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Determination of material constants for high strain rate constitutive model of high entropy alloys

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Abstract: The paper deals with constitutive models determination for a group of high entropy materials (coded BI-x). The procedure includes the following steps: experimental determination of mechanical characteristics for quasi-static domain through compression tests, correction of experimental data of quasi-static tests using numerical simulation, acquisition of experimental data obtained on the Split Hopkinson Pressure Bars and their simulated interpretation. Using the processed data, the constitutive patterns of the materials under investigation are drawn. The experimental data correction for quasi-static tests performed using numerical simulation involves drawing the engineering stress/strain diagram, the transition to the plastic characteristic diagram in the hypothesis of perfectly cylindrical shape of the specimen throughout the test, simulation of the compression deformation process in the presence of friction force on the end specimen surfaces and the correction of the characteristic plastic diagram using the calculated error. Dynamic data obtained on Split Hopkinson Pressure Bar were interpreted using a methodology previously published. Experimental data under real conditions are compared with data obtained by numerical simulation on an elasto-plastic model. The viscous component of the dynamic response is the difference between the real and simulated on the elasto-plastic model responses.

1. Introduction

Materials for military applications, such as armors for collective ballistic protection, are subjected, mainly, to the threat of kinetic impact, a complex phenomenon which implies large deformations at high strain rates [1] that more often cannot be modeled through analytic relations. Thus, the design and the development of new candidate materials imply a great research effort, laborious and expensive experiments and a long period of time [2]. A way to speed up the designing process is to combine experiments with simulations [2]. For the success of such approach there is necessary to build numerical models able to provide results of a high fidelity to real tests [3]. In this respect, a critical



element of the numerical models is the constitutive models used to describe in detail the mechanical behavior of the candidate materials [4]. Dynamic constitutive models are mathematical expressions of physical law which govern high strain rate processes [5]. Given the specific application, “in detail” implies knowing the influence of the strain rate on mechanical and thermal properties [6]. In high strain rate processes, mechanical and thermal properties depend on following thermodynamic quantities: strain state, strain rate tensor, local temperature of material and pressure. Generally, constitutive models are functions of stress and strain state and strain rate. Moreover, local temperature can be a variable in constitutive models, especially when high energies are involved. The complete constitutive model of a material, in a generally formulation, include elastic, plastic and even fluid behavior when phases changes occur.

HEA represents a new class of materials, candidate for ballistic protection applications, obtained through unconventional techniques [7]. For a complete assessment of these materials is necessary to determine the characteristic morphostructure [8] and mechanical behavior.

In the followings we will present a methodology used for constitutive laws determination of a new family of HEA metallic alloys (coded BI-x) intended for armor applications that combines the experiments with numerical simulations.

2. Constitutive models

2.1. Plastic Model with Symonds Cowper viscosity model [9]

Plastic model with Symonds Cowper viscosity model include plastic with isotropic hardening, kinematic and combined behavior having the possibility to include the effects of strain rate.

Strain rate effect is given using Cowper-Symonds model, which scale the yield stress with the factor $\left[1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{\frac{1}{p}}\right]$, where $\dot{\varepsilon}$ is the strain rate.

2.2. Johnson-Cook model [10]

Johnson-Cook model is used for problems where the materials are subjected to high plastic strain with high strain rate and to high temperature thermal conditions. If the thermal effects are unimportant the model can be simplified by elimination of temperature terms.

Mathematical expression of Johnson-Cook model is given by:

$$\sigma_y = [A + B(\varepsilon_{ef}^p)^n](1 + C \ln \dot{\varepsilon}^*)[1 - (T^*)^m] \quad (1)$$

where A,B,C,n and m are material constants, experimentally determined, σ_y yield stress and ε_{ef}^p effective plastic strain. Variables $\dot{\varepsilon}^*$ and T^* are relative parameters. In a more complete model of equation (1) a second power law term is introduced $[1 + C_1 \ln \dot{\varepsilon}^* + C_2 (\ln \dot{\varepsilon}^*)^2]$.

Strain rate is defined in two ways, depending on the viscosity formulation:

- for visco-plastic formulation:

$$\dot{\varepsilon}^* = \frac{\dot{\varepsilon}_{ef}^p}{\dot{\varepsilon}_0} \quad (2)$$

- for yield stress multiplication formulation:

$$\dot{\varepsilon}^* = \frac{\dot{\varepsilon}_{ef}}{\dot{\varepsilon}_0} \quad (3)$$

In equation (2) and equation (3) the following notations are used:

- $\dot{\varepsilon}_{ef}$ —total effective strain rate;

- $\dot{\epsilon}_{ef}^p$ —effective plastic strain rate;
- $\dot{\epsilon}_0$ —reference strain rate, usually quasi-static load strain rate.

Relative temperature is given by equation (4):

$$T^* = \frac{T - T_0}{T_{top} - T_0} \quad (4)$$

where:

- T is local temperature,
 - T_{top} is melting temperature,
 - T_0 is a reference temperature, by example ambient temperature,
- For strain rate influence Johnson-Cook model use the linear term $(1 + C \ln \dot{\epsilon}^*)$.

3. Determination of HEA constitutive models

Formulation methodology for determination of constitutive models for high entropy alloys (coded BI-x) consists of the following steps:

1. Static tests performing;
2. Determination of constitutive model in quasi-static conditions by numerical simulation of compression test;
3. Dynamic tests performing using SHPB;
4. Formulation of constitutive model by numerical simulation of SHPB test.

3.1. Static tests

Due to the small size of batches obtained in the metallurgical furnace the material samples shape allows only the compression test performing. The acquired data consist in a force variation as a function of displacement that can be transformed in engineering stress vs. strain as it is shown in figure 1.

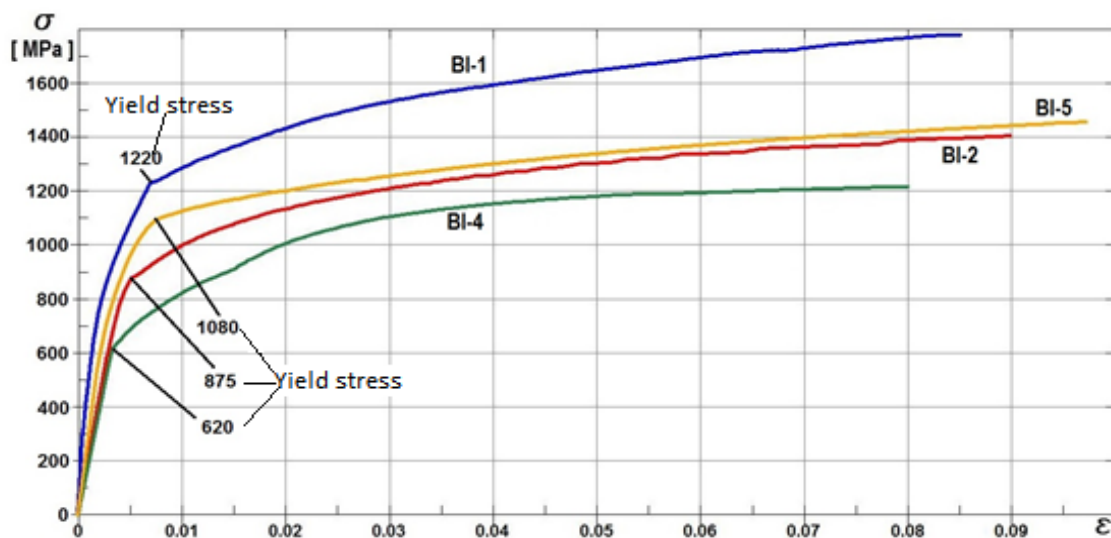


Figure 1. Raw characteristic diagrams obtained after compression tests.

The Young Modulus of the alloys was determined as a mass weighted average of all components. The machine compliance was corrected, based on a Young Modulus value determined and assuming that the elastic process of compression machine was linear.

The corrected conventional characteristic diagrams, as it is shown in figure 3, were obtained by taking into account the compliance of the test machine, assumed linear elastic, and imposing on the linear part of the curves, the real slope of curves, equal with the real Young Modulus, as it is shown in figure 2 for BI-5. The correction is applied in all raw curves points, resulting the characteristics curves presented in figure 3.

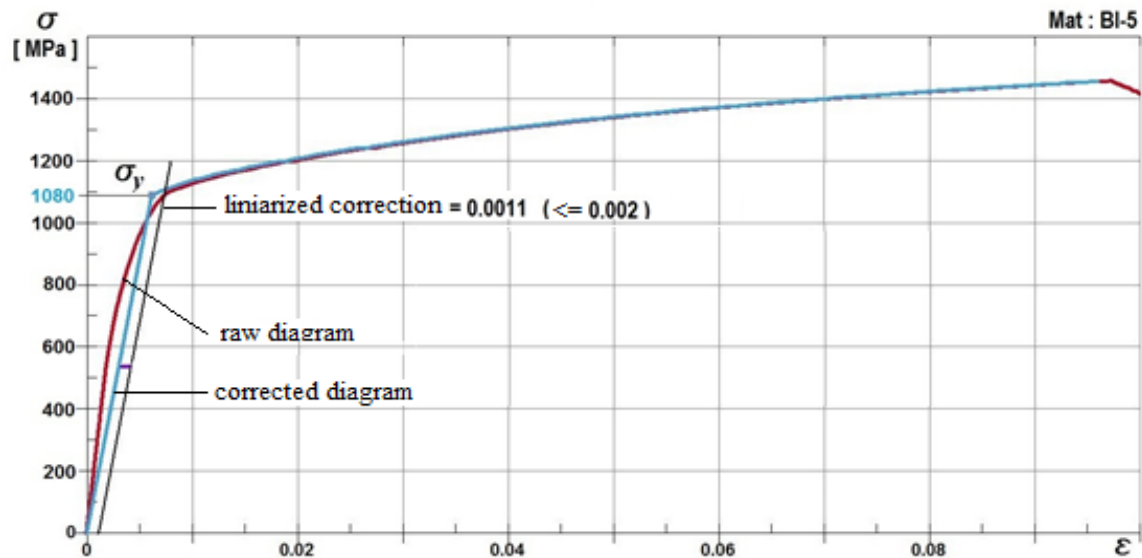


Figure 2. Construction of conventional characteristics diagrams.

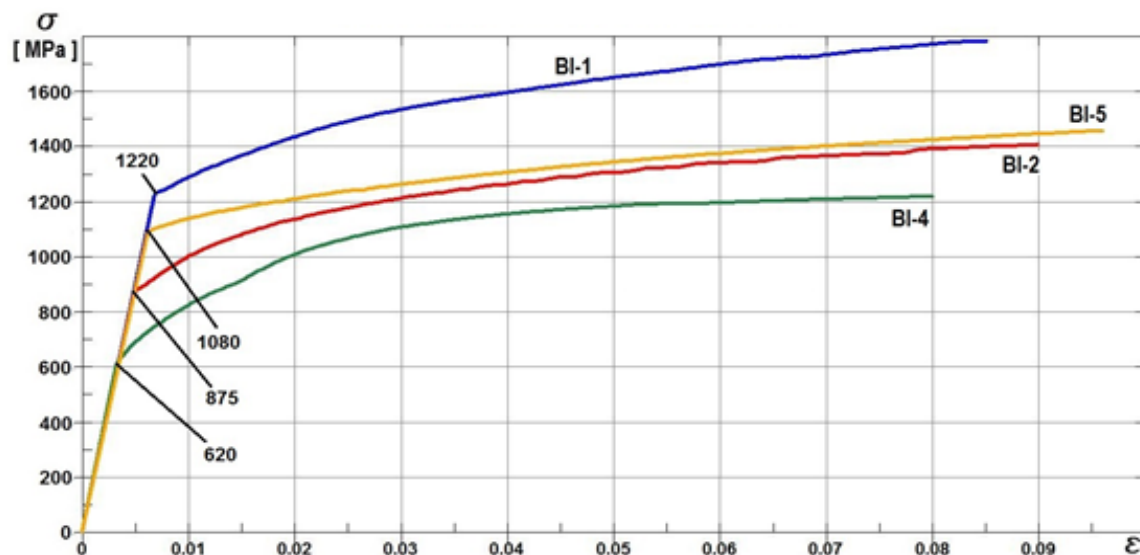


Figure 3. Conventional characteristics diagrams.

Taking into account that in plasticity with large deformations, for accuracy, it is necessary to use true characteristics diagrams, in which the reference state is the current state, the transition to real diagrams was made. The transition from conventional characteristics diagrams to true ones was done in two steps. First was assumed that the volume is invariable (true to plastic strains that are of an order of magnitude larger than the elastic strains) and that there are no friction forces on the sample/platforms interfaces. In this condition the sample remains perfectly cylindrical. The transformation procedure from conventional to true it is shown in figure 4.

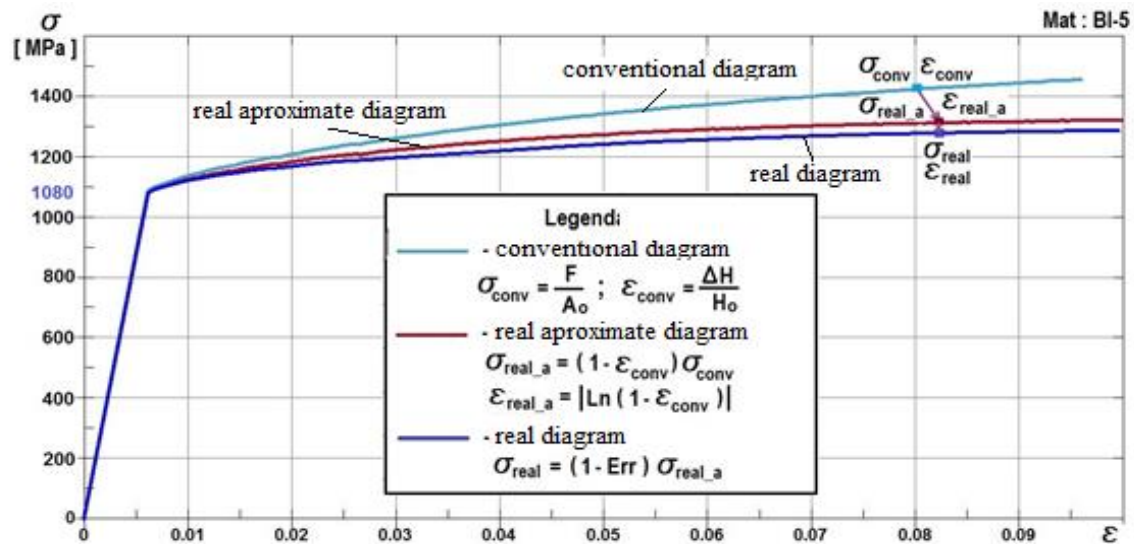


Figure 4. Construction of true diagram.

In the previous assumptions, a first form of the real-approximate characteristic curve is obtained (figure 4). But in real conditions the shape of deformed specimen, because of the friction, it doesn't remain cylindrical but swells more in the median section and less at the ends (barrel shape), thus being necessary to make a new step from true-approximate to true diagrams, as it is shown in figure 4. The true characteristic diagrams for the entire BI-x family are shown in figure 5.

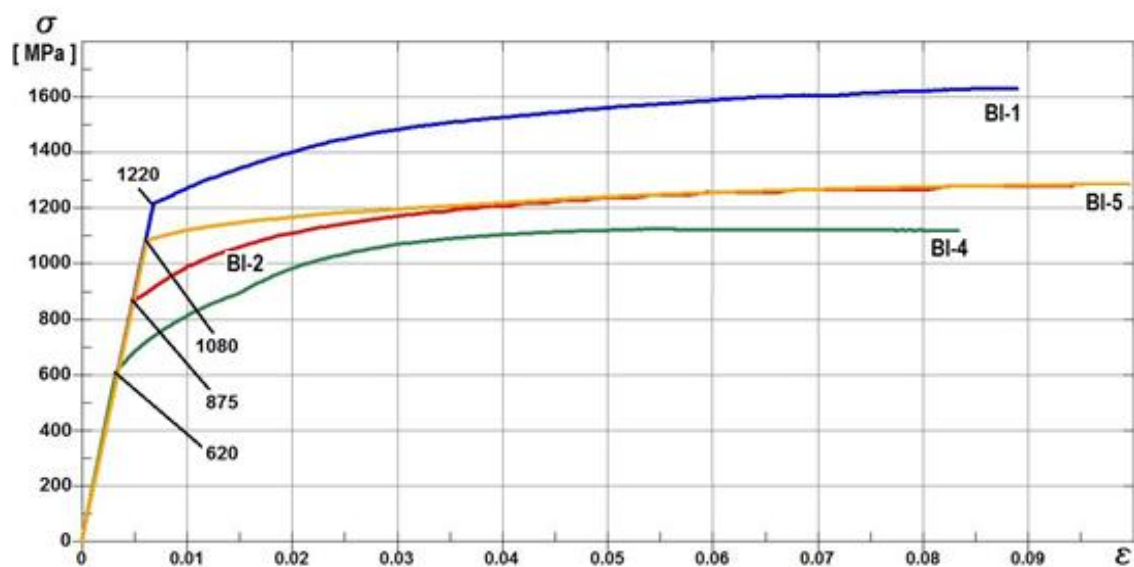


Figure 5. True characteristic diagrams.

By removing the elastic components of the strain from the true characteristic diagrams the plastic characteristic approximate diagrams of tested materials are obtained (figure 6).

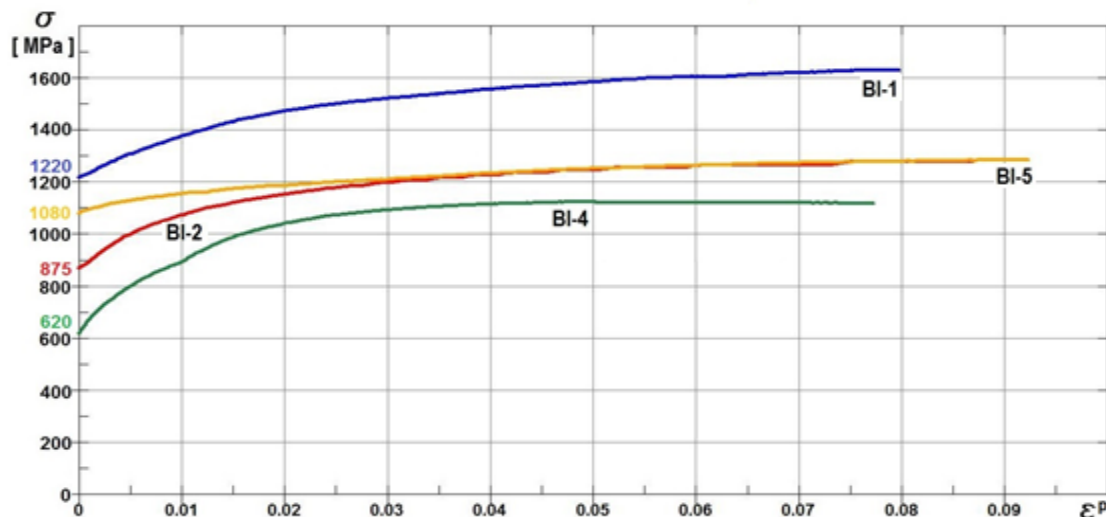


Figure 6. Plastic characteristics diagrams – plasticity function.

The adjustment of approximate characteristic diagrams was made using a method of numerical simulation of plastic strain process of the sample in the compression test, using approximate material properties. The simulated solution obtained, expressed in force versus displacement, is compared with experimental data. The calculated error for the force was used to correct the approximate curves for the enhancement of the solution accuracy.

The simulation of compression test on the cylindrical samples was made using a 2D axial symmetric model. The loading program used in the numerical simulation was identical with the simulated one, controlling the relative displacement of the machine platforms.

In figure 7 it is shown the plastic state contour plot in the longitudinal section of a sample, a few moments before the failure. The analysis of the two contours plots presented in figure, effective plastic strain and effective stress, reveals large deviations from the uniformity hypothesis, resulting that the experimental determinations in compression tests gives only mean values of stress and strain. The simulation shows that in the presence of friction the maximum values of effective plastic strain and effective stress attained in sample exceed the calculated experimental values. For instance, stress and strain at failure are maximum on sample edges. Traditional interpretation of compression tests experimental data does not cover the deviations of stress and strain contours from mean values, a drawback that may be overcome by the numerical simulation approach.

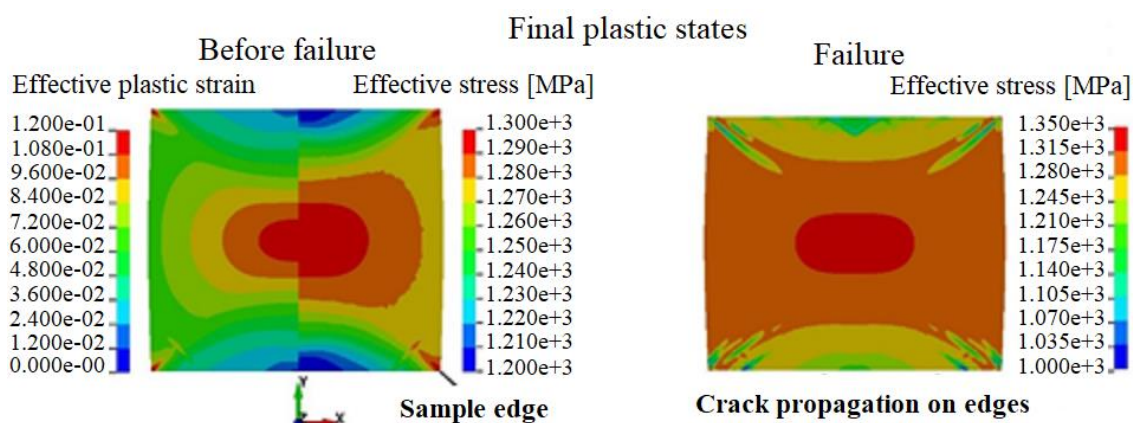


Figure 7. 2D Axial symmetric model used in numerical simulation of BI-5 sample.

In numerical simulation the compression force as a function of displacement was tracked. In figure 8 the simulated force curve F_1 and corrected experimental force curve F_0 for B1-5 material were plotted. The simulation error, E_{rr} , is determined using the formula shown on figure 8 box. To simplify the calculation, the obtained error was linearized and the correction was applied to true characteristic diagrams, presented in figure 5, and to plastic characteristic diagram, presented in figure 6.

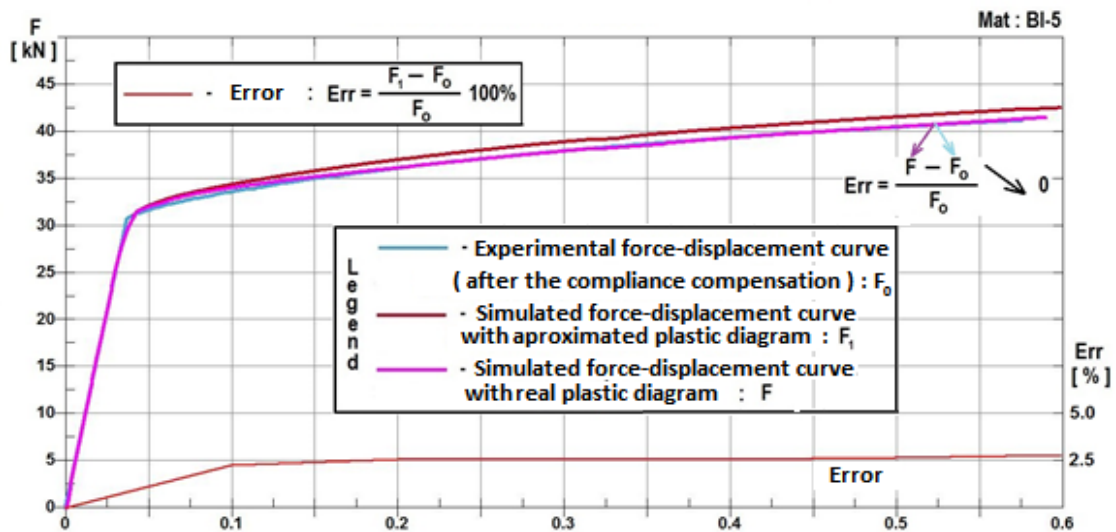


Figure 8. Force vs. displacement diagrams.

To confirm the solution, numerical simulation of compression test was repeated using corrected material properties. The curve force vs. displacement, F , obtained in last simulation overlaps experimental diagram, F_0 , as it shown in figure 8.

After the last simulation, taken into account the calculated error, true diagram of material obtained was validated, thus resulting plastic characteristic diagram on static conditions. This curve is a particular case of dynamic low strain rate and also the initial data used in dynamic tests simulations.

To determine the constitutive model of materials for static conditions the plasticity functions, $f_p(\epsilon^p)$, must be defined as functions that approximate the real plastic diagrams previously represented in figure 6. The experimental plasticity function can be approximated in several ways: ideal plastic model, with linear hardening model, poly-linear model and binomial model. The above mentioned variants for the plasticity function are plotted in figure 9 for B1-5 alloy. As can be seen, the poly-linear and binomial models give the best approximation. Since the plasticity function of Johnson-Cook model is expressed as:

$$f_p = A + B(\epsilon_{ef}^p)^n \quad (5)$$

the binomial approximation of equation (5) gives for B1-5 alloy the model coefficients:

$$A = 1000 \text{ MPa}, B = 1200 \text{ MPa and } n = 0,24$$

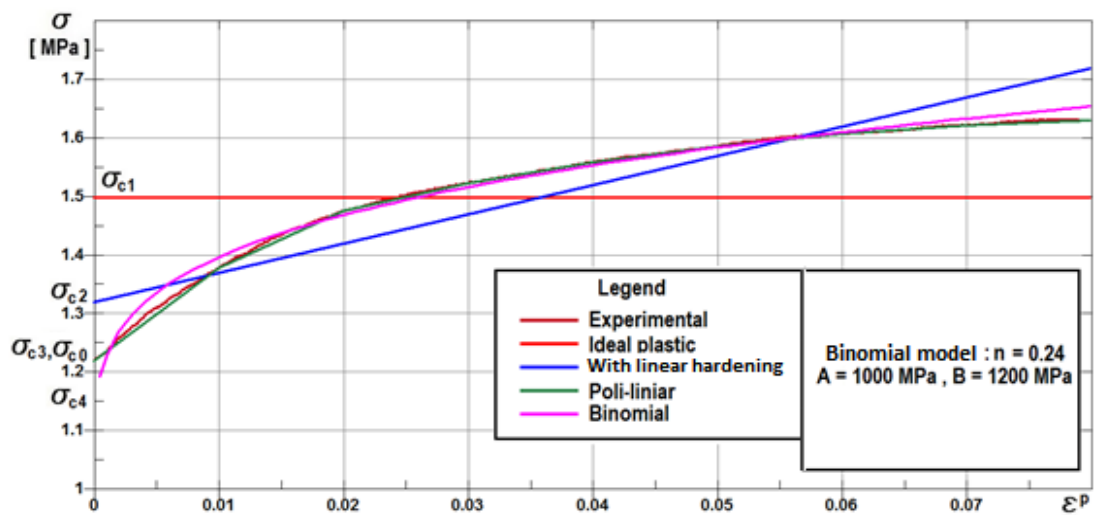


Figure 9. Plastic characteristic diagram and its approximations for BI-5.

3.2. Dynamic tests

For the determination of viscosity function, $f_v(\dot{\epsilon}^p)$ of the constitutive laws there is the need to perform a dynamic test. The dynamic tests on BI-x samples were made using SHPB from MTA laboratory, presented schematically in figure 10.

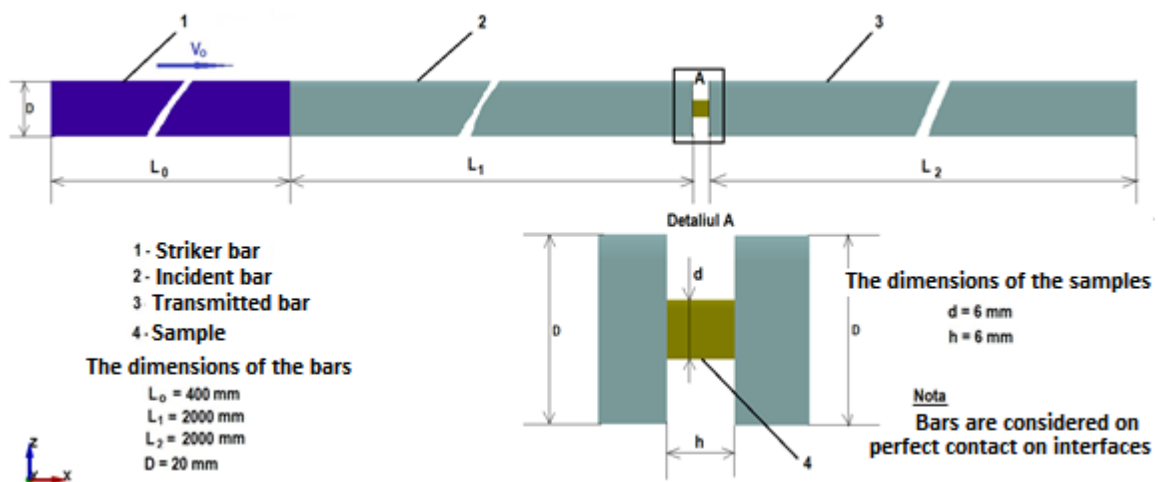


Figure 10. Split Hopkinson Pressure Bar.

As long as the viscosity function may use one or two coefficients, two tests at different impact velocity were made for each alloy BI-x. For each test a pair of values $(\dot{\epsilon}_m^p, f_v)$ was determined. Such pairs were used to calculate the viscosity function coefficient/coefficients.

To determine viscosity function for BI-x alloys, it has proceeded as follows.

Experimental tests on SHPB system were performed and the elastic stresses in bars were obtained through mathematical processing of electrical signal acquired with Wheatstone bridges (figure 11).

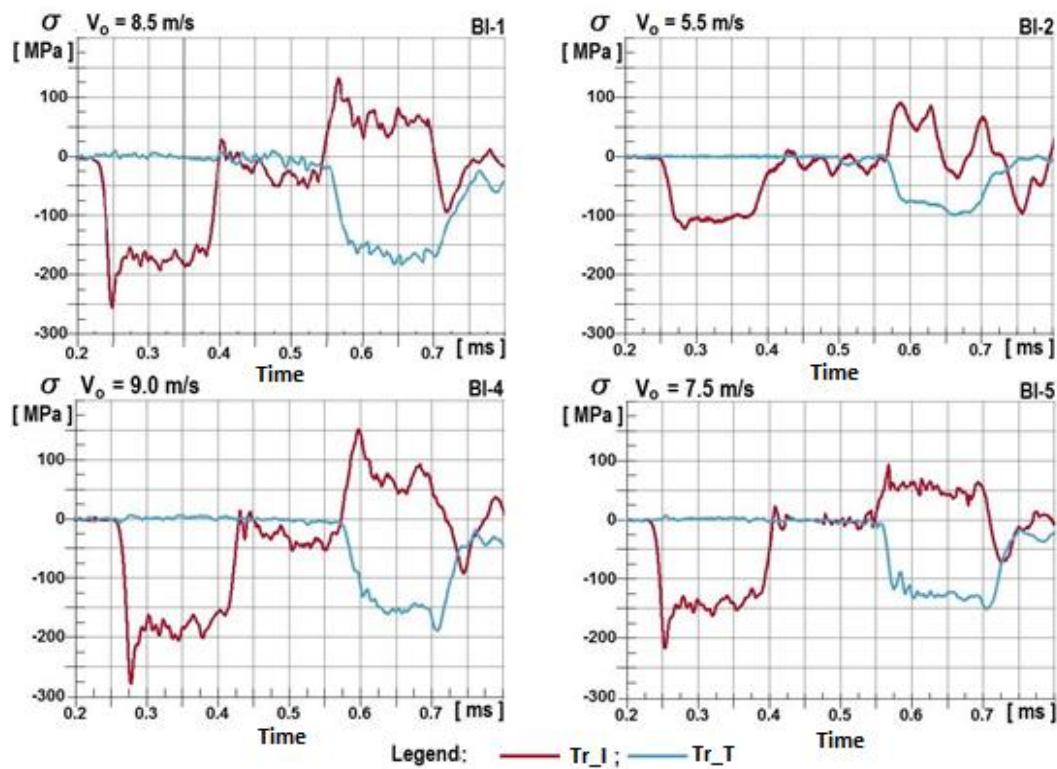


Figure 11. Elastic stress obtained in SHPB system.

For each SHPB test a 3D numerical model was build in LS-DYNA [11] using for sample constitutive law only the previously obtained plasticity function, and the simulation results were compared to experimental ones, as in figure 12.

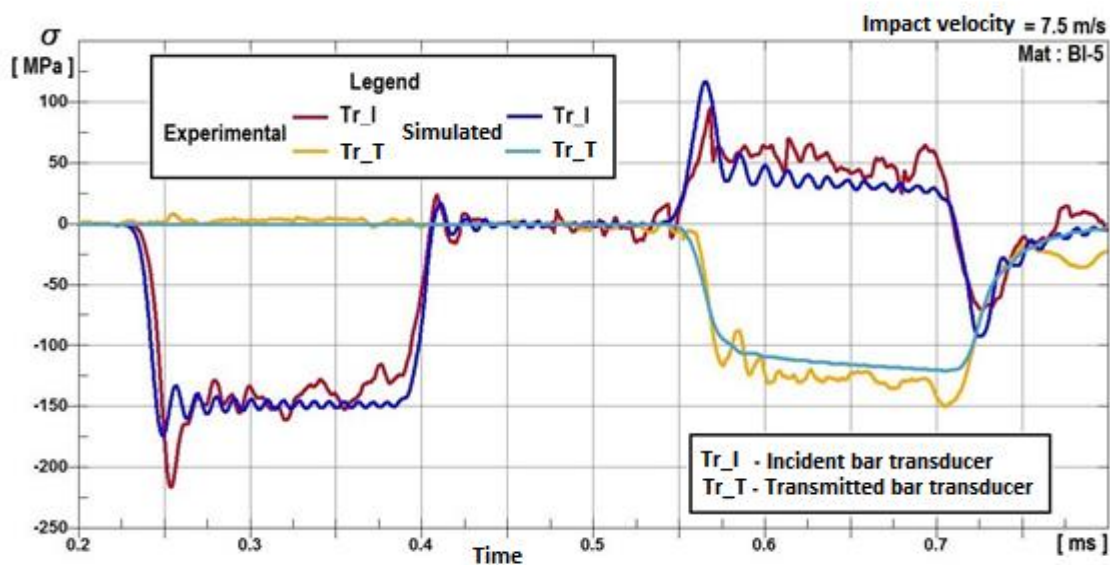


Figure 12. Experimental and simulated stress in SHPB system for BI-5 sample.

To get the values of f_v the simulated and experimental stresses occurred in transmitted bar were averaged and then divided (figure 13 and 14).

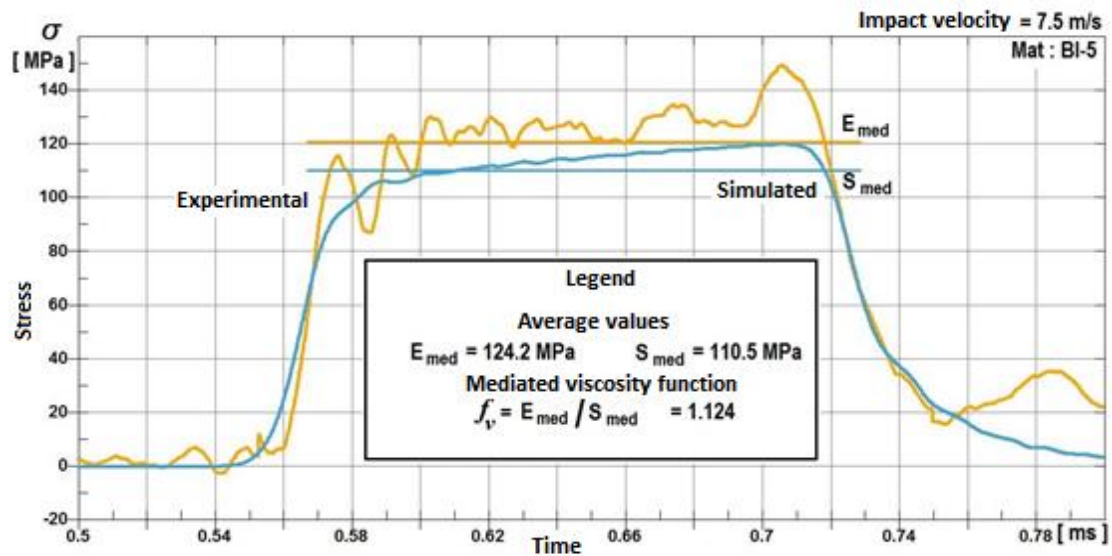


Figure 13. Viscosity function determination for BI-5 at 7.5 m/s impact velocity.

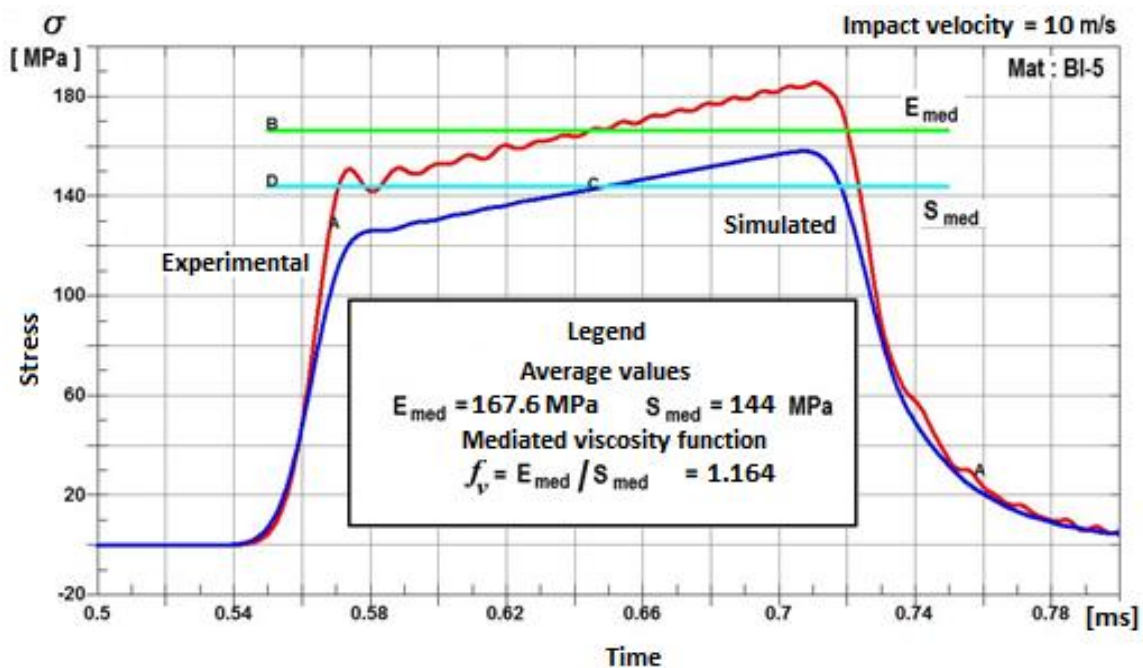


Figure 14. Viscosity function determination for BI-5 at 10 m/s impact velocity.

To get the mean values of $\dot{\epsilon}$ we have proceed in the same way as the above. In figures 15 and 16 the effective plastic strain rates are presented for BI-x alloys in specified impact velocities. Mean values are also shown in these figures for each case.

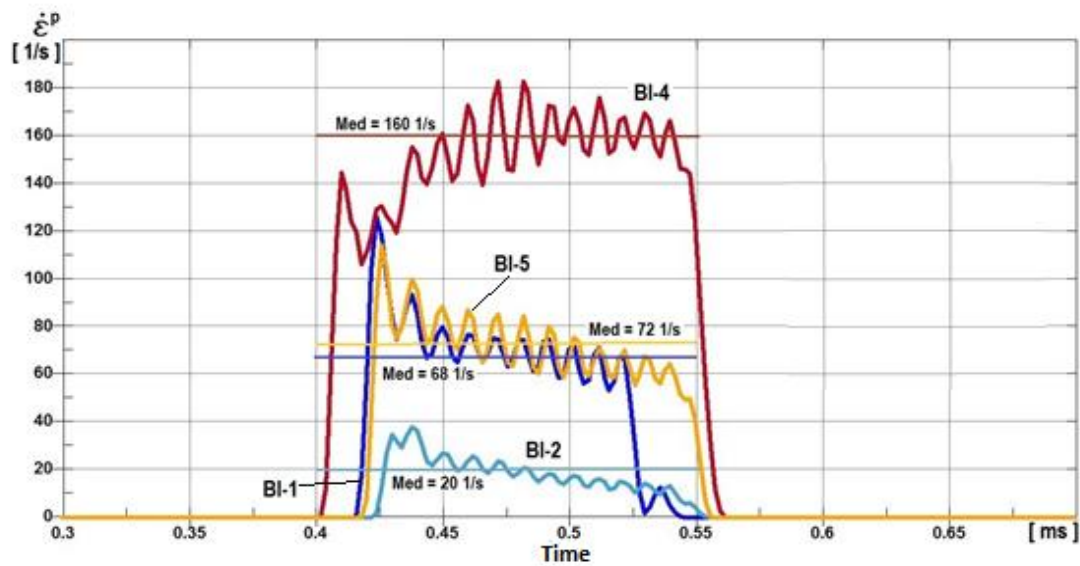


Figure 15. Effective plastic strain rate corresponding to cases from figure 10.

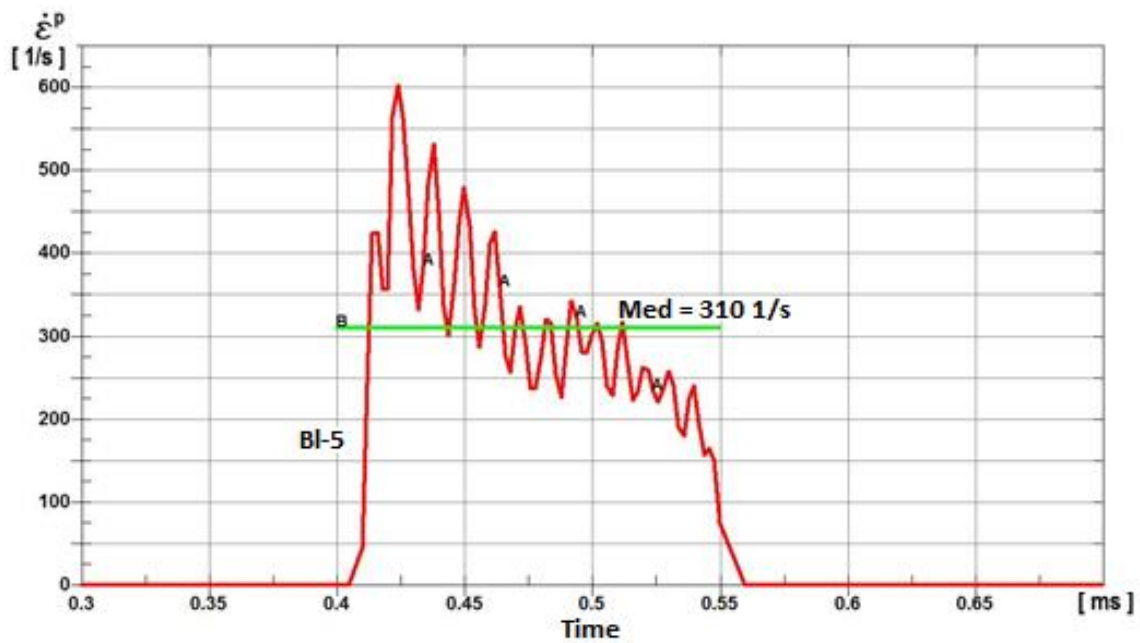


Figure 16. Effective plastic strain rate for BI-5 at 10 m/s impact velocity.

Viscosity functions of Johnson Cook and Cowper Symonds constitutive models are expressed:

$$f_v = (1 + \ln \dot{\epsilon}) \quad (6)$$

$$f_v = 1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{p}} \quad (7)$$

To determine the constitutive models, the following coefficients need to be determined: C for Johnson-Cook model respectively D and p for Cowper Symonds model.

Having the $(\dot{\epsilon}_m^p, f_v)$ values previously determined we may write and solve the equations necessary to calculate the coefficients of viscous components of the Johnson Cook and Cowper Symonds constitutive models.

For example, for B1-5 alloy, the equation (6) and equation (7) are in the following form:

$$\{1 + C \cdot \ln 72 = 1.124 \quad (8)$$

$$\begin{cases} 1 + \left(\frac{72}{D}\right)^{\frac{1}{p}} = 1.124 \\ 1 + \left(\frac{310}{D}\right)^{\frac{1}{p}} = 1.164 \end{cases} \quad (9)$$

By solving the equation (8) and equation (9) the values determined for the coefficients are:

$$C = 0.0287$$

$$D = 3.8 \cdot 10^6$$

$$p = 5.216$$

4. Conclusions

Constitutive material models determination in high strain rate dynamic loads is an important step in military development of products.

In the present paper Johnson Cook and Cooper Symonds constitutive models were determined for several new high entropy alloys. An expedient methodology was applied, based on dynamic and static laboratory tests and numerical simulation.

The used procedure for constitutive material models determination gives an approximate solution, since the average values of stress and strain rates are used in coefficient calculus.

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