)})\right] ; \tag{11}$$

$$\tilde{f}_{yx}(m,n) = (2G_1)^{-1}\sqrt{-1}(D^{(1)} + \bar{D}^{(1)}) ; \tag{12}$$

$$\tilde{f}_{zx}(m,n) = -(1-\nu_1)(C^{(1)} + \bar{C}^{(1)})/G_1 ; \tag{13}$$

$$\tilde{f}_{xy}(m,n) = \tilde{f}_{yx}(m,n) ; \quad \tilde{f}_{yy}(m,n) = \tilde{f}_{xx}(n,m) ; \quad \tilde{f}_{zy}(m,n) = \tilde{f}_{zx}(n,m) . \tag{14}$$

The parameters involved in equations (8) - (10) are fully described in [20], whereas those in relations (11) - (13) are given in the companion paper. Once a method for displacement calculation considering the contribution of all contact tractions is made available, the solution of the slip-stick contact problem can be achieved with the contact model described in the previous section.

4. Results and discussions

For validation purposes, the newly proposed computer program is first benchmarked against the closed-form solution [19] of the Cattaneo-Mindlin problem for the spherical contact in a fretting loop. To this end, the contacting bodies are assumed homogenous and similarly elastic. A steel ball of radius $R = 18$ mm is first pressed into a half-space with a normal force $W_{\max} = 1$kN. A tangential force T_x, oscillating between $T_{\lim}$ and $-T_{\lim}$, with $T_{\lim} = 0.9\mu W_{\max}$, is subsequently applied. This type of loading history, depicted in figure 1, is specific to a fretting loop. As the contacting materials are similarly elastic, the problems in the normal and in the tangential directions are uncoupled, and the contact solution is achieved on a single level of iterations. However, the tangential force needs to be applied incrementally, with the reproduction of the states in which the tangential load increment changes sign.

In figure 2, the frictional coefficient is assumed uniform over the whole contact area, $\mu = 0.1$, and constant during load application to allow comparison with the analytical framework. However, the computer program can handle mapped distributions of μ. The Hertz frictionless theory for this contact scenario predicts a central pressure $p_H = 3.1684$ GPa and a contact radius $a_H = 0.388$ mm, which are used as normalizers. The time period of the simulation window is denoted by τ (seconds).

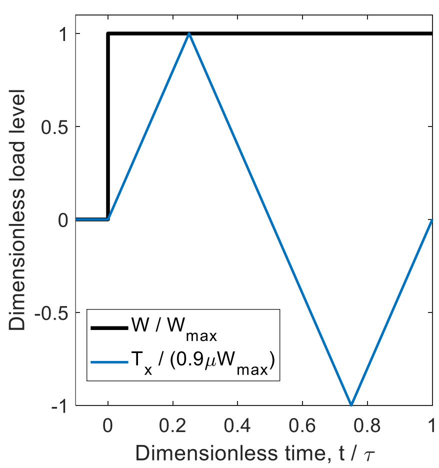

Figure 1. The loading history in a fretting loop.

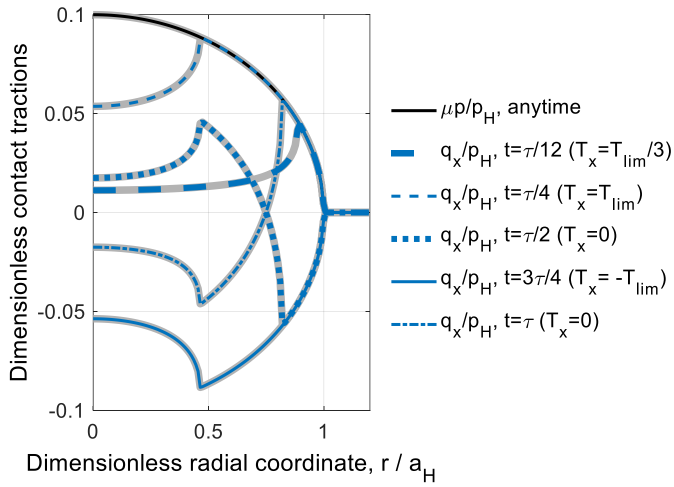

Figure 2. Contact tractions in a fretting loop. The closed-form solution [19] is displayed using grey lines.

The slip-stick spherical contact involving a coated material is simulated next by pressing a rigid ball into a coated half-space. The Young moduli and the Poisson's ratio are denoted by E_i and ν_i, respectively, with $i = 1$ for the coating and $i = 2$ for the substrate. A set of contact scenarios is simulated by keeping E_2, ν_1 and ν_2 constant, whereas E_1 is varied. The Hertz contact parameters for the case when $E_1 = E_2$ are used as normalizers. The frictional coefficient is also fixed at $\mu = 0.1$, as well as the coating thickness, $h = a_H$. The loading program only retains the ascending branch of the tangential force depicted in figure 1.

In this contact scenario, the materials of the contacting bodies are dissimilarly elastic, and consequently the contact problems in the normal and in the tangential directions are coupled. Moreover, the normal load cannot be applied in one step, as in the case of the frictionless contact problem, where the final state depends only on the loading level. The numerical simulations suggest that small loading increments (for both normal and tangential forces) are of paramount importance to obtain well-converged numerical solutions. The contact tractions profiles in the plane $y = 0$, for different loading levels and E_1/E_2 ratios, are depicted in figures 3 and 4.

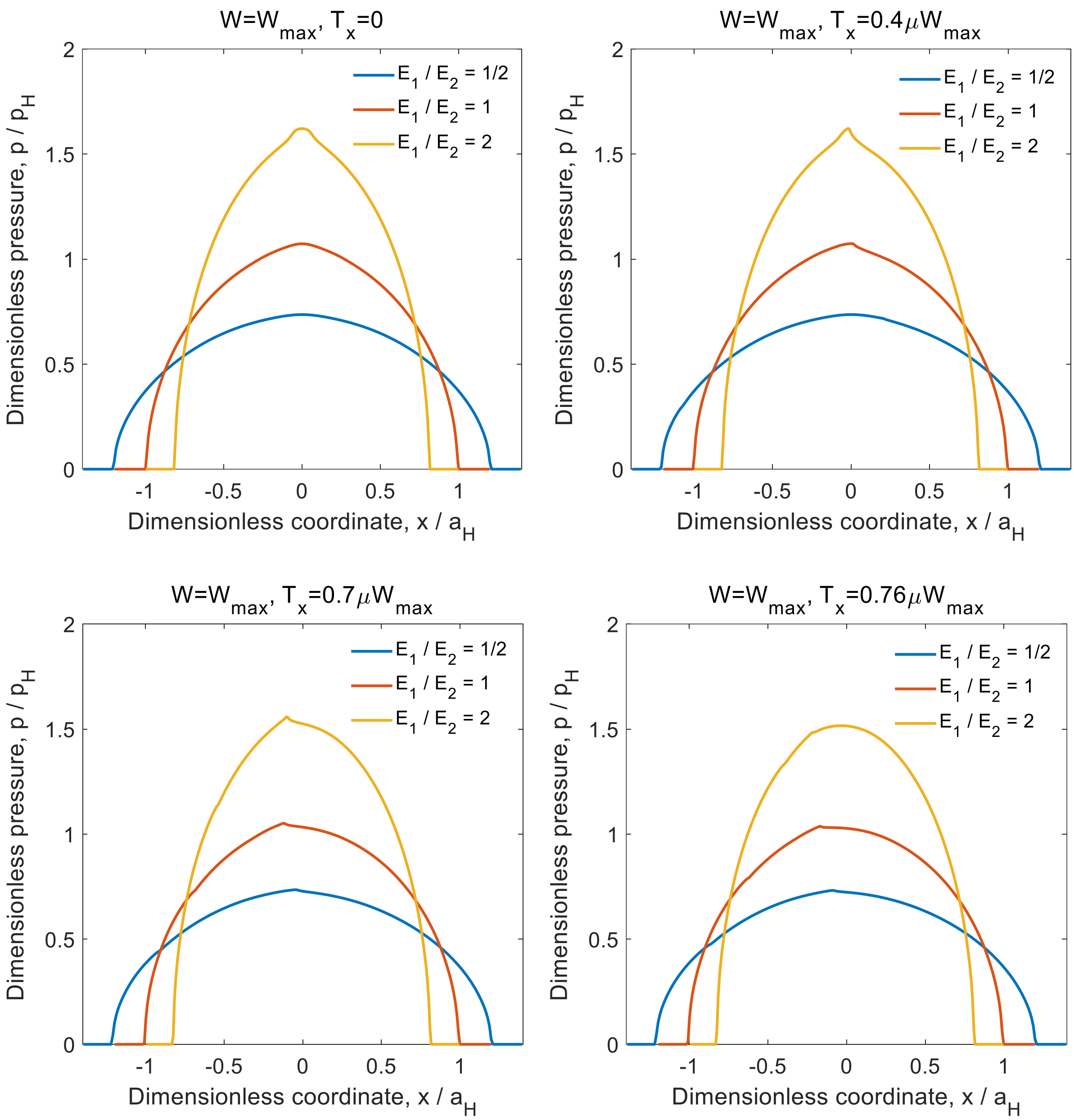

Figure 3. The influence of the elastic mismatch E_1/E_2 on the pressure profiles in the plane $y = 0$.

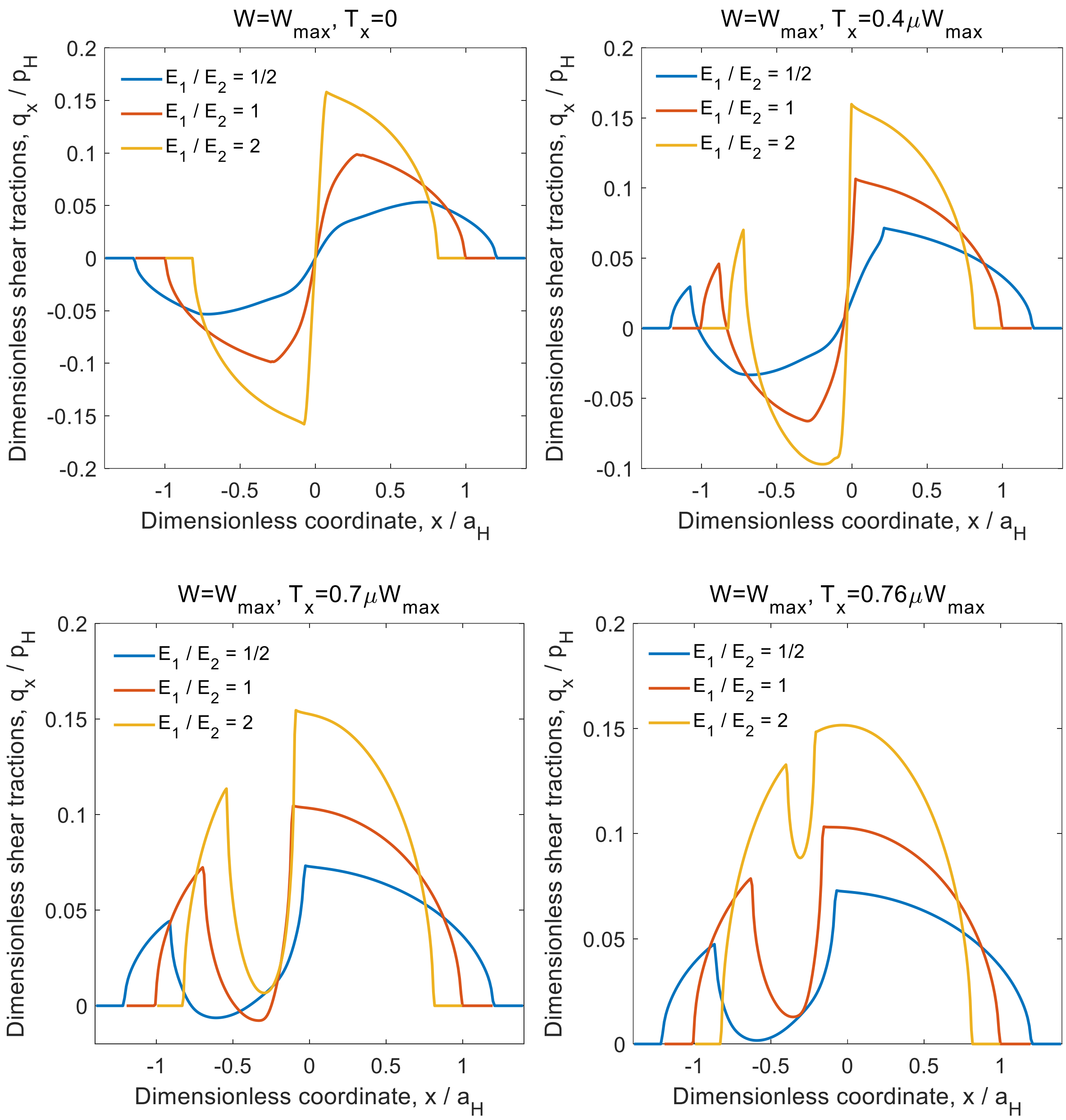

Figure 4. The influence of elastic mismatch E_1/E_2 on the shear tractions profiles in the plane $y=0$.

For the loading level $W=W_{\max}$ and $T_x=0$, although no tangential force is applied, there exist self-equilibrating shear tractions due to the mismatch in the tangential displacements between the surface particles of the contacting bodies. Further increase of the tangential force past the threshold of $T_{\lim}=0.76\mu W_{\max}$ lead to gross sliding for the case when $E_1/E_2=2$, whereas the theoretical model [1, 2] allows for a limit value $T_{\lim}=\mu W_{\max}$ before the stick region vanish completely. This discrepancy is attributed to the coupling between the normal and the tangential problem, which is not accounted for in the analytical framework, and was also observed in other studies concerning the slip-stick contact [7, 16].

The knowledge of the contact tractions arising in the slip-stick contact further allows the computation of the stress state in the coated body, and formulation of design recommendations concerning the coated system.

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5. Conclusions

The tractions arising in a frictional contact involving a coated material are obtained using a semi-analytical simulation technique. The elastic response of the coated material is computed based on closed-form expressions derived in the frequency domain, also known as the frequency response functions. The spectral calculation of the displacement has the additional advantage of increased computational efficiency, related to the application of the fast Fourier transform and the discrete convolution theorem.

The contact process is simulated using a numerical scheme based on two levels of iterations. The algorithm accounts for the coupling between the normal and the tangential contact problems, and therefore is well adapted to the contact of bodies with dissimilarly elastic properties. As the normal displacement required for the normal contact solution requires knowledge of the shear stresses, contact parameters are obtained in an iterative process that stabilizes the shear tractions with respect to pressure.

The advanced computer program was benchmarked against the solution of the Cattaneo-Mindlin problem during a fretting loop. The pressure and shear tractions in the contact between a rigid sphere and a coated half-space are obtained for increasing tangential force. The influence of the elastic mismatch between the coating and the substrate is investigated numerically. The numerical method suggests that the full slip regime is attained at loading levels smaller than the limiting friction force.

The presented results suggest the method ability to simulate the behavior of the frictional and coated contact, thus assisting to the design of improved tribological components for practical engineering applications.

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